Refutation of the ordinal Turing machine (OTM) on set theory

Abstract: From the sections on OTM-realizabilty and intuitionistic provability and axioms and systems of constructive set theories, we evaluate an inference rule and two propositions. None is tautologous. The refutes OTM on set theory in Hilbert space for intuitionistic logic. Therefore that approach produces *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⤚ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Carl, M. (2019). A note on OTM-realizability and constructive set theories. arxiv.org/pdf/1903.08945.pdf merlin.carl@uni-flensburg.de

Abstract: We define an ordinalized version of Kleene's realizability interpretation of intuitionistic logic by replacing Turing machines with Koepke's ordinal Turing machines (OTMs) ...

3 OTM-Realizability and Intuitionistic Provability

ii Given φ → ψ, where x does not appear freely in φ, one may infer φ → ∀xψ (3.2.1)

LET p, q, r, s: φ, ψ, x, X

(~(r<p)&(p>q))>(p>(#r&q)) ; TTT**F** TTTN TTT**F** TTTN (3.2.2)

4 Axioms and systems of constructive set theories

We now discuss the OTM-realizability of the axioms of ZFC set theory and their most prominent constructive variants ... . It is easy to see that the axioms of Empty Set Existence, Extensionality, Pairing, Union and Infinity are OTM-realizable.

Proposition 6. The separation schema ∀a∃x∀y(y ∈x ↔ (y ∈a ∧φ(y)) has instantiations with $\in$-formulas φ that are not OTM-realizable. However, every instantiation by a $\Delta 0$-formula is OTM-realizable. (4.6.1)

LET p, q, r, s: φ, ψ, x, X

`(#q<%p)=((#q<r)&(#s&#q)) ;` `TTCT TTCT TTTC TTCT` (4.6.2)

The following may come as a small surprise; however, noting its dependence on the reading assigned here to implication, it is quite natural.

Proposition 7. Every instance of the collection axiom $\forall x \in X \exists y \varphi(x, y) \rightarrow \exists Y \forall x \in X \exists y \in Y \varphi(x, y)$, and thus of the replacement axiom and the strong collection axiom, is OTM-realizable. (4.7.1)

LET p, s, t, x, y: φ, X, Y, x, y

`((#x<s)&(%y&(p&(x&y))))>((%y&(#x<s))&((%y<t)&(p&(x&y)))) ;`

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TTTT TTTT TTTT TTTT(56),
TCTC TCTC TTTT TTTT( 8)
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(4.7.2)

Eqs. 3.2.2, 4.6.2, and 4.7.2 as rendered are *not* tautologous. This denies the application of ordinal Turing machines (OTM) to set theory, which is also refuted elsewhere.