Refutation of finitary, non-deterministic, inductive definitions of ECST and denial of CZF

Abstract: From the elementary constructive set theory (ECST) of intuitionistic logic, we evaluate six axioms of equality for system CZF. None is tautologous. This refutes those axioms in set theory and by extension denies intuitionistic logic.

Therefore are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⇀ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hirata, A.; Ishihara, H.; Kawai, T.; Nemoto, T. (2019).
Equivalents of the finitary non-deterministic inductive definitions.
arxiv.org/pdf/1903.05852.pdf tatsuji.kawai@jaist.ac.jp

2 Elementary constructive set theory

We work in a weak subsystem of **CZF**, called the *elementary constructive set theory* **ECST** [..], where none of the known fragments of the NID principle [*non-deterministic inductive definitions*] seems to be derivable.

The language of **ECST** contains variables for sets and binary predicates = and ∈. The axioms and rules of **ECST** are the axioms and rules of intuitionistic predicate logic with equality, and the following set-theoretic axioms:

Extensionality: $\forall a \forall b\, (\forall x\, (x \in a \leftrightarrow x \in b) \rightarrow a = b)$. (2.1.1)

LET p, q, r, s: a, b, x or y, u.

((#r<#p)=(#r<#q))>(#p=#q) ; `TTTT TCCT TTTT TCCT` (2.1.2)

Paring: $\forall a \forall b \exists y \forall u\, (u \in y \leftrightarrow u = a \lor u = b)$. (2.2.1)

`(#s<%r)=(#s=((#p+#s)=#q)) ;` `TCCT TCCT CCTT TTCC` (2.2.2)

Union: $\forall a \exists y \forall x\, (x \in y \leftrightarrow \exists u \in a\, (x \in u))$. (2.3.1)

LET p, q, r, s: a, u, x, y.

`(#r<%s)=((%q<#p)&(#r<%q)) ;` `TTTT CCCC TTTT TTTT` (2.3.2)

Restricted Separation: $\forall a \exists b \forall x\, (x \in b \leftrightarrow x \in a \land \phi(x))$ where $\phi(x)$ is restricted. Here, a formula is said to be *restricted* if all quantifiers in the formula occur in the forms $\forall x \in a$ or $\exists x \in a$. (2.4.1)

LET p, q, r, s: ϕ , a, b, x

`(#s<%r)=((#s<#q)&(p&#s)) ;` `TTTT TTTT CTCC TCTT` (2.4.2)

Replacement:
$\forall a(\forall x \in a \exists! y\, \phi(x, y) \rightarrow \exists b \forall y\, (y \in b \leftrightarrow \exists x \in a\, \phi(x, y)))$ where $\phi(x, y)$ is any formula. (2.5.1)

LET p, q, r, x, y: ϕ, a, b, x, y

`((#x<#q)&(p&(#x&%y)))>((#y<%r)=((%x<#q)&(p&(%x&#y)))) ;`
`TTTT TTTT TTTT TTTT(48),`
`TTTT TCTT TTTT TCTT(16)` (2.5.2)

Strong Infinity:
$\exists a[0 \in a \land \forall x(x \in a \rightarrow x + 1 \in a) \land \forall y\, (0 \in y \land \forall x(x \in y \rightarrow x + 1 \in y) \rightarrow a \subseteq y)]$
where $x + 1$ denotes $x \cup \{x\}$ and 0 is the empty set $\varnothing$. ... (2.6.1)

LET p, q, r: a, x, y

`(((p@p)<%p)&((#q<%p)>((#q+(%p>#p))<%p)))&(((p@p)<#r)&`
`(((#q<#r)>((#q+(%p>#p))<#r))>~(#r<%p))) ;` **`FFFF FFFF FFFF FFFF`** (2.6.2)

This completes the description of **ECST**.

The constructive Zermelo–Fraenkel set theory **CZF** [..] is obtained from **ECST** by substituting Strong Collection for Replacement and adding Subset Collection and $\in$-Induction. ...

Eqs. 2.1.2-2.6.2 as rendered are *not* tautologous. This refutes those axioms on **ECST** and hence denies intuitionistic predicate logic and **CZF** set theory.