Energy in Closed Systems

Anamitra Palit

palit.anamitra@gmail.com

Abstract

Is there any form of energy that develops surreptitiously/mystically from within a closed system that does not exchange energy with its environment? The article investigates such an issue.

Introduction

What is the possibility of energy[1] developing from within a closed system that does not exchange energy? Our aim here is to explore such an issue.

Basic calculations

In a closed system of interacting particles where there is no exchange of energy with the external environment, $dw = 0$. We have,

$$dw = \sum_i dw_i = \sum_i \vec{F}_i d\vec{r}_i = \sum_i m_i \frac{d\vec{v}_i}{dt} d\vec{r}_i \quad (1)$$

dw:total infinitesimal work done on the system, covering all the particles

$$dw = \sum_i m_i \frac{d\vec{v}_i}{dt}\frac{d\vec{r}_i}{dt} dt = \sum_i \frac{m_i \vec{v}_i d\vec{v}_i}{dt} dt$$

$$= \sum_i m_i \vec{v}_i d\vec{v}_i = \frac{1}{2}\sum_i m_i d(\vec{v}_i \vec{v}_i)$$

$$dw = \frac{1}{2}\sum_i m_i d{v_i}^2$$

$$dw = d\left(\sum_i \frac{1}{2} m_i {v_i}^2\right) \quad (2)$$

For an isolated system[energy is not flowing into the system or flowing out of it]:

$$dw = 0 \Rightarrow \sum_i \frac{1}{2} m_i {v_i}^2 = constant \quad (3)$$

The above implies

$$\frac{d\left(\sum_i \frac{1}{2} m_i {v_i}^2\right)}{dt} dt = 0 \Rightarrow \frac{d\left(\sum_i \frac{1}{2} m_i {v_i}^2\right)}{dt} = 0$$

Therefore the constant is time independent of time and similarly of other variables.

Let's now consider a particle in a conservative field:

$$\vec{E} = -\nabla U$$

$$dW = \vec{E}.d\vec{r} = -\nabla U.d\vec{r} = -dU$$

if the potential function is independent of time

If U depends on time then $dU = \frac{\partial U}{\partial t} dt + \nabla U.d\vec{r} \Rightarrow \nabla U.d\vec{r} = dU - \frac{\partial U}{\partial t} dt \neq dU$

We have for time independent potentials

$$dW = -dU$$

Again

$$dW = KE_f - KE_i$$

The last equation is the work energy theorem which is universally valid

$$KE_f - KE_i = -dU$$

$$KE_f - KE_i = U_i - U_f$$

$$KE_i + U_i = KE_f + U_f \quad (4)$$

The above is valid for a <u>time independent</u> conservative field. *If the mass[magnitude of source] of one body is much larger than the other bodies involved then the above two conditions are satisfied to a goodhigh degree of approximation. The potentials of the smaller bodies in motion are ignorable. Thus time independence is achieved.*

Inverse square Law, Gravitation

Potential energy in a three body gravitational system:

$$dW = \vec{F}_1 d\vec{r}_1 + \vec{F}_2 d\vec{r}_2 + \vec{F}_3 d\vec{r}_3$$

$$= \left(\vec{F}_{12} + \vec{F}_{13}\right) d\vec{r}_1 + \left(\vec{F}_{21} + \vec{F}_{23}\right) d\vec{r}_2 + \left(\vec{F}_{31} + \vec{F}_{32}\right) d\vec{r}_3$$

where $\vec{F}_{ij}$ is the force on the ith particle from the $j\ th$ one.

$dW = \left(\vec{F}_{12} + \vec{F}_{13}\right) d\vec{r}_1 + \left(\vec{F}_{21} + \vec{F}_{23}\right) d\vec{r}_2 + \left(\vec{F}_{31} + \vec{F}_{32}\right) d\vec{r}_3$

$$= \left(\vec{F}_{12}d\vec{r}_1 + \vec{F}_{21}d\vec{r}_2\right) + \left(\vec{F}_{13}d\vec{r}_1 + \vec{F}_{31}d\vec{r}_3\right) + \left(\vec{F}_{23}d\vec{r}_2 + \vec{F}_{32}d\vec{r}_3\right)$$

$$= \left(\vec{F}_{12}d\vec{r}_1 - \vec{F}_{12}d\vec{r}_2\right) + \left(\vec{F}_{13}d\vec{r}_1 - \vec{F}_{13}d\vec{r}_3\right) + \left(\vec{F}_{23}d\vec{r}_2 - \vec{F}_{23}d\vec{r}_3\right)$$

$$= \vec{F}_{12}(d\vec{r}_1 - d\vec{r}_2) + \vec{F}_{13}(d\vec{r}_1 - d\vec{r}_3) + \vec{F}_{23}(d\vec{r}_2 - d\vec{r}_3)$$

$$= \vec{F}_{12}d(\vec{r}_1 - \vec{r}_2) + \vec{F}_{13}d(\vec{r}_1 - \vec{r}_3) + \vec{F}_{23}d(\vec{r}_2 - \vec{r}_3)$$

$$= \vec{F}_{12}d\vec{r}_{12} + \vec{F}_{13}d\vec{r}_{13} + \vec{F}_{23}d\vec{r}_{23}; \vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

$$= -Gm_1m_2\frac{\vec{r}_{12}}{{r_{12}}^3}d\vec{r}_{12} - Gm_2m_3\frac{\vec{r}_{13}}{{r_{13}}^3}d\vec{r}_{13} - Gm_3m_1\frac{\vec{r}_{23}}{{r_{23}}^3}d\vec{r}_{23}$$

$$= -Gm_1m_2\frac{1}{2{r_{12}}^3}d(\vec{r}_{12}.\vec{r}_{12}). - Gm_2m_3\frac{\vec{r}_{13}}{2{r_{13}}^3}d(\vec{r}_{13}.\vec{r}_{13}) - Gm_3m_1\frac{\vec{r}_{23}}{2{r_{23}}^3}d(\vec{r}_{23}.\vec{r}_{23})$$

$$dw = -Gm_1m_2\frac{1}{2{r_{12}}^3}d{r_{12}}^2. - Gm_2m_3\frac{\vec{r}_{13}}{2{r_{13}}^3}d{r_{13}}^2 - Gm_3m_1\frac{\vec{r}_{23}}{2{r_{23}}^3}d{r_{23}}^2$$

$$w = -Gm_1m_2\frac{1}{2r_{12}} - Gm_2m_3\frac{1}{2r_{13}} - Gm_3m_1\frac{1}{2r_{23}} + constant \ \ (5)$$

If w=0

$$Gm_1m_2\frac{1}{r_{12}} + Gm_2m_3\frac{1}{r_{13}} + Gm_3m_1\frac{1}{r_{23}} = constant \ (6)$$

for an –body interaction with $dw = 0$ we have

$$\sum_{i,j;i\neq j} Gm_im_j\frac{1}{r_{ij}} = constant \ (7)$$

the above constant is time independent.

We recall

$$dw = -Gm_1m_2\frac{1}{2{r_{12}}^3}d({r_{12}}^2). - Gm_2m_3\frac{\vec{r}_{13}}{2{r_{13}}^3}d({r_{13}}^2) - Gm_3m_1\frac{\vec{r}_{23}}{2{r_{23}}^3}d({r_{23}}^2)$$

$$w = -Gm_1m_2\frac{1}{2r_{12}} + C_1(r_{13}, r_{23}) - Gm_2m_3\frac{1}{2r_{13}} + C_1(r_{23}, r_{12}) - Gm_3m_1\frac{1}{2r_{23}} + C_1(r_{13}, r_{12})$$

$$w = 0 \Rightarrow = -Gm_1m_2\frac{1}{r_{12}} + C_1(r_{13}, r_{23}) - Gm_2m_3\frac{1}{r_{13}} + C_2(r_{23}, r_{12}) - Gm_3m_1\frac{1}{r_{23}} + C_3(r_{13}, r_{12}) = 0$$

Differentiating the above with respect to time

$$Gm_1m_2\frac{1}{{r_{12}}^2}\frac{dr_{12}}{dt} + Gm_2m_3\frac{1}{{r_{23}}^2}\frac{dr_{23}}{dt} + Gm_1m_3\frac{1}{{r_{13}}^2}\frac{dr_{13}}{dt} = 2\big(C_1(r_{13}, r_{23}) + C_2(r_{23}, r_{12}) + C_3(r_{13}, r_{12})\big)$$

$$Gm_1m_2\frac{r_{12}}{{r_{12}}^3}\frac{dr_{12}}{dt}+Gm_2m_3\frac{r_{23}}{{r_{23}}^3}\frac{dr_{23}}{dt}+Gm_1m_3\frac{r_{13}}{{r_{13}}^3}\frac{dr_{13}}{dt}$$
$$=2\frac{d\big(C_1(r_{13},r_{23})+C_2(r_{23},r_{12})+C_3(r_{13},r_{12})\big)}{dt}\neq 0$$

$$Gm_1m_2\frac{\vec{r}_{12}}{{r_{12}}^3}\frac{d\vec{r}_{12}}{dt}+Gm_2m_3\frac{\vec{r}_{23}}{{r_{23}}^3}\frac{d\vec{r}_{23}}{dt}+Gm_1m_3\frac{\vec{r}_{13}}{{r_{13}}^3}\frac{d\vec{r}_{13}}{dt}$$
$$=2\frac{d\big(C_1(r_{13},r_{23})+C_2(r_{23},r_{12})+C_3(r_{13},r_{12})\big)}{dt}\neq 0$$

$$Gm_1m_2\frac{\vec{r}_{12}}{{r_{12}}^3}d\vec{r}_{12}+Gm_2m_3\frac{\vec{r}_{23}}{{r_{23}}^3}d\vec{r}_{23}+Gm_1m_3\frac{\vec{r}_{13}}{{r_{13}}^3}d\vec{r}_{13}$$
$$=2\frac{d\big(C_1(r_{13},r_{23})+C_2(r_{23},r_{12})+C_3(r_{13},r_{12})\big)}{dt}\quad(8)$$

The functions $C_1, C_2 and\ C_3$ are arbitrary: we can fix them up according to our choice

The above is not true when w=0

Therefore the constant in(7) is time independent.

$\sum_{i,j;i\neq j} Gm_im_j\frac{1}{r_{ij}}=constant$,independent of time. Else the right side (8) will be non zero when we require it to be zero[for w=0].

From the Lagrangian formulation

If the potential function is independent of the generalized velocities then we have the relation[2][3] $T+V$ =constant; T: total kinetic energy; Vpotential function of the system of particles; we consider the potential function to be velocity independent.:

$$\frac{\partial V}{\partial x_{ij}}=-F_{ij}$$

i;particle index

j:component index[$j=x,y,z$]

$$\sum_{i,j;i\neq j} Gm_im_j\frac{1}{r_{ij}}=V\quad(9)$$

satisfies

$$\frac{\partial V}{\partial x_{ij}}=-F_{ij}$$

$$\sum_i \frac{1}{2} m_i {v_i}^2 + \frac{1}{2} \sum_{i,j} \frac{{}' G m_i m_j}{r_{ij}} = constant \ (10)$$

$\sum_i \frac{1}{2} m_i {v_i}^2$ and $\sum_{i,j} G m_i m_j \frac{1}{r_{ij}}$ are not constant independently. Energy flows in and out of the system

Conclusion

If dw=0 breaks down internally in a closed system, we have to believe in a surreptitious/mystic source of energy. Else we have to do away with the notion of a closed system that does not exchange energy..

References

1. Goldstein H., Poole C, Safko J, Classical Mechanics, Third edition, Addison Wesley,p61-62
2. Goldstein H., Poole C, Safko J, Classical Mechanics, Third edition, Addison Wesley,p61-62
3. Goldstein H., Poole C, Safko J, Classical Mechanics, Third edition, Addison Wesley,p 21