Refutation of Euclidean geometry embedded in hyperbolic geometry for equal consistency

Abstract: The following conjecture is refuted: "[An] n-dimensional Euclidean geometry can be embedded into (n+1)-dimensional hyperbolic non Euclidean geometry. Therefore hyperbolic non Euclidean geometry and Euclidean geometry are equally consistent, that is, either both are consistent or both are inconsistent." Hence, the conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, → ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

"[An] n-dimensional Euclidean geometry can be embedded into (n+1)-dimensional hyperbolic non Euclidean geometry. The case n = 2 was already known to Gauss in the early 1800's. Therefore hyperbolic non Euclidean geometry and Euclidean geometry are equally consistent, that is, either both are consistent or both are inconsistent."

We rewrite the above excluding the Gaussian reference as:

"[An] n-dimensional Euclidean geometry can be embedded into (n+1)-dimensional hyperbolic non Euclidean geometry. (1.1.1)
Therefore hyperbolic non Euclidean geometry and Euclidean geometry are equally consistent, that is, either both are consistent or both are inconsistent." (1.2.1)

We map Eq. 1.1.1 as "dimensional-n Euclidean planar geometry can be embedded into dimensional-n+1 hyperbolic non Euclidean geometry":

LET p, q, r, s, t:
Euclidean geometry, dimensional-n, planar, hyperbolic, elliptical;
consistent (p=p) Tautology; inconsistent ~(p=p), (p@p) Contradiction.

(q&(r&p))<((q+(%q>#q))&(s&~p)) ; **FFFF FFF**T **FFFF FFF**T (1.1.2)

We map Eq. 1.2.1 as "(hyperbolic non Euclidean geometry and planar Euclidean geometry are equally consistent) is equivalent to, that is, either both are consistent or both are inconsistent":

(((s&~p)&(r&~p))=(p=p)) =
((((s&~p)&(r&~p))=(p=p))+(((s&~p)&(r&~p))=(p@p))) ; **FFFF FFFF FFFF** T**F**T**F** (1.2.2)

Eqs. 1.1.2 and 1.2.2 respectively serve as antecedent and consequent for the full conjecture that 1.1.2 implies 1.2.2. (2.1)

We map Eq. 2.1 as: "(dimensional-n Euclidean planar geometry can be embedded into dimensional-n+1 hyperbolic non Euclidean geometry) implies ((hyperbolic non Euclidean geometry and planar Euclidean geometry are equally consistent) is equivalent to, that is, either both are consistent or both are inconsistent):

((q&(r&p))<((q+(%q>#q))&(s&~p))) >
((((s&~p)&(r&~p))=(p=p)) =
((((s&~p)&(r&~p))=(p=p))+(((s&~p)&(r&~p))=(p@p)))) ;
TTTT TTT**F** TTTT TTT**F** (2.2)

Eqs. 1.1.2, 1.2.2, and 2.2 as rendered are *no*t tautologous. This means the conjecture is refuted.

Remark 2.2: Eq. 2.2 can be coerced into a tautology by changing 1.2 to read "(hyperbolic non Euclidean geometry and planar Euclidean geometry are equally consistent) *implies*, that is, either both are consistent or both are inconsistent", to rely on the abstract (**F**>T)=T rather than on the original (**F**=T)=**F**.