Here, we propose a new type of quantum algorithm for determining the values of a function. By measuring the output state, we determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus.

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Keywords: Quantum algorithms, Quantum computation
I. INTRODUCTION

Articles on the history of research into quantum computing [1] are mentioned as follows: An implementation of a quantum algorithm to solve Deutsch's problem [2–4] on a nuclear magnetic resonance quantum computer is reported [5]. An implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is reported [6]. Oliveira et al. implements Deutsch’s algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [7]. Single-photon Bell states are prepared and measured [8]. The decoherence-free implementation of Deutsch’s algorithm is introduced by using such a single-photon and by using two logical qubits [9]. A one-way based experimental implementation of Deutsch’s algorithm is reported [10].

In 1993, the Bernstein-Vazirani algorithm was published [11, 12]. In 1994, Simon’s algorithm [13] and Shor’s algorithm [14] were discussed. In 1996, Grover [15] provided the motivation for exploring the computational possibilities offered by quantum mechanics. An implementation of a quantum algorithm to solve the Bernstein-Vazirani parity problem without entanglement in an ensemble quantum computer is mentioned [16]. Fiber-optics implementation of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits is discussed [17]. The question whether or not quantum learning is robust against noise is a subject of a study [18].

A quantum algorithm for approximating the influences of Boolean functions and its applications are studied [19]. Quantum computation with coherent spin states and the close Hadamard problem are reported [20]. Transport implementation of the Bernstein-Vazirani algorithm with ion qubits is studied [21]. Quantum Gauss-Jordan elimination and simulation of accounting principles on quantum computers are discussed [22]. The dynamical analysis of Grover’s search algorithm in arbitrarily high-dimensional search spaces is studied [23]. The relation between quantum computer and secret sharing with the use of quantum principles is discussed [24]. An application of quantum Gauss-Jordan elimination code to quantum secret sharing code is studied [25]. Designing quantum circuit by one step method and similarity with neural network are discussed [26].

There are many researches concerning quantum computing, quantum algorithm, and their experiments. However, a complete understanding of a fundamental structure of quantum computing is not given.

In this contribution, we propose a new type of quantum algorithm for determining the values of a function. By measuring the output state, we determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus.

II. A NEW TYPE OF QUANTUM ALGORITHM

Our discussion is based on Nielsen and Chuang [27]. Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function $f(x)$ for many different $x$ simultaneously. Suppose

$$f : \{0, 1\} \rightarrow \{0, 1\}$$

is a function with a one-bit domain and range. A convenient way of computing the function on a quantum computer is to consider a two-qubit quantum computer that starts in the state $|x, y\rangle$. With an appropriate sequence of logic gates, it is possible to transform this state into

$$|x, y \oplus f(x)\rangle,$$

where $\oplus$ indicates addition modulo 2. We denote by $U_f$ the transformation defined by the map

$$U_f : |x, y\rangle \rightarrow |x, y \oplus f(x)\rangle.$$

Here, the input state is as follows:

$$|\psi_0\rangle = \alpha|0\rangle \frac{1}{\sqrt{2}} \left[|0\rangle - |1\rangle\right] + \beta|1\rangle \frac{1}{\sqrt{2}} \left[|0\rangle - |1\rangle\right],$$

$$\alpha^2 + \beta^2 = 1.$$
We have the following formula:

\[
U_f(0)(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|0\rangle(|f(0)) - i|f(0)\rangle)/\sqrt{2}
\]

\[
= \begin{cases} 
(-i)^f(0)(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(0) = 0, \\
(-i)^f(0)(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(0) = 1.
\end{cases}
\]

\[
U_f(1)(|0\rangle - |1\rangle)/\sqrt{2} \rightarrow +|1\rangle(|f(1)) - i|f(1)\rangle)/\sqrt{2}
\]

\[
= \begin{cases} 
(-i)^f(1)(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(1) = 0, \\
(-i)^f(1)(|0\rangle + |1\rangle)/\sqrt{2} & \text{if } f(1) = 1.
\end{cases}
\]

Applying \(U_f\) to \(|\psi_0\rangle\) therefore leaves us with one of four possibilities:

\[
|\psi_1\rangle = \begin{cases} 
\alpha|0\rangle \frac{|0\rangle - i|1\rangle}{\sqrt{2}} + \beta|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} & \text{if } f(0) = 0, f(1) = 0, \\
-i\alpha|0\rangle \frac{|0\rangle + i|1\rangle}{\sqrt{2}} - \beta|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} & \text{if } f(0) = 1, f(1) = 1, \\
\alpha|0\rangle \frac{|0\rangle - i|1\rangle}{\sqrt{2}} - \beta|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} & \text{if } f(0) = 0, f(1) = 1, \\
-i\alpha|0\rangle \frac{|0\rangle + i|1\rangle}{\sqrt{2}} + \beta|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} & \text{if } f(0) = 1, f(1) = 0.
\end{cases}
\]

So, by measuring \(|\psi_1\rangle\), we may determine all the values of \(f(x)\) for all \(x\). This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of \(f(x)\), namely, \(f(x)\). This is faster than a classical apparatus, which would require at least two evaluations.

### III. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE VALUES OF A FUNCTION

We propose a quantum algorithm for determining the values of a function. Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function \(f(x)\) for many different \(x\) simultaneously. Suppose

\[
f : \{0, 1, 2, 3\} \rightarrow \{0, 1\}
\]

is a function.

Here, the input state is as follows:

\[
|\psi_0\rangle = a_1|00\rangle \frac{|0\rangle - i|1\rangle}{\sqrt{2}} + a_2|01\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + a_3|10\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + a_4|11\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}},
\]

\[
(a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1).
\]

We have the following formula:

\[
U_f|00\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|00\rangle(|f(0)) - i|f(0)\rangle)/\sqrt{2}
\]

\[
= \begin{cases} 
(-i)^f(00)|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(00) = 0, \\
(-i)^f(00)|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(00) = 1.
\end{cases}
\]

\[
U_f|01\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|01\rangle(|f(01)) - i|f(01)\rangle)/\sqrt{2}
\]

\[
= \begin{cases} 
(-i)^f(01)|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(01) = 0, \\
(-i)^f(01)|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(01) = 1.
\end{cases}
\]
\[
U_f|10\rangle(|0\rangle - |1\rangle)/\sqrt{2} \rightarrow +|10\rangle(|f(10)\rangle - |\overline{f}(10)\rangle)/\sqrt{2}
\]
\[
= \begin{cases}
(-1)^{f(10)}|10\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(10) = 0, \\
(-1)^{f(10)}|10\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(10) = 1.
\end{cases}
\]
\[
(12)
\]
\[
U_f|11\rangle(|0\rangle - |1\rangle)/\sqrt{2} \rightarrow +|11\rangle(|f(11)\rangle - |\overline{f}(11)\rangle)/\sqrt{2}
\]
\[
= \begin{cases}
(-1)^{f(11)}|11\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(11) = 0, \\
(-1)^{f(11)}|11\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(11) = 1.
\end{cases}
\]
\[
(13)
\]
Applying \(U_f\) to \(|\psi_0\rangle\), \(U_f|\psi_0\rangle = |\psi_1\rangle\), therefore leaves us with one of \(2^4\) possibilities:

\[
a_1|00\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_2|01\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] + a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 0, f(01) = 0, f(10) = 0, f(11) = 0,
\]
\[
(14)
\]
\[
-a_1|00\rangle \left[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right] + a_2|01\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] + a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 1, f(01) = 0, f(10) = 0, f(11) = 0,
\]
\[
(15)
\]
\[
a_1|00\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] - ia_2|01\rangle \left[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right] + a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] + a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 0, f(01) = 1, f(10) = 0, f(11) = 0,
\]
\[
(16)
\]
\[
a_1|00\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_2|01\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] - a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] + a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 0, f(01) = 0, f(10) = 1, f(11) = 0,
\]
\[
(17)
\]
\[
a_1|00\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_2|01\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] - a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 0, f(01) = 0, f(10) = 0, f(11) = 1,
\]
\[
(18)
\]
\[
-ia_1|00\rangle \left[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right] - ia_2|01\rangle \left[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right] + a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] + a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 1, f(01) = 1, f(10) = 0, f(11) = 0,
\]
\[
(19)
\]
\[
-ia_1|00\rangle \left[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right] + a_2|01\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] - a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] + a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 1, f(01) = 0, f(10) = 1, f(11) = 0,
\]
\[
(20)
\]
\[
-ia_1|00\rangle \left[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right] + a_2|01\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + a_3|10\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] - a_4|11\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]
\]
\[
\text{if } f(00) = 1, f(01) = 0, f(10) = 0, f(11) = 1,
\]
\[
(21)
\]
So, by measuring apparatus, which would require at least four evaluations. 

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to determine a perfect property of a function. We propose a quantum algorithm for determining the $2^N$ values of a function $f(x)$ for many different $x$ simultaneously. Suppose

$$ f : \{0, 1, \ldots, 2^N - 1\} \to \{0, 1\} $$

is a function.

Here, the input state is as follows:

$$ |\psi_0\rangle = \sum_{j=0}^{2^{(N-1)}-1} a_j |j\rangle + \sum_{k=2^{N-1}}^{2^N-1} a_k |k\rangle, $$

$$ (a_0^2 + a_1^2 + \ldots + a_{2^N-1}^2 = 1). $$

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least four evaluations.

IV. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE $2^N$ VALUES OF A FUNCTION

We propose a quantum algorithm for determining the $2^N$ values of a function.

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function $f(x)$ for many different $x$ simultaneously. Suppose

$$ f : \{0, 1, \ldots, 2^N - 1\} \to \{0, 1\} $$

is a function.

Here, the input state is as follows:

$$ |\psi_0\rangle = \sum_{j=0}^{2^{(N-1)}-1} a_j |j\rangle + \sum_{k=2^{N-1}}^{2^N-1} a_k |k\rangle, $$

$$ (a_0^2 + a_1^2 + \ldots + a_{2^N-1}^2 = 1). $$

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least four evaluations.
Applying $U_f$ to $|\psi_0\rangle$, $U_f|\psi_0\rangle = |\psi_1\rangle$, therefore leaves us with one of $2^{2N}$ possibilities:

$$|\psi_1\rangle = \sum_{j=0}^{2^{(N-1)}-1} (-i)^{f(j)} a_j |j\rangle \left[ \frac{|0\rangle - (-i)^{f(j)} |1\rangle}{\sqrt{2}} \right] + \sum_{k=2^{(N-1)}}^{2^{N} - 1} (-1)^{f(k)} a_k |k\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right].$$

(32)

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least $2^N$ evaluations.

V. CONCLUSIONS

In conclusion, a new type of quantum algorithm has been proposed. By measuring the output state, we have determined all the values of $f(x)$ for all $x$. This has been faster than a classical apparatus.

NOTE

On behalf of all authors, the corresponding author states that there is no conflict of interest.