Denial of Suzko's problem

Abstract: We examine a sentential logic description, as based on set theory, in support of Suzko's theorem that only two truth values are required as a universal logic. Under syntactic notions, we evaluate three definitions (monotonicity, transivity, permeability) out of six definitions (trivial are substitution-invariance, reflexivity, combined consequence relation). Monotonicity and transivity are *not* tautologous. Right-to-left permeability is *not* tautologous. What follows is that a Malinowski extension of *mixed-consequence* by relaxation of the two values for three logical values is spurious, especially due to the fact that Suzko's theorem is a conjecture based on the *assumption* of set theory. What also follows is that compositionality as based on Suzko-Scott reductions are *not* bivalent and exact, but rather a vector space and probabilistic. Our results point further to the equations analyzed as being *non* tautologous fragments of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⇾ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Chemla, E.; Egrvé, P. (2019).
Suszko’s problem: mixed consequence and compositionality.
arxiv.org/pdf/1707.08017.pdf paul.egre@ens.fr

Definition 2.5 (Monotonicity)**.** A consequence relation ⊢ is *monotonic* if:

$\forall \Gamma_1 \subseteq \Gamma_2, \Delta_1 \subseteq \Delta_2 : \Gamma_1 \vdash \Delta_1$ implies $\Gamma_2 \vdash \Delta_2$. (2.5.1)

LET p, q, r, s, t: Γ_1 or Γ, Γ_2 or Γ', Δ_1 or Δ, Δ_2 or Δ' or Σ, L.

(~(#q<#p)&~(#s<#r))>((#r>#p)>(#s>#q)) ; `TTTT TTTT TTTT TTCT` (2.5.2)

Definition 2.7 (Transitivity)**.** A consequence relation ⊢ is *transitive* iff:

if Γ⊬Δ, then there are Γ'⊇Γ , Δ'⊇Δ such that Γ'⊬Δ' and Γ'∪Δ'=L. (2.7.1)

((q>p)>(~(p>q)&~(r<s)))>((~(s>q)&(q+s))=t) ;
TTTT TTTT T**F**TT T**F**TT,
T**FF**T TT**F**T TT**F**T TT**F**T (2.7.2)

We introduce here a formal property that a consequence relation should *not* have:

Definition 2.9 (Permeability)**.** A consequence relation is *permeable* if it is *left-to-right* or *right-to-left permeable*, in the following sense: ...

Right-to-left permeability: $\forall \Gamma, \Delta, \Sigma : \Gamma \vdash \Sigma, \Delta \Rightarrow \Gamma, \Sigma \vdash \Delta$ (2.9.2.1)

((#s&#q)>#p)>(#q>(#p&#s)) ; TTCC TTCC TTTT TTTT (2.9.2.2)

By extension, a logic is called *permeable* if its consequence relation is permeable. If a logic is not permeable, then its consequence relation is neither universal nor trivial ...

Eqs. 2.5.2, 2.7.2, and 2.9.2 are *not* tautologous. This means those three of six equations refute the goal of mixed-consequence before subsequent machinations including entertainment of 3- or 4-values and the Appendix A compositionality of Suzko-Scott reductions which are *not* bivalent but a vector space.