Refutation of standard induction, coinduction and mutual induction, coinduction

Abstract: From the summary of standard and mutual induction and coinduction, we evaluated four formulas with *non* tautologous and hence refutations. Therefore these are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moez A. AbdelGawad, M.A. (2019). Mutual coinduction. arxiv.org/pdf/1903.06514.pdf moez@cs.rice.edu

LET p, q, r, s, t, u: *P, F, μ, ν, O, G*; ⊑ is equivalent to ≤.

The formulation of standard induction and standard coinduction, and related concepts, that we present here is a summary [presented elsewhere].

• (*standard induction*) if $F(P) \leq F$, then $\mu_F \leq P$,

~(p<(q&p))>~(p<(r&q)) ; TTTT TTT**F** TTTT TTT**F** (1.2)

• (*standard coinduction*) if $P \leq F(P)$, then $P \leq \nu_F$,

~((q&p)<p)>~((s&q)<p) ; TTTT TTTT TT**F**T TT**F**T (2.2)

[G]iven that *μF* and *μG* are the *least* simultaneous pre-fixed points of *F* and *G* and *νF* and *νG* are the *greatest* simultaneous post-fixed points of *F* and *G*, for any element *O* ∈O and *P* ∈P we have:

• (*mutual induction*) if $F(O) \sqsubseteq P$ and $G(P) \leq O$, then $\mu_F \leq O$ and $\mu_G \sqsubseteq P$ (3.1)

(~(p<(q&t))&~(t<(u&p)))>(~(t<(r&q))&~(p<(r&u))) ;

TTTT TTTT TTTT TTTT(3),
TTT**F** TTTT TTT**F** TTTT(1) (3.2)

• (*mutual coinduction*) if $P \sqsubseteq F(O)$ and $O \leq G(P)$, then $O \leq \nu_F$ and $P \sqsubseteq \nu_G$ (4.1)

(~((q&t)<p)&~((u&p)<t))>(~((s&q)<t)&~((s&u)<p)) ;
TTTT TTTT TT**FF** TT**FF**(1),
TTTT TTTT TTTT TTTT(1),
TTTT TTTT **F**T**F**T **F**T**F**T(1),
TTTT TTTT **F**TTT **F**TTT(1) (4.2)

Eqs. 1.2 - 4.2 as rendered are *not* tautologous. This refutes standard induction, coinduction and mutual induction, coinduction.