Here, we propose a new type of quantum algorithm for determining the values of a function. By measuring the output state, we determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus.

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I. INTRODUCTION

Articles on the history of research into quantum computing [1] are mentioned as follows: An implementation of a quantum algorithm to solve Deutsch’s problem [2—4] on a nuclear magnetic resonance quantum computer is reported [5]. An implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is reported [6]. Oliveira et al. implements Deutsch’s algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [7]. Single-photon Bell states are prepared and measured [8]. The decoherence-free implementation of Deutsch’s algorithm is introduced by using such a single-photon and by using two logical qubits [9]. A one-way based experimental implementation of Deutsch’s algorithm is reported [10].

In 1993, the Bernstein-Vazirani algorithm was published [11, 12]. In 1994, Simon’s algorithm [13] and Shor’s algorithm [14] were discussed. In 1996, Grover [15] provided the motivation for exploring the computational possibilities offered by quantum mechanics. An implementation of a quantum algorithm to solve the Bernstein-Vazirani parity problem without entanglement in an ensemble quantum computer is mentioned [16]. Fiber-optics implementation of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits is discussed [17]. The question whether or not quantum learning is robust against noise is a subject of a study [18].

A quantum algorithm for approximating the influences of Boolean functions and its applications are studied [19]. Quantum computation with coherent spin states and the close Hadamard problem are reported [20]. Transport implementation of the Bernstein-Vazirani algorithm with ion qubits is studied [21]. Quantum Gauss-Jordan elimination and simulation of accounting principles on quantum computers are discussed [22]. The dynamical analysis of Grover’s search algorithm in arbitrarily high-dimensional search spaces is studied [23]. The relation between quantum computer and secret sharing with the use of quantum principles is discussed [24]. An application of quantum Gauss-Jordan elimination code to quantum secret sharing code is studied [25]. Designing quantum circuit by one step method and similarity with neural network are discussed [26].

There are many researches concerning quantum computing, quantum algorithm, and their experiments. However, a complete understanding of a fundamental structure of quantum computing is not given.

In this contribution, we propose a new type of quantum algorithm for determining the values of a function. By measuring the output state, we determine all the values of \( f(x) \) for all \( x \). This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of \( f(x) \), namely, \( f(x) \). This is faster than a classical apparatus.

II. A NEW TYPE OF QUANTUM ALGORITHM

Our discussion is based on Nielsen and Chuang [27]. Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function \( f(x) \) for many different \( x \) simultaneously. Suppose

\[
f : \{0, 1\} \to \{0, 1\}
\]

is a function with a one-bit domain and range. A convenient way of computing the function on a quantum computer is to consider a two-qubit quantum computer that starts in the state \( |x, y\rangle \). With an appropriate sequence of logic gates, it is possible to transform this state into

\[
|x, y \oplus f(x)\rangle,
\]

where \( \oplus \) indicates addition modulo 2. We denote by \( U_f \) the transformation defined by the map

\[
U_f : |x, y\rangle \to |x, y \oplus f(x)\rangle.
\]

Here, the input state is as follows:

\[
|\psi_0\rangle = \alpha|0\rangle \left[ \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right] + \beta|1\rangle \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right],
\]

\[
(\alpha^2 + \beta^2 = 1).
\]
We have the following formula:

\[ U_f(0)(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|0\rangle(|f(0)\rangle - i\overline{|f(0)\rangle})/\sqrt{2} \]

\[ = \begin{cases} 
(-i)^f(0)(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(0) = 0, \\
(-i)^f(0)(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(0) = 1. 
\end{cases} \tag{5} \]

\[ U_f(1)(|0\rangle - |1\rangle)/\sqrt{2} \rightarrow +|1\rangle(|f(1)\rangle - i\overline{|f(1)\rangle})/\sqrt{2} \]

\[ = \begin{cases} 
(-1)^f(1)(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(1) = 0, \\
(-1)^f(1)(|0\rangle + |1\rangle)/\sqrt{2} & \text{if } f(1) = 1. 
\end{cases} \tag{6} \]

Applying \( U_f \) to \( |\psi_0\rangle \) therefore leaves us with one of four possibilities:

\[ |\psi_1\rangle = \begin{cases} 
\alpha|0\rangle \left[ |0\rangle - i|1\rangle \right] + \beta|1\rangle \left[ |0\rangle - |1\rangle \right] & \text{if } f(0) = 0, f(1) = 0, \\
-i\alpha|0\rangle \left[ |0\rangle + i|1\rangle \right] - \beta|1\rangle \left[ |0\rangle - |1\rangle \right] & \text{if } f(0) = 1, f(1) = 1, \\
\alpha|0\rangle \left[ |0\rangle - i|1\rangle \right] - \beta|1\rangle \left[ |0\rangle - |1\rangle \right] & \text{if } f(0) = 0, f(1) = 1, \\
-i\alpha|0\rangle \left[ |0\rangle + i|1\rangle \right] + \beta|1\rangle \left[ |0\rangle - |1\rangle \right] & \text{if } f(0) = 1, f(1) = 0. 
\end{cases} \tag{7} \]

So, by measuring \( |\psi_1\rangle \), we may determine all the values of \( f(x) \) for all \( x \). This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of \( f(x) \), namely, \( f(x) \). This is faster than a classical apparatus, which would require at least two evaluations.

### III. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE VALUES OF A FUNCTION

We propose a quantum algorithm for determining the values of a function. Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function \( f(x) \) for many different \( x \) simultaneously. Suppose

\[ f : \{0, 1, 2, 3\} \rightarrow \{0, 1\} \]  

is a function.

Here, the input state is as follows:

\[ |\psi_0\rangle = a_1|00\rangle \left[ |0\rangle - i|1\rangle \right] + a_2|01\rangle \left[ |0\rangle - |1\rangle \right] + a_3|10\rangle \left[ |0\rangle - i|1\rangle \right] + a_4|11\rangle \left[ |0\rangle - |1\rangle \right], \]

\[ (a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1). \tag{9} \]

We have the following formula:

\[ U_f|00\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|00\rangle(|f(0)\rangle - i\overline{|f(0)\rangle})/\sqrt{2} \]

\[ = \begin{cases} 
(-i)^f(00)(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(00) = 0, \\
(-i)^f(00)(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(00) = 1. 
\end{cases} \tag{10} \]

\[ U_f|01\rangle(|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow +|01\rangle(|f(01)\rangle - i\overline{|f(01)\rangle})/\sqrt{2} \]

\[ = \begin{cases} 
(-i)^f(01)(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(01) = 0, \\
(-i)^f(01)(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(01) = 1. 
\end{cases} \tag{11} \]
\[
U_f|10\rangle (|0\rangle - |1\rangle) / \sqrt{2} \rightarrow +|10\rangle(|f(10)\rangle - |f(10)\rangle) / \sqrt{2}
\]
\[
= \begin{cases} 
(-1)^{f(10)}|10\rangle (|0\rangle - |1\rangle) / \sqrt{2} & \text{if } f(10) = 0, \\
(-1)^{f(10)}|10\rangle (|0\rangle - |1\rangle) / \sqrt{2} & \text{if } f(10) = 1.
\end{cases}
\] (12)

\[
U_f|11\rangle (|0\rangle - |1\rangle) / \sqrt{2} \rightarrow +|11\rangle(|f(11)\rangle - |f(11)\rangle) / \sqrt{2}
\]
\[
= \begin{cases} 
(-1)^{f(11)}|11\rangle (|0\rangle - |1\rangle) / \sqrt{2} & \text{if } f(11) = 0, \\
(-1)^{f(11)}|11\rangle (|0\rangle - |1\rangle) / \sqrt{2} & \text{if } f(11) = 1.
\end{cases}
\] (13)

Applying \(U_f\) to \(|\psi_0\rangle\), \(U_f|\psi_0\rangle = |\psi_1\rangle\), therefore leaves us with one of \(2^4\) possibilities:

\[
a_1|00\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 0, f(01) = 0, f(10) = 0, f(11) = 0,
\] (14)

\[
-a_1|00\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 1, f(01) = 0, f(10) = 0, f(11) = 0,
\] (15)

\[
a_1|00\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 0, f(01) = 1, f(10) = 0, f(11) = 0,
\] (16)

\[
a_1|00\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 0, f(01) = 0, f(10) = 1, f(11) = 0,
\] (17)

\[
a_1|00\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 0, f(01) = 0, f(10) = 0, f(11) = 1,
\] (18)

\[
-ia_1|00\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 1, f(01) = 1, f(10) = 0, f(11) = 0,
\] (19)

\[
-ia_1|00\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 1, f(01) = 0, f(10) = 1, f(11) = 0,
\] (20)

\[
-ia_1|00\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]
\]
\[
\text{if } f(00) = 1, f(01) = 0, f(10) = 0, f(11) = 1,
\] (21)
So, by measuring evaluate the values of a function
apparatus, which would require at least four evaluations.

\[ a_1|00\rangle \left( \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right) - \sqrt{2} a_2|01\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) - a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 0, f(01) = 1, f(10) = 1, f(11) = 0, \) \( (22) \)

\[ a_1|00\rangle \left( \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right) - \sqrt{2} a_2|01\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) + a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 0, f(01) = 1, f(10) = 0, f(11) = 1, \) \( (23) \)

\[ a_1|00\rangle \left( \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right) + a_2|01\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 0, f(01) = 1, f(10) = 1, f(11) = 1, \) \( (24) \)

\[ -i a_1|00\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) + a_2|01\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 1, f(01) = 0, f(10) = 1, f(11) = 1, \) \( (25) \)

\[ -i a_1|00\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) - i a_2|01\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) + a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 1, f(01) = 1, f(10) = 0, f(11) = 1, \) \( (26) \)

\[ -i a_1|00\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) - i a_2|01\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) - a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 1, f(01) = 1, f(10) = 1, f(11) = 0, \) \( (27) \)

\[ -i a_1|00\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) - i a_2|01\rangle \left( \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) - a_3|10\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) - a_4|11\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \]

if \( f(00) = 1, f(01) = 1, f(10) = 1, f(11) = 1. \) \( (29) \)

So, by measuring \(|\psi_1\rangle\), we may determine all the values of \( f(x) \) for all \( x \). This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of \( f(x) \), namely, \( f(x) \). This is faster than a classical apparatus, which would require at least four evaluations.

IV. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE \( 2^N \) VALUES OF A FUNCTION

We propose a quantum algorithm for determining the \( 2^N \) values of a function.

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the values of a function \( f(x) \) for many different \( x \) simultaneously. Suppose

\[ f : \{0, 1, \ldots, 2^N - 1\} \rightarrow \{0, 1\} \]

is a function.

Here, the input state is as follows:

\[ |\psi_0\rangle = \sum_{j=0}^{2^{N-1}-1} a_j |j\rangle \left( \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right) + \sum_{k=2^{N-1}}^{2^N-1} a_k |k\rangle \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right), \]

\[ \left( a_0^2 + a_1^2 + \ldots + a_{2^{N-1}}^2 = 1 \right). \]

(31)
Applying $U_f$ to $|\psi_0\rangle$, $U_f|\psi_0\rangle = |\psi_1\rangle$, therefore leaves us with one of $2^N$ possibilities:

$$|\psi_1\rangle = \sum_{j=0}^{2^{(N-1)}-1} (-i)^{f(j)} a_j |j\rangle + \sum_{k=2^{(N-1)}}^{2^N} (-1)^{f(k)} a_k |k\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}. \quad (32)$$

So, by measuring $|\psi_1\rangle$, we may determine all the values of $f(x)$ for all $x$. This is very interesting indeed: the quantum circuit gives us the ability to determine a perfect property of $f(x)$, namely, $f(x)$. This is faster than a classical apparatus, which would require at least $2^N$ evaluations.

V. CONCLUSIONS

In conclusion, a new type of quantum algorithm has been proposed. By measuring the output state, we have determined all the values of $f(x)$ for all $x$. This has been faster than a classical apparatus.

NOTE

On behalf of all authors, the corresponding author states that there is no conflict of interest.