Refutation of induction formulas in elementary arithmetic EA

Abstract: From the introduction, we evaluate EA elementary arithmetic for induction formulas which are *not* tautologous. This further refutes the reflection property upon which subsequent assertions are based. These formulas constitute a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ⤚ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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arxiv.org/pdf/1903.03331.pdf jjoosten@ub.edu

The theory EA of Elementary Arithmetic is given by the defining axioms for the arithmetical symbols together with the induction formulas

$$I_\phi := [\ \phi(0) \land \forall x\ \phi(x) \rightarrow \phi(x+1)\ ] \rightarrow \forall x\ \phi(x) \qquad (1.1)$$

for each bounded formula ϕ.

LET p, q, r, s: ϕ , x, r, s.

(((p&(p@p))&(p&#q))>(p&(#q+(p=p))))>(p&#q) ;
FFFN FFFN FFFN FFFN (1.2)
using T as value for 1

(((p&(p@p))&(p&#q))>(p&(#q+(%p>#p))))>(p&#q) ;
FFFN FFFN FFFN FFFN (1.3)
using N as value for 1

Eqs. 1.2 and 1.3 are *not* tautologous. This refutes the induction formulas of EA. This further refutes the reflection property upon which subsequent assertions are based. These formulas constitute a *non* tautologous fragment of the universal logic VŁ4.