

Chemical onium Hamiltonians & quantum separability

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Abstract

The existence of a disentangling mathematical transformation of wave functions in a Coulomb entangled state of charged molecular radicals, reveals a new chapter to the Einstein - Schrödinger discussion about entanglement.

1. Introduction

In the philosophy of physics there is every now and then some debate about the exact meaning of Einstein's seminal criticism [1] on the completeness of the quantum theory. Einstein's idea of entanglement was to let two particles A and B have a brief interaction and then to separate them [2] and [3]. Einstein reformulated his criticism, which still contains the Heisenberg uncertainty in [1], into the following. If the wave function of A , denoted by ψ_A , can be manipulated by observer O_A , then, the wave function to be observed by the distant O_B is not uniquely attached to B . This is Einstein's *inseparability* criticism. Don Howard [2] argues that, because of e.g. Bose statistics, already far before the publication of the EPR paper, Einstein had his doubts about the separability of quantum particles.

In later developments Bohm [4] replaced the brief interaction between A and B with the singlet spin state to entangle spins of particles A and B . Bohm's paradigmatic particle was para Positronium. Bohm's work gave rise to Bell his formula and the inequality derived thereof [5].

The present author has shown a critical flaw in Bell's work [6]. This flaw is in fact a reference to concrete mathematical incompleteness: the Gödel phenomenon in concrete mathematics [7]. Non inequality research supported the idea that Bell inequalities are perhaps not the only way to determine that the quantum mechanical non-relativistic analogue of classical mechanics, is different from classical mechanics. Historically, one of the earliest proofs thereof found is by Kocher and Commins [11]. The study of Kocher and Commins is an example of a photon correlation experiment without

the need for a Bell correlation formula based inequality. Recently, Nordén [12] discussed the question how wrong Einstein was after all. The mathematical incompleteness of Bell's inequalities in [6] questions ≈ 40 years of experimental research into non-locality. It will not receive a warm welcome in certain quarters of research. It does, however, not invalidate quantum mechanics as a statistical theory. The latter view on quantum mechanics was Einstein's conception [2]. In a sense, Nordén [13] supports this idea with an explanation that comes close to a Hanbury-Brown Twiss view of spin-spin correlation. It must, in addition, be noted also that Wenneström [14] advances physics criticism on the non-locality conclusions that are derived from experiment. A famous experiment that deserves mentioning here is Aspect's [15].

To continue, it must be noted that in his letter to Schrödinger, Einstein was interested in the physics of a brief interaction followed by a spatial separation. This means a transformation of the joint wave function $\psi(\mathbf{x}_A, \mathbf{x}_B)$ into a product of two separate wave functions $\psi_A(\mathbf{x}_A)\psi_B(\mathbf{x}_B)$ for distant particles A and B .

Einstein was displeased with the EPR paper [3]: ...die Hauptsache ist sosusagen durch Gelehrsamkeit verschüttet¹..". In my humble opinion this was not because of a more or less artistic need for simplicity. The EPR paper formulated something close to, but definitely not identical with, Einstein's inseparability criticism.

In an earlier letter to Schrödinger, Einstein writes down after correction [2], a Schrödinger equation for entangled particles in the sense discussed here in the above. The conversation between the two giants of physics continues with Schrödinger noting that with the non-relativistic quantum analogue of classical mechanics, separability cannot be conceived [3, page 177]. One can with a more modern view imagine virtual photons carrying the Coulomb interaction between two opposite charged particles. Because the absence of relativity, the Schrödinger equation with Coulomb potential function is, apparently according to Schrödinger, unfit to describe the separability that Einstein was looking for. End of story according to Schrödinger [3, page 177].

Einstein directed his arrows of criticism to the unexpected inseparability in non-relativistic quantum theory. Let us accept the words of Schrödinger for the quantum analogue of classical mechanics. But what about unexpected *separability* in non-relativistic quantum theory? What would *that*

¹the main point is burried under erudition..

tell us about the quantum analogue of classical mechanics.

Before entering into the mathematics of this question we first may note that this question is most likely not pure philosophy. It is possible to design a real experiment with e.g. charged molecule radicals and perform separation within the boundaries of distances where Coulomb potentials can be felt. This means, we can employ the kind of Schrödinger equation that Einstein considered. However, now we look for unexpected *separability*. We look for disentangling transformation in the realm where an inevitable entangling Coulomb potential function rules. The matter of separability and therefore disentanglement because of temperature [16] will be discussed later.

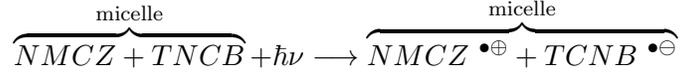
In order to find the properly charged radicals we can look at e.g. the interesting field of spin-chemistry and make use of their experimental techniques. Let us look at Figure - 1. The use of nearly equal mass radicals make sense when we want to approximate a kind of chemical / molecular onium-type of "atom" on the meso scale where quantum theory is still valid. The Schrödinger equation can be similar to the one which approximates the Positronium [8]. In Figure-1 the two radicals are presented. The (ideal) molecular mass with $M_C = 12$, $M_N = 14$, $M_H = 1$, is for N-Methyl Carbazole, $M_{C_{13}H_{11}N} = M_{NMCZ} = 181$. For Tetra Cyano Benzene we have $M_{C_{10}H_2N_4} = M_{TCNB} = 178$. The two charged molecular radicals come close to a meso scale type of "onium" atom approach that can also be found with e.g. Positronium but then for electron and positron.

In a paper of Tanimoto and Fujiwara, [9, page 440] we learn about a charged radicals generating reaction for N-Ethyl Carbazole *NECZ* and *TCNB*. This is what we want to accomplish for *NMCZ* and *TCNB*. The method is to capture *NECZ* and *TCNB* in a micelle. Subsequently, the micellar solution is subjected to light irradiation. The generation of the radicals is performed with the use of photons denoted by $\hbar\nu$. In our case we capture *NMCZ* and *TCNB* (Figure -1) in micelles and use light to generate the radicals.



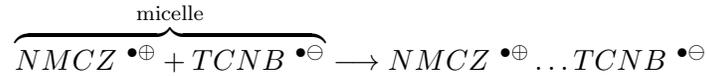
Figure 1. Left, N-Methyl Carbazole radical R denotes $N^{\bullet\oplus} - CH_3$ abbreviated as $NMCZ^{\bullet\oplus}$ and right Tetra Cyano Benzene radical with R' denoting $C \equiv N^{\bullet\ominus}$, abbreviated as $TCNB^{\bullet\ominus}$. In fact, the position of \oplus in the $NMCZ$ radical and \ominus in the $TCNB$ radical is unknown.

The generation of charged molecular radicals in micelles, is presented below



The right hand side of this equation serves as the entangled pair (better: pairs) in a micelle. The spin-chemistry literature shows that this step is possible.

A subsequent step is the separation of $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$. This can be done by e.g. "destroying" the micelle confinement and simultaneously separate $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ with e.g. dipole radiation. The latter separation method can be compared to the way e.g. Wigner described the separation of entangled electron and positron from Positronium [10]. Hence,



The dots denote the spatial separation. We claim that Einstein's treatment of the Schrödinger equation for a number of $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ in the micelle, is correct here. What is needed is that, perhaps for a short moment in time, $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ are in a state where a joint wave function exists. Let us call this the onium state. Most likely, similar to spin-chemistry, there will be a loss of free $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ through chemical reaction. Moreover, the number of "onium" typed but afterwards free $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ must not be too small compared to the $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ that never were in the "onium" state.

In relation to that we may note that spin-chemistry experiments [9] do show that separate molecular radicals can be in the spin singlet state. So in case of spin-spin entanglement a comparable "onium" is possible. We believe that therefore the "onium" without explicit consideration of the spin, living in the micelle state and under the blessing of Einstein his compound Schrödinger equation [2, page 26], is not just sheer fantasy.

2. Hamiltonian in a non-relativistic quantum analogue of classical mechanics

Let us start with the, normalized in form, stationary Schrödinger equation for the Coulomb bound state of two particles. The structure in a sense resembles a Positronium [8] and coincide with [3, page 26]. In the lowest non-relativistic approximation the binding energy is determined by the instantaneous electrostatic interaction, similar to the hydrogen atom but then for, for

instance, $1 \equiv NMCZ \bullet^{\oplus}$ and $2 \equiv TCNB \bullet^{\ominus}$. The reduced molecular mass $M = M_1 M_2 / (M_1 + M_2)$ is close to $179.49/2$, e.g. $m \approx m_{NMCZ}/2$ in e.g. kg , the reduced mass is $m \approx 1.4987 \times 10^{-25}$. In SI units, $\hbar \approx 1.055 \times 10^{-34}$ J.s and $e \approx 1.602 \times 10^{-19}$ coulomb.

$$\left[\nabla_1^2 + \nabla_2^2 + \frac{\alpha'}{r_{1,2}} + \epsilon_{1,2} \right] \psi(\mathbf{x}_1, \mathbf{x}_2) = 0 \quad (1)$$

In this equation $\nabla_1^2 = \frac{\partial^2}{\partial \mathbf{x}_1^2}$, with, $\mathbf{x}_1 = (x_{1,1}, x_{1,2}, x_{1,3})$. Similarly for ∇_2^2 , with, $\mathbf{x}_2 = (x_{2,1}, x_{2,2}, x_{2,3})$. Furthermore, $r_{1,2}^{-1} = \|\mathbf{x}_1 - \mathbf{x}_2\|^{-1}$ and $\alpha' = \frac{2e^2 m}{4\pi\hbar^2}$ and e the unit of charge. $\epsilon_{1,2}$ is $2m/\hbar^2$ times the energy eigenvalue [8, page 182] if \hbar is not in units giving \hbar is unity.

2.1. Unexpected independence

Here we ask can there be a transformation of the approximate Schrödinger equation such that, despite the presence and validity of the Coulomb force, we have mutual independence between distant particles? Of course temperature effects are here crucial to the question [16].

In the experiment we have a number of subsequent stages. Each stage is described by a stationary Schrödinger equation. This is what is intended by time-sliced change. First we have the situation where interacting particles are in a micelle. Then, secondly, the initial separation sets in. Here $\xi = (x_{1,2} + x_{2,1})/2$ is, momentarily, a constant despite changes in respectively, $x_{1,1}$ and $x_{2,1}$. This can be accomplished when e.g. $x_{1,1} \rightarrow x_{1,1} + \Delta x$ and $x_{2,1} \rightarrow x_{2,1} - \Delta x$. In the third next stage, we freeze $x_{1,1}$ and let $y = x_{1,2}$ and $z = x_{1,3}$ vary on the particle 1 side. On the particle 2 side we let $x_{2,1}$ increase. The ξ in that stage or moment of time is no longer a constant because $x_{1,1}$ is "frozen". The mathematics below makes things clear about the arrangement of the experiment for "free" but within Coulomb range $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ radical pairs.

In order to study this we first look at the Coulomb potential function itself. Now suppose that there is a $\xi \in \mathbb{R}$ and the $(x_{1,1} - x_{2,1})^2$ dominate the $\sum_{k=2}^3 (x_{1,k} - x_{2,k})^2$ such that $r_{1,2} = \sqrt{(x_{1,1} - x_{2,1})^2 + \epsilon^2}$. If we then subsequently arrange it such that $x_{1,1} > \xi$ and $x_{2,1} < \xi$ we approximate $r_{1,2}^{-1}$ with a $0 < \beta$ using

$$r_{1,2}^{-1} \approx 2^{-\beta-1} \left\{ (x_{1,1} - \xi)^{-\beta} + (\xi - x_{2,1})^{-\beta} \right\} \quad (2)$$

If we take e.g. $\xi = (x_{1,1} + x_{2,1})/2$, then $r_{1,2}^{-1} \approx (x_{1,1} - x_{2,1})^{-\beta}$. This amounts to an approximation of the Coulomb potential in the "amount of space and

time” where a Coulomb potential rightfully may be employed. We have

$$\frac{\alpha'}{r_{1,2}} \approx 2^{-\beta-1} \frac{2e^2 m}{4\pi\hbar^2} \left\{ (x_{1,1} - \xi)^{-\beta} + (\xi - x_{2,1})^{-\beta} \right\} \quad (3)$$

Let us define $\gamma = 2^{-\beta-1} \frac{2e^2 m}{4\pi\hbar^2}$. If we then accept that at a certain point in time the stationary Schrödinger equation (1) for a product wave function $\psi(\mathbf{x}_1, \mathbf{x}_2) = \psi_1(\mathbf{x}_1)\psi_2(\mathbf{x}_2)$ can be written as

$$\left[\nabla_1^2 + \nabla_2^2 + \gamma \left\{ \frac{1}{(x_{1,1} - \xi)^\beta} + \frac{1}{(\xi - x_{2,1})^\beta} \right\} + \epsilon_{1,2} \right] \psi_1(\mathbf{x}_1|\xi)\psi_2(\mathbf{x}_2|\xi) = 0 \quad (4)$$

This equation (4) can be split into two equations given below. The time-slice development in time allows us to momentarily take ξ is constant: $x_{1,1} \rightarrow x_{1,1} + \Delta x$ and $x_{2,1} \rightarrow x_{2,1} - \Delta x$. Both the $x_{,1}$ coordinates change but ξ does not. The idea is to separate the particles with the use of ”dipole radiation” such as described by Wigner [10, around equation (46) of Wigner’s lecture]. This appears in principle to be possible with the charged radicals $NMCZ^{\bullet\oplus}$ and $TCNB^{\bullet\ominus}$. Hence,

$$\psi_2(\mathbf{x}_2|\xi) \left\{ \nabla_1^2 + \frac{\gamma}{(x_{1,1} - \xi)^\beta} + \epsilon_1 \right\} \psi_1(\mathbf{x}_1|\xi) = 0 \quad (5)$$

and

$$\psi_1(\mathbf{x}_1|\xi) \left\{ \nabla_2^2 + \frac{\gamma}{(\xi - x_{2,1})^\beta} + \epsilon_2 \right\} \psi_2(\mathbf{x}_2|\xi) = 0 \quad (6)$$

with, $\epsilon_{1,2} = \epsilon_1 + \epsilon_2$. It is supposed that the two separate equations describe the situation in a stationary form just after the separation split. Then one may imagine that in experiment it is possible to restrict the stationary description of particle one with wave function $\psi_1(\mathbf{x}_1)$ to the directions $x_{1,2} = y$ and $x_{1,3} = z$. The y and z notation are introduced for ease of the presentation of computation.

If we then introduce the transformation of ψ_1 with

$$\varphi_1(\mathbf{x}_1, \xi) = \left(\frac{\partial}{\partial \xi} + \chi(\mathbf{x}_1, \xi) \right) \psi_1(\mathbf{x}_1|\xi) = D\psi_1(\mathbf{x}_1|\xi) \quad (7)$$

the question can be asked if it is possible to find a transformation (7) such that φ_1 does not depend on ξ . The symmetric propagation where $x_{1,1} \rightarrow x_{1,1} + h$ and $x_{2,1} \rightarrow x_{2,1} - h$ and the $x_{,1}$ coordinates change but ξ does not, is broken in that time frame. In turn ξ depends on the $x_{2,1}$ coordinate of

particle 2. It is assumed that the value of ξ with fixed $x_{1,1}$ can vary with $x_{2,1}$.

Of course, in experiment one can fix $x_{1,1}$ without fixing $x_{2,1}$. In addition ξ can vary because nobody knows exactly where we are allowed to start talking about two separate particles. At the "split" the ξ is approximatedly fixed when looking at e.g. $x_{1,1}$ for the equation of particle 1. Similar case for particle 2. But when the stationary equations for particle 1 and 2 evolve for later "time slices" ξ varies.

Given equation (5) we then may have

$$\left(\text{div grad} + \frac{\gamma}{(x - \xi)^\beta} + \epsilon_1 \right) \psi_1(\mathbf{x}|\xi) = 0 \quad (8)$$

with $\mathbf{x} = (x, y, z) = (x_{1,1}, x_{1,2}, x_{1,3})$ and $x = x_{1,1}$ fixed. For ease of notation denote grad for the gradient $(\frac{\partial}{\partial y}, \frac{\partial}{\partial z})$. Therefore, $\text{div grad} = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. Obviously we also have,

$$D \left(\text{div grad} + \frac{\gamma}{(x - \xi)^\beta} + \epsilon_1 \right) \psi_1(\mathbf{x}|\xi) = 0 \quad (9)$$

2.2. Transformation

Let us first look at the term $D \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi_1$ in (8). Subsequently observe $\text{div grad} \varphi_1 = \text{div grad} \left\{ \left(\frac{\partial}{\partial \xi} + \chi \right) \psi_1 \right\}$. Hence,

$$\begin{aligned} \text{div grad} \left(\frac{\partial}{\partial \xi} + \chi \right) \psi_1 = \\ \left(\frac{\partial}{\partial \xi} \text{div grad} \psi_1 + \chi \text{div grad} \psi_1 \right) + \psi_1 \text{div grad} \chi + 2 \text{grad} \psi_1 \cdot \text{grad} \chi = (10) \\ \left(\frac{\partial}{\partial \xi} + \chi \right) \text{div grad} \psi_1 + \psi_1 \text{div grad} \chi + 2 \text{grad} \psi_1 \cdot \text{grad} \chi \end{aligned}$$

Therefore the first term in the differential equation (8) transforms like

$$D \text{div grad} \psi_1 = \text{div grad} \varphi_1 - \psi_1 \text{div grad} \chi - 2 \text{grad} \psi_1 \cdot \text{grad} \chi \quad (11)$$

The second term contains a ξ . We have

$$D \frac{\gamma}{(x - \xi)^\beta} \psi_1 = \frac{\partial}{\partial \xi} \left(\frac{\gamma}{(x - \xi)^\beta} \psi_1 \right) + \frac{\gamma}{(x - \xi)^\beta} \chi \psi_1 \quad (12)$$

Or, the second term can be written down as,

$$D \frac{\gamma}{(x - \xi)^\beta} \psi_1 = \frac{\beta\gamma}{(x - \xi)^{\beta+1}} \psi_1 + \frac{\gamma}{(x - \xi)^\beta} \varphi_1 \quad (13)$$

For completeness,

$$\frac{\partial}{\partial \xi} (x - \xi)^{-\beta} = (-\beta)(-1) (x - \xi)^{-\beta-1}$$

Note the definition of φ_1 provided in (7) and used in (13). The third term of (9) is a simple transformation

$$D\epsilon_1\psi_1 = \epsilon_1\varphi_1 \quad (14)$$

If we then add equations (11), (13) and (14) we are back at (9) and note that

$$\left(\text{div grad} + \frac{\gamma}{(x - \xi)^\beta} + \epsilon_1 \right) \varphi_1(\mathbf{x}|\xi) = 0 \quad (15)$$

provided

$$\psi_1 \text{ div grad } \chi + 2\text{grad } \psi_1 \cdot \text{grad } \chi - \frac{\beta\gamma}{(x - \xi)^{\beta+1}} \psi_1 = 0 \quad (16)$$

If we observe the previous equation (16), then by construction, $\varphi_1(\mathbf{x}|\xi)$ is a solution to (8) operator equation as is $\psi_1(\mathbf{x}|\xi)$.

2.3. A transformation that allows φ_1 independent of ξ

We start with the assumption that the experiment is such that, after the split. Remembering, $\text{div grad} = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\left\{ \text{div grad} + \frac{\gamma}{(x - \xi)^\beta} + \epsilon_1 \right\} \psi_1(x, y, z|\xi) = 0 \quad (17)$$

Let us assume an $O(m^2)$ approximation theory for m from γ in the potential function and take

$$\psi_1(x, y, z|\xi) = g(y, z)(x - \xi)^{\beta+1} \quad (18)$$

Hence, the equation (17) results into

$$(\text{div grad } g)(x - \xi)^{\beta+1} + \gamma g(x - \xi) + \epsilon_1 g(x - \xi)^{\beta+1} = 0 \quad (19)$$

or

$$(\operatorname{div} \operatorname{grad} g)(x - \xi)^\beta + \gamma g + \epsilon_1 g(x - \xi)^\beta = 0 \quad (20)$$

Now because we approximate in $O(m^2)$ it is possible to e.g. have $g(y, z) = mh(y, z)$ and so, $\gamma g = O(m^2)$. The possibility to have a function with only $g = g(y, z)$ is therewith acknowledged and this is what is needed to have a φ_1 that, according to our aim, does not in $O(m^2)$ approximation depend on ξ . The equation (20) then turns into

$$\operatorname{div} \operatorname{grad} g(y, z) + \epsilon_1 g(y, z) = 0 \quad (21)$$

in $O(m^2)$. Subsequently we can have a look at

$$\psi_1(x, y, z|\xi) = g(y, z)(x - \xi)^{\beta+1} \quad (22)$$

$$\chi(x, y, z|\xi) = \frac{f(y, z) + (\beta + 1)(x - \xi)^\beta}{(x - \xi)^{\beta+1}}$$

The first of the equations obeys the Schrödinger equation (19) and holds a $g = g(y, z)$ which is in $O(m^2)$ approximation, independent of ξ . The second of the equations in (22) is not trivial even though we can multiply nominator and denominator with $g(y, z)$ to, obviously, obtain

$$\chi = \frac{fg - \frac{\partial}{\partial \xi} \psi_1}{\psi_1}$$

It is a part of D transformation and combines with the ψ_1 , for convenience again given in the first of (22), to form a φ_1 as

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial}{\partial \xi} + \chi \right) \psi_1 = \quad (23) \\ &= -(\beta + 1)g(y, z)(x - \xi)^\beta + f(y, z)g(y, z) + (\beta + 1)g(y, z)(x - \xi)^\beta \end{aligned}$$

Hence, $\varphi_1 = g(y, z)f(y, z)$ and φ_1 is, clearly, independent of ξ . For completeness,

$$\frac{\partial}{\partial \xi} \psi_1(\mathbf{x}|\xi) = -g(y, z)(\beta + 1)(x - \xi)^\beta$$

and

$$\chi(\mathbf{x}|\xi)\psi_1(\mathbf{x}|\xi) = g(y, z)f(y, z) + g(y, z)(\beta + 1)(x - \xi)^\beta$$

3. Discussion and conclusion

3.1. Verification

We need to verify if the condition in (16) is fulfilled in a way that warrants φ_1 independence of ξ . If (22) is substituted in (16) we find, remembering, $\text{grad} = \left(\frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ and x, ξ constant for grad

$$\begin{aligned} & g \text{ div grad} \left(f + (\beta + 1)(x - \xi)^\beta \right) + \\ & 2 \text{ grad}(g) \cdot \text{grad} \left(f + (\beta + 1)(x - \xi)^\beta \right) - \beta \gamma g = 0 \end{aligned} \quad (24)$$

Therefore,

$$g \text{ div grad} (f) + 2 \text{ grad}(g) \cdot \text{grad} (f) - \beta \gamma g = 0 \quad (25)$$

with, $\text{grad} = \left(\frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ and, $\xi = (x_{1,1} - x_{2,1})/2$. For x fixed,

$$\text{grad} \left((\beta + 1)(x - \xi)^\beta \right) = (0, 0) \quad (26)$$

We also note that it is assumed: $\gamma g = O(m^2)$. Moreover we note that (25) allows the conclusion that $f = f(y, z)$ is indeed possible. This implies that $\varphi_1 = \varphi_1(y, z)$ is a solution of (15) despite the presence of the potential $\frac{\gamma}{(x-\xi)^\beta}$ in that equation. Please also observe that $\varphi_1(y, z) = g(y, z)f(y, z)$ which need not be $O(m^2)$. In addition, $\gamma g(y, z)$ is $O(m^2)$ but that does also not imply that ψ_1 , defined in (22), is small of $O(m^2)$ in all cases as well. This so because of the occurrence of the $(x - \xi)^{\beta+1}$ as a factor in ψ_1 .

When $x_{1,1} - x_{2,1}$ is large, the potential in (15) decreases. But some wave function descriptions of the same particle, like ψ_1 , still "feel" the effect while others $\varphi_1 = D\psi_1$ may become "immune" to it. Observe that the D transformation can transform $\psi(\mathbf{x}|\xi)$, which is ξ *dependent*, into $\varphi(\mathbf{x})$, which is ξ *independent*. Both functions in (8) and (15) are a solution to

$$\left(\text{div grad} + \frac{\gamma}{(x - \xi)^\beta} + \epsilon_1 \right) \Psi_1 = 0$$

Here, $\Psi_1 \in \{\psi_1(\mathbf{x}|\xi), D\psi_1(\mathbf{x}|\xi), \dots\}$.

Looking at the potential $\frac{\gamma}{(x-\xi)^\beta}$ in (8) we see that

$$2^{-\beta-1} \frac{2me^2}{4\pi\hbar^2} g(y, z)(x - \xi) = 2^{-\beta-1} \frac{me^2}{4\pi\hbar^2} g(y, z)(x_{1,1} - x_{2,1}) \quad (27)$$

is obtained. Noting $mg(y, z) = mg(x_{1,2}, x_{1,3}) = O(m^2)$, the

$$(x - \xi) = (x_{1,1} - x_{2,1})/2$$

in (27) i.e. in (8) can compensate for $O(m^2)$ if e.g. $|x_{1,1} - x_{2,1}|$ is increased sufficiently.

3.2. Schrödinger's end of story

A "relatively" large $|x_{1,1} - x_{2,1}|$, with fixed $x_{1,1}$, maintains the influence of the $x_{2,1}$ coordinate of the second particle on the ψ_1 function. This is true despite the $1/(x_{1,1} - x_{2,1})$ form of the potential function in the Hamiltonian.

It allows also a transformation to $\varphi_1 = D\psi_1$ which is order $O(m^2)$ independent of ξ . Suppose that we in our analysis remain in the distances where Schrödinger implicitly talked about in his letter to Einstein. The ξ represents the influence, via the Coulomb force, of the coordinate of the second entangled charged molecular radical on the wave function of the first.

Apparently, quantum mechanics allows a transformation of a wave function where the inevitable Coulombic entanglement, according to Schrödinger's end of story, is "immunized". The use of charged molecular radicals makes the description with the Schrödinger equation more natural. Note that the charged radicals are taken to be particles with wave functions of their own. The latter are likely related to the intrinsic molecular wave function. Nevertheless, a description of the role of the particle in the experimental environment is aimed for, not the intrinsic molecular wave function. We are looking at a meso scale, but still quantum, behavior.

The mathematics of the Schrödinger equation implies that one can transform away the Coulombic influence of the second particle on the first. We can also ask the question what it means when not a single onium pair can be found. E.g. what does it mean for radical chemistry, when this transformation cannot be accomplished.

A possible explanation for the effect can be a transformation to classical levels. Can temperature effects disallow all kinds of "non temperature based" mathematics to be realised in the real world. From [16] we may learn that asymptotic behavior of atom-atom interaction at sufficiently large separation, -which is perhaps needed in our present case-, are profoundly influenced by excitations in the radiation field. This has the effect that the initial quantum interaction goes over to its classical analogue. On the other hand, working with molecular radicals perhaps lowers the thermal noise that could as well spoil the physical realization of the "non temperature based" mathematics [17]. It is also noted that other forces [16] of the order $1/r_{1,2}^6$

might play a role as well in the physics of a possible experiment with photon generated micelle based charged radicals.

In any case, the transformation $\varphi_1 = D\psi_1$ appears to open a new chapter in the Schrödinger-Einstein story. One may, firstly, wonder what Einstein would have thought of a description of a particle that can transform itself in such a way that it does not "feel" the Coulomb force anymore. If it is a temperature effect then we actually have a temperature effect associated wave function transformation to classical levels. But still the question may linger if in classical domain the Coulomb force at the distance in the experiment will persist. Secondly, the author believes he is halfway a criticism of the quantum analogue of classical mechanics and a genuine discovery in the physical organic chemistry of charged radicals. The conditions for a phenomenon of physical organic chemistry are,

- is it possible to create onium for most of the $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ in the micelle
- is it possible to separate $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ and to stop one radical along the x axis but still allow movement along y and z for the first particle, while the other charged radical is separated along the x axis in the opposite direction
- is it possible to, initially, orderly x-axis separate $NMCZ \bullet^{\oplus}$ and $TCNB \bullet^{\ominus}$ with $x_{1,1} \rightarrow x_{1,1} + \Delta x$ and $x_{2,1} \rightarrow x_{2,1} - \Delta x$
- is it possible to bring either $NMCZ \bullet^{\oplus}$ or $TCNB \bullet^{\ominus}$, considered mesoscopic, in a state wave function $\psi_1(x, y, z|\xi) = g(y, z)(x - \xi)^{\beta+1}$.
 - what is the influence of the intrinsic wave function on the possibility to arrive at mesoscopic ψ_1
 - what does it mean when it is impossible to bring the, or even any, charged radical in mesoscopic ψ_1
- is it possible to find a physical equivalent for $D = \frac{\partial}{\partial \xi} + \chi(\mathbf{x}|\xi)$ so that the "Coulomb immunity" mesoscopic state wave function actually can be determined

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