Denial of Płonka sums in logics of variable inclusion and the lattice of consequence relations

Abstract: From the section on Płonka sums, we evaluate an equation derived therefrom. It is *not* tautologous, hence coloring subsequent assertions in the conjecture. This means the *non* tautologous conjecture is a fragment of the the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↣ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↢, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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2.2. **Płonka sums.** The main mathematical tool that allows for a systematic study of logics of variable inclusion is an algebraic construction coming from universal algebra, and more specifically from the study of regular varieties ... . Such construction, known as *Płonka sums*, originates in the late 1960's from a series of papers published by the Polish mathematician J. Płonka, who first provided a general representation theorem for regular varieties. ... A *semilattice* is an algebra **A** = < A, ∨ >, where ∨ is a binary commutative, associative and idempotent operation. Given a semilattice **A** and a, b ∈ A, we set

$$a \le b \iff a \vee b = b. \qquad (2.2.1)$$

It is easy to see that ≤ is a partial order on A.

LET p, q: a, b.

~(q<p)=((p+q)=q) ; T**FF**T T**FF**T T**FF**T T**FF**T (2.2.2)

Eq. 2.2.2 is *no*t tautologous, thus coloring the entire conjecture.