Violation of Conservation of Momentum by Lorentz Transformation

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An isolated physical system of elastic collision between two identical objects is chosen to verify the conservation of momentum in two inertial reference frames. In the first reference frame, the center of mass (COM) is stationary. In the second reference frame, the center of mass moves at a constant velocity. By applying Lorentz transformation to the velocities of both objects, total momentum before and during the collision in the second reference frame can be compared. The comparison shows that conservation of momentum fails to hold when both objects move together at the same velocity.

I. INTRODUCTION

Elastic collision between two identical objects is an excellent physics system for demonstration of the conservation of momentum. The collision will be examined in two inertial reference frames, the center of mass (COM) frame and a random inertial reference frame. Conservation of momentum is expected to hold in both reference frames.

Lorentz transformation[1] transforms the velocities of both objects from COM frame to a random inertial reference frame. The total momentum, before and during the collision, will be calculated in this random inertial reference frame to verify if conservation of momentum still holds when both objects move together at the same velocity.

The concept of relativistic mass becomes less popular in modern physics. The momentum of an object is represented by either \( \gamma(v) \ast m(0) \ast v \) or \( m(v) \ast v \). Both representations are equivalent to each other mathematically. In this paper, \( \gamma(v) \ast m \ast v \) is chosen to emphasize Lorentz Factor, \( \gamma(v) \), in Lorentz Transformation. Mass is not a function of velocity.

II. PROOF

Consider one-dimensional motion.

A. Elastic Collision

Two identical objects move toward each other to make head-on collision. In the COM frame (Center Of Mass), both objects move at the identical speed but in the opposite direction. At the moment when two objects make contact, there is a repulsive force between them. Both objects eventually slow down to become stationary. The repulsive force continues to push both objects away until there is no further contact between objects.

B. Before Collision

Let a reference frame \( F_1 \) be stationary relative to COM frame.

<table>
<thead>
<tr>
<th>Object</th>
<th>Frame</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of object 1, ( O_1 ), in ( F_1 )</td>
<td>is ( V )</td>
<td></td>
</tr>
<tr>
<td>Velocity of object 2, ( O_2 ), in ( F_1 )</td>
<td>is ( -V )</td>
<td></td>
</tr>
<tr>
<td>Momentum of ( O_1 ) in ( F_1 )</td>
<td>is ( \gamma(V) \ast m \ast V )</td>
<td></td>
</tr>
<tr>
<td>Momentum of ( O_2 ) in ( F_1 )</td>
<td>is ( \gamma(-V) \ast m \ast (-V) )</td>
<td></td>
</tr>
</tbody>
</table>

C. Lorentz Transformation

Let another reference frame \( F_2 \) move at the velocity \(-U\) relatively to \( F_1 \). The velocity of \( F_1 \) relative to \( F_2 \) is \( U \). According to Lorentz Transformation, the velocities of \( O_1 \) and \( O_2 \) in \( F_2 \) are transformed as,

<table>
<thead>
<tr>
<th>Object</th>
<th>Frame</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of ( O_1 ) in ( F_1 )</td>
<td>( F_2 )</td>
<td>is ( \frac{V + U}{\sqrt{1 + \frac{U^2}{c^2}}} )</td>
</tr>
<tr>
<td>Velocity of ( O_2 ) in ( F_1 )</td>
<td>( F_2 )</td>
<td>is ( \frac{-V + U}{\sqrt{1 + \frac{U^2}{c^2}}} )</td>
</tr>
</tbody>
</table>

D. Collision

Upon collision, both objects in \( F_1 \) will slow down and come to stand still in \( F_1 \) before moving away. As both objects become stationary in \( F_1 \), both objects move at the same velocity \( U \) in \( F_2 \).
E. Conservation of Momentum

The total momentum in $F_1$ is always zero during the collision. However, the total momentum in $F_2$ is not zero. Let $v_1$ be the velocity of $O_1$ in $F_2$ before collision.

$$v_1 = \frac{V + U}{1 + \frac{V U}{c^2}}$$ (1)

Let $v_2$ be the velocity of $O_2$ in $F_2$ before collision.

$$v_2 = \frac{-V + U}{1 - \frac{V U}{c^2}}$$ (2)

Total momentum in $F_2$ before collision is $P_{before}$.

$$P_{before} = \gamma(v_1) \ast m \ast v_1 + \gamma(v_2) \ast m \ast v_2$$ (3)

Total momentum in $F_2$ during collision when both objects move at the same velocity $U$ is $P_{after}$.

$$P_{after} = \gamma(U) \ast m \ast U + \gamma(U) \ast m \ast U$$ (4)

$$= 2 \ast \gamma(U) \ast m \ast U$$ (5)

Conservation of momentum demands

$$P_{after} = P_{before}$$ (6)

However, conservation of momentum is violated because equation (6) fails to hold for $U = V = \frac{C}{2}$.

$$P_{after} = 2 \ast \gamma(U) \ast U \ast m = \frac{2}{\sqrt{3}} \ast C \ast m$$ (7)

From equations (7,9), $P_{before}$ is not equal to $P_{after}$.

Total momentum before elastic collision is different from total momentum during collision when both objects move at the same velocity if

$$U = V = \frac{C}{2}$$ (10)

III. Conclusion

Lorentz Transformation violates the conservation of momentum.

Conservation of momentum fails to hold if Lorentz Transformation is applied to an isolated system of elastic collision in COM frame. The failure of this physics law is due to the addition of velocity from Lorentz Transformation. The correct formula for velocity transformation has been derived by Su in 2018[2][10].

Lorentz Transformation leads to the assumption[3] that the speed of light is independent of inertial reference frame. In addition to the violation of conservation of momentum, Lorentz Transformation also violates translation symmetry[4] in physics. Translation symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of elapsed time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is not a valid transformation in physics. Consequently, any theory based on Lorentz transformation is incorrect in physics. For example, theory of special relativity[8][9]