Refutation of Coq proof assistant to map Euclidean geometry to Hilbert space

Abstract: For the relation of "a point is incident to a straight line", we find that proposition is *not* tautologous. This denies the conjectured approach of a constructive mapping Euclidean geometry into a Hilbert space and also refutes the Coq proof assistant as a bivalent tool. The conjecture and Coq are therefore *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ivashkevidh, E.V. (2019). On constructive-deductive method for plane Euclidean geometry.
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Constructive-deductive method for plane Euclidean geometry is proposed and formalized within Coq Proof Assistant. This method includes both *postulates* that describe elementary constructions by idealized geometric tools (pencil, straightedge and compass), and *axioms* that describes properties of basic geometric figures (points, lines, circles and triangles). The proposed system of postulates and axioms can be considered as a constructive version of the Hilbert's formalization of plane Euclidean geometry.

Remark 1.4: The law of excluded middle as presented is unclear as to its order of operations; to be tautologous, the main connective is the Or operator.

2.2. Incidence relation

The main undefined relation, that determines the relative position of points and straight lines on the plane, is the relation of *incidence*. ... It is easy to see that if at some scale the spots that represent a point and a straight line do not intersect and do not touch each other, then they will be distinguishable on all larger scales. In this case, we say that the point is *apart* from the line. It is impossible to confirm *empirically* the fact that a point belongs to a straight line, because for this we would have to make sure that the graphite spots that represent this point and this straight line overlap or touch each other on all scales.

Note that expressions often used in geometry: "a point lies on a line", "a point belongs to a line", "a line passes through a point" — all of them are equivalent to the proposition "a point is incident to a straight line".

$(A\ B\ \ldots : \text{Point})(x\ y\ \ldots : \text{Line}),$
$(A \in xy\ \ldots) \equiv A \in x \wedge A \in y \wedge \ldots;$
the lines $x, y, \ldots$ pass through point A (2.2.2.1)

LET p, q, r, s: A, B, x, y

$(p<(r\&s))=((p<r)\&(p<s))$; TTTT T**F**T**F** T**F**T**F** TTTT (2.2.2.2)

Remark 2.2.2.2: The only way to coerce Eq. 2.2.2.1 into a tautology is to specify that:

Point A is lesser than points x or y is equivalent to A is lesser than x and y, as $(p<(r+s))=((p<r)\&(p<s))$; or (2.2.2.3)

Point A lesser than points x and y, is equivalent to A is lesser than x or y, as $(p<(r\&s))=((p<r)+(p<s))$. (2.2.2.4)

Eq. 2.2.2.2 as rendered is *not* tautologous. This refutes Coq proof assistant and further denies the conjecture of a constructive and deductive mapping Euclidean geometry into a Hilbert space.

We also hasten to add that the former is bivalent and exact, but the latter is a vector space and probabilistic, meaning the approach is *not* tenable.