New type of quantum algorithm

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Here, we propose a new type of quantum algorithm for determining the values of a function. By measuring the output state, we determine all the values of $f(x)$ for all x. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus.

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I. INTRODUCTION

Articles on the history of research into quantum computing [1] are mentioned as follows: An implementation of a quantum algorithm to solve Deutsch's problem [2–4] on a nuclear magnetic resonance quantum computer is reported [5]. An implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is reported [6]. Oliveira *et al.* implements Deutsch's algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [7]. Single-photon Bell states are prepared and measured [8]. The decoherence-free implementation of Deutsch's algorithm is introduced by using such a single-photon and by using two logical qubits [9]. A one-way based experimental implementation of Deutsch's algorithm is reported [10].

In 1993, the Bernstein-Vazirani algorithm was published [11, 12]. In 1994, Simon's algorithm [13] and Shor's algorithm [14] were discussed. In 1996, Grover [15] provided the motivation for exploring the computational possibilities offered by quantum mechanics. An implementation of a quantum algorithm to solve the Bernstein-Vazirani parity problem without entanglement in an ensemble quantum computer is mentioned [16]. Fiber-optics implementation of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits is discussed [17]. The question whether or not quantum learning is robust against noise is a subject of a study [18].

A quantum algorithm for approximating the influences of Boolean functions and its applications are studied [19]. Quantum computation with coherent spin states and the close Hadamard problem are reported [20]. Transport implementation of the Bernstein-Vazirani algorithm with ion qubits is studied [21]. Quantum Gauss-Jordan elimination and simulation of accounting principles on quantum computers are discussed [22]. The dynamical analysis of Grover's search algorithm in arbitrarily high-dimensional search spaces is studied [23]. The relation between quantum computer and secret sharing with the use of quantum principles is discussed [24]. An application of quantum Gauss-Jordan elimination code to quantum secret sharing code is studied [25]. Designing quantum circuit by one step method and similarity with neural network are discussed. [26].

There are many researches concerning quantum computing, quantum algorithm, and their experiments. However, a complete understanding of a fundamental structure of quantum computing is not given.

In this contribution, we propose a new type of quantum algorithm for determining the values of a function. By measuring the output state, we determine all the values of $f(x)$ for all x. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus.

II. A NEW TYPE OF QUANTUM ALGORITHM

Our discussion is based on Nielsen and Chuang [27]. Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function $f(x)$ for many different x simultaneously. Suppose

$$f : \{0,1\} \rightarrow \{0,1\} \tag{1}$$

is a function with a one-bit domain and range. A convenient way of computing the function on a quantum computer is to consider a two-qubit quantum computer that starts in the state $|x, y\rangle$. With an appropriate sequence of logic gates, it is possible to transform this state into

$$|x, y \oplus f(x)\rangle, \tag{2}$$

where $\oplus$ indicates addition modulo 2. We denote by U_f the transformation defined by the map

$$U_f : |x, y\rangle \rightarrow |x, y \oplus f(x)\rangle. \tag{3}$$

Here, the input state is as follows:

$$|\psi_0\rangle = \alpha|0\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + \beta|1\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], \\ (\alpha^2 + \beta^2 = 1). \tag{4}$$

We have the following formula:

$$\begin{aligned} & U_f|0\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|0\rangle(|f(0)\rangle - i|\overline{f(0)}\rangle)/\sqrt{2} \\ & = \begin{cases} (-i)^{f(0)}|0\rangle(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if} \quad f(0) = 0, \\ (-i)^{f(0)}|0\rangle(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if} \quad f(0) = 1. \end{cases} \end{aligned} \tag{5}$$

$$\begin{aligned} & U_f|1\rangle(|0\rangle - |1\rangle)/\sqrt{2} \rightarrow +|1\rangle(|f(1)\rangle - |\overline{f(1)}\rangle)/\sqrt{2} \\ & = \begin{cases} (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if} \quad f(1) = 0, \\ (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if} \quad f(1) = 1. \end{cases} \end{aligned} \tag{6}$$

Applying U_f to $|\psi_0\rangle$ therefore leaves us with one of four possibilities:

$$|\psi_1\rangle = \begin{cases} \alpha|0\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + \beta|1\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ \text{if} \quad f(0) = 0, f(1) = 0, \\ \\ -i\alpha|0\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - \beta|1\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ \text{if} \quad f(0) = 1, f(1) = 1, \\ \\ \alpha|0\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - \beta|1\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ \text{if} \quad f(0) = 0, f(1) = 1, \\ \\ -i\alpha|0\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + \beta|1\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ \text{if} \quad f(0) = 1, f(1) = 0. \end{cases} \tag{7}$$

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all x. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least two evaluations.

III. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE VALUES OF A FUNCTION

We propose a quantum algorithm for determining the values of a function.

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function $f(x)$ for many different x simultaneously. Suppose

$$f : \{0, 1, 2, 3\} \rightarrow \{0, 1\} \tag{8}$$

is a function.

Here, the input state is as follows:

$$|\psi_0\rangle = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], \\ (a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1). \tag{9}$$

We have the following formula:

$$\begin{aligned} & U_f|00\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|00\rangle(|f(00)\rangle - i|\overline{f(00)}\rangle)/\sqrt{2} \\ & = \begin{cases} (-i)^{f(00)}|00\rangle(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if} \quad f(00) = 0, \\ (-i)^{f(00)}|00\rangle(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if} \quad f(00) = 1. \end{cases} \end{aligned} \tag{10}$$

$$\begin{aligned} & U_f|01\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|01\rangle(|f(01)\rangle - i|\overline{f(01)}\rangle)/\sqrt{2} \\ & = \begin{cases} (-i)^{f(01)}|01\rangle(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if} \quad f(01) = 0, \\ (-i)^{f(01)}|01\rangle(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if} \quad f(01) = 1. \end{cases} \end{aligned} \tag{11}$$

$$\begin{aligned} &U_f|10\rangle(|0\rangle-|1\rangle)/\sqrt{2} \rightarrow +|10\rangle(|f(10)\rangle-|\overline{f(10)}\rangle)/\sqrt{2} \\ &= \begin{cases} (-1)^{f(10)}|10\rangle(|0\rangle-|1\rangle)/\sqrt{2} & \text{if} \quad f(10)=0, \\ (-1)^{f(10)}|10\rangle(|0\rangle-|1\rangle)/\sqrt{2} & \text{if} \quad f(10)=1. \end{cases} \end{aligned} \tag{12}$$

$$\begin{aligned} &U_f|11\rangle(|0\rangle-|1\rangle)/\sqrt{2} \rightarrow +|11\rangle(|f(11)\rangle-|\overline{f(11)}\rangle)/\sqrt{2} \\ &= \begin{cases} (-1)^{f(11)}|11\rangle(|0\rangle-|1\rangle)/\sqrt{2} & \text{if} \quad f(11)=0, \\ (-1)^{f(11)}|11\rangle(|0\rangle-|1\rangle)/\sqrt{2} & \text{if} \quad f(11)=1. \end{cases} \end{aligned} \tag{13}$$

Applying U_f to $|\psi_0\rangle$, $U_f|\psi_0\rangle = |\psi_1\rangle$, therefore leaves us with one of 2^4 possibilities:

$$\begin{aligned} &a_1|00\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]+a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=0, f(01)=0, f(10)=0, f(11)=0, \end{aligned} \tag{14}$$

$$\begin{aligned} &-ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right]+a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]+a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=1, f(01)=0, f(10)=0, f(11)=0, \end{aligned} \tag{15}$$

$$\begin{aligned} &a_1|00\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]-ia_2|01\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right]+a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]+a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=0, f(01)=1, f(10)=0, f(11)=0, \end{aligned} \tag{16}$$

$$\begin{aligned} &a_1|00\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]-a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]+a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=0, f(01)=0, f(10)=1, f(11)=0, \end{aligned} \tag{17}$$

$$\begin{aligned} &a_1|00\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]-a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=0, f(01)=0, f(10)=0, f(11)=1, \end{aligned} \tag{18}$$

$$\begin{aligned} &-ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right]-ia_2|01\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right]+a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]+a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=1, f(01)=1, f(10)=0, f(11)=0, \end{aligned} \tag{19}$$

$$\begin{aligned} &-ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right]+a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]-a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]+a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=1, f(01)=0, f(10)=1, f(11)=0, \end{aligned} \tag{20}$$

$$\begin{aligned} &-ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right]+a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right]+a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]-a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \\ &\text{if} \quad f(00)=1, f(01)=0, f(10)=0, f(11)=1, \end{aligned} \tag{21}$$

$$a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 0, f(01) = 1, f(10) = 1, f(11) = 0, \tag{22}$$

$$a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 0, f(01) = 1, f(10) = 0, f(11) = 1, \tag{23}$$

$$a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 0, f(01) = 0, f(10) = 1, f(11) = 1, \tag{24}$$

$$a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 0, f(01) = 1, f(10) = 1, f(11) = 1, \tag{25}$$

$$-ia_1|00\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 1, f(01) = 0, f(10) = 1, f(11) = 1, \tag{26}$$

$$-ia_1|00\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 1, f(01) = 1, f(10) = 0, f(11) = 1, \tag{27}$$

$$-ia_1|00\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 1, f(01) = 1, f(10) = 1, f(11) = 0, \tag{28}$$

$$-ia_1|00\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{if} \quad f(00) = 1, f(01) = 1, f(10) = 1, f(11) = 1. \tag{29}$$

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all x. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least four evaluations.

IV. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE 2^N VALUES OF A FUNCTION

We propose a quantum algorithm for determining the 2^N values of a function.

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function $f(x)$ for many different x simultaneously. Suppose

$$f : \{0, 1, ..., 2^N - 1\} \to \{0, 1\} \tag{30}$$

is a function.

Here, the input state is as follows:

$$|\psi_0\rangle = \sum_{j=0}^{2^{(N-1)}-1} a_j|j\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + \sum_{k=2^{(N-1)}}^{2^N-1} a_k|k\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right],$$
$$(a_0^2 + a_1^2 + ... + a_{2^N-1}^2 = 1). \tag{31}$$

Applying U_f to $|\psi_0\rangle$, $U_f|\psi_0\rangle = |\psi_1\rangle$, therefore leaves us with one of 2^{2^N} possibilities:

$$|\psi_1\rangle = \sum_{j=0}^{2^{(N-1)}-1} (-i)^{f(j)} a_j |j\rangle \left[\frac{|0\rangle - (-i)^{f(j)}|1\rangle}{\sqrt{2}}\right] + \sum_{k=2^{(N-1)}}^{2^N-1} (-1)^{f(k)} a_k |k\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{32}$$

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all x. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least 2^N evaluations.

V. CONCLUSIONS

In conclusion, a new type of quantum algorithm has been proposed. By measuring the output state, we have determined all the values of $f(x)$ for all x. This has been faster than a classical apparatus.

NOTE

On behalf of all authors, the corresponding author states that there is no conflict of interest.

[1] R. Rennie (Editor), *Oxford dictionary of physics* (Oxford University Press, 2015), Seventh ed.
[2] D. Deutsch, *Proc. Roy. Soc. London Ser. A* **400**, 97 (1985).
[3] D. Deutsch and R. Jozsa, *Proc. Roy. Soc. London Ser. A* **439**, 553 (1992).
[4] R. Cleve, A. Ekert, C. Macchiavello, and M. Mosca, *Proc. Roy. Soc. London Ser. A* **454**, 339 (1998).
[5] J. A. Jones and M. Mosca, J. Chem. Phys. **109**, 1648 (1998).
[6] S. Gulde, M. Riebe, G. P. T. Lancaster, C. Becher, J. Eschner, H. Häffner, F. Schmidt-Kaler, I. L. Chuang, and R. Blatt, Nature (London) **421**, 48 (2003).
[7] A. N. de Oliveira, S. P. Walborn, and C. H. Monken, J. Opt. B: Quantum Semiclass. Opt. **7**, 288-292 (2005).
[8] Y.-H. Kim, Phys. Rev. A **67**, 040301(R) (2003).
[9] M. Mohseni, J. S. Lundeen, K. J. Resch, and A. M. Steinberg, Phys. Rev. Lett. **91**, 187903 (2003).
[10] M. S. Tame, R. Prevedel, M. Paternostro, P. Böhi, M. S. Kim, and A. Zeilinger, Phys. Rev. Lett. **98**, 140501 (2007).
[11] E. Bernstein and U. Vazirani, Proceedings of the Twenty-Fifth Annual ACM Symposium on Theory of Computing (STOC '93), pp. 11-20 (1993).
[12] E. Bernstein and U. Vazirani, SIAM J. Comput. 26-5, pp. 1411-1473 (1997).
[13] D. R. Simon, Foundations of Computer Science, (1994) Proceedings., 35th Annual Symposium on: 116-123, retrieved 2011-06-06.
[14] P. W. Shor, Proceedings of the 35th IEEE Symposium on Foundations of Computer Science. 124 (1994).
[15] L. K. Grover, Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing. 212 (1996).
[16] J. Du, M. Shi, X. Zhou, Y. Fan, B. J. Ye, R. Han, and J. Wu, Phys. Rev. A **64**, 042306 (2001).
[17] E. Brainis, L.-P. Lamoureux, N. J. Cerf, Ph. Emplit, M. Haelterman, and S. Massar, Phys. Rev. Lett. **90**, 157902 (2003).
[18] A. W. Cross, G. Smith, and J. A. Smolin, Phys. Rev. A **92**, 012327 (2015).
[19] H. Li and L. Yang, Quantum Inf. Process. **14**, 1787 (2015).
[20] M. R. A. Adcock, P. Hoyer, and B. C. Sanders, Quantum Inf. Process. **15**, 1361 (2016).
[21] S. D. Fallek, C. D. Herold, B. J. McMahon, K. M. Maller, K. R. Brown, and J. M. Amini, New J. Phys. **18**, 083030 (2016).
[22] D. N. Diep, D. H. Giang, and N. Van Minh, Int. J. Theor. Phys. **56**, 1948 (2017).
[23] W. Jin, Quantum Inf. Process. **15**, 65 (2016).
[24] D. N. Diep and D. H. Giang, Int. J. Theor. Phys. **56**, 2797 (2017).
[25] D. N. Diep, D. H. Giang, and P. H. Phu, Int. J. Theor. Phys. **57**, 841 (2018).
[26] G. Resconi and K. Nagata, International Journal of General Engineering and Technology, Vol. 7, Issue 1 (2018) Page 1 – 20.
[27] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).