Refutation of domain theory

Abstract: We evaluate six equations for conjectures in five subsections of origins, bases of objects, axiomatic conditions, adjunctions, finite domains, and join-approximable relations. None is tautologous, hence refuting the domain theory of Dana Scott. Therefore, Scott's domain theory is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊸ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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karen.barnes@cs.ox.ac.uk A.Jung@cs.bham.ac.uk

1. Introduction and origins, 1.1 Origins, 1.1.2.2. Recursive types.

Domain Theory ... began in 1969, [attribured to] Dana Scott [in] the following key insight ...

2. Recursive types. Scott’s key construction was a solution to the “domain equation” ... thus giving the first mathematical model of the type-free λ-calculus.

$$D \simeq [D \rightarrow D] \quad (1.1.2.1)$$

D=(D>D) ;
`FFFF FFFF FFFF FFFF``(4),`
`CCCC CCCC CCCC CCCC(4),`
`NNNN NNNN NNNN NNNN(4),`
`TTTT TTTT TTTT TTTT(4)` (1.1.2.2)

Remark 1.1.2.2: The "domain equation" is *not* tautologous, hence refuting domain theory at the outset. However, we press on with evaluations of six other equations as keyed to sections.

2 Domains individually, 2.2 Approximation, 2.2.6 Bases as objects

Definition 2.2.20. *An (*abstract*)* basis *is given by a set* B *together with a transitive relation* ≺ *on* B*, such that ... holds for all elements* x *and finite subsets* M *of* B.

(INT) $M \prec x \Rightarrow \exists y \in B. M \prec y \prec x$ (2.2.20.1)

LET p, q, r, s: M, X, y, B.

(p<q)>((%r<s)&(p<(r<q))) ; TCTT T**F**TT T**F**TT T**F**TT (2.2.20.2)

There is one further problem to overcome, namely, the fact that continuous functions do not preserve the order of approximation. The only way out is to switch from functions to relations, where we relate a basis element c to all basis elements approximating f(c). This can be axiomatized as follows.

Definition 2.2.27. *A relation* R *between abstract bases* B *and* C *is called* approximable *if the following conditions are satisfied: ...*

4. $\forall x \in B\ \forall y \in C. (xRy \Longrightarrow (\exists z \in B.\ x \succ zRy))$. (2.2.27.4.1)

LET p, q, r, s, x, y, z:
A, B, R, C, x, y, z

((#x<q)&(#y<r))&((#x&(r&#y))>((%z<q)&(x>(%z&(r&#y))))) ;
FFFF FFFF FFFF FFFF(48),
NN**FF FFFF** NN**FF FFFF**(16) (2.2.27.4.2)

3. Domains collectivley, 3.1 Comparing domains, 3.1.3 Adjunctions,

Proposition 3.1.10. Let P *and* Q *be posets and* l: P → Q *and* u: Q → P *be monotone functions. Then the following are equivalent: ...*

4. $\forall x \in P\ \forall y \in Q.(x \sqsubseteq u(y) \Leftrightarrow l(x) \sqsubseteq y)$. (3.1.10.4.1)

LET p, q, r, s:
P, Q, x, y; l=(p>q) and u=(q>p).

((#r<p)&(#s<q))&(~(((q>p)&#s)<#r)=~(s<((p>q)&#r))) ;
FFFF FFFF FFFF N**FFF** (3.1.10.4.2)

4. Cartesian closed categories of domains, 4.2 Finite choice: compact domains, 4.2.1 Bifinite domains

Lemma 4.2.3. *If* D *is a bifinite domain and* E *is pointed and algebraic, then every joinable subset of* K(D) × K(E) *gives rise to a compact element of* [D → E]*. If* F *and* G *are joinable families then the corresponding functions are related if and only if*

$$\forall(d, e) \in G\ \exists(d', e') \in F.\ d' \sqsubseteq d \textit{ and } e \sqsubseteq e'. \quad (4.2.3.1)$$

LET p, q, r, s, t, u:
d, e,d',e',F, G.

((#(p&q)<u)&(%(r&s)<t))&(~(p<r)&~(s<q)) ;
FFFF FFFF FFFF FFFN(1),
FFFF FFFF FFFF FFFF(3) (4.2.3.2)

7. Domains and logic, 7.2 Some equivalences, 7.2.5 Compact-open set and spectral spaces

Two additional axioms are needed, however, because frame-homomorphisms are more special than Scott-continuous functions.

Definition 7.2.24. *A relation* R *between lattices* V *and* W *is called* join-approximable *if the following conditions are satisfied:*

1. $\forall x, x' \in V\ \forall y, y' \in W.\ (x' \geq x\ R\ y \geq y' \Rightarrow x'\ R\ y')$; (7.2.24.1.1)

LET r, v, w, x, y, p, q
R,V,W, x, y, x',y'.

((#(x&q)<v)&(#(y&q)<w))&
((~q>(~((#x&(r&#y))>p)))>(#p&(r&#q))) ;
FFFF FFFF FFFF FFFF(4),
FFFF FFFN **FFFF FFF**N(12) (7.2.24.1.2)

The six Eqs. 1, 2.2.20, 2.2.27, 3, 4, 7 above are *not* tautologous, to deny those subsections and hence refute the domain theory of Dana Scott.