Uncertainty and the *Zitterbewegung* interpretation of an electron

Jean Louis Van Belle, Drs, MA*Ec*, BA*Ec*, B*Phil*

Email: jeanlouisvanbelle@outlook.com

19 March 2019

Abstract : This paper explores how the *Zitterbewegung* interpretation and the Uncertainty Principle might mesh. It also further details our geometric interpretation of the *de Broglie* wavelength.

Contents

Uncertainty and the *Zitterbewegung* interpretation of an electron

The *Zitterbewegung* interpretation of an electron

The diagram below show the idealized *Zitterbewegung* of a moving pointlike charge. The *combined* idea of a pointlike charge and its presumed motion make up what we think of as an electron.

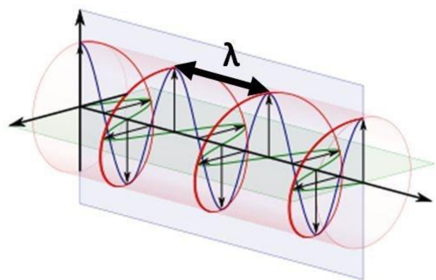

Figure 1: An idealized *Zitter* trajectory[1]

The model is exceedingly simple. The pointlike charge itself has no rest mass and, hence, rotates around at the speed of light.[2] We also think of the circular motion as some kind of two-dimensional oscillation and – because the two oscillations are independent and 90 degrees out of phase – we just add the energy of the two oscillators and, therefore, get an $E = m \cdot a^2 \cdot \omega^2$ equation. We then use the Planck-Einstein relation ($\omega = E/\hbar$) and Einstein's mass-energy equivalence ($E = m \cdot c^2$), to get the Compton radius (a):

$$\mathrm{E} = \mathrm{m} \cdot c^2 = \mathrm{m} \cdot a^2 \cdot \omega^2 = \mathrm{m} \cdot a^2 \cdot \frac{\mathrm{E}^2}{\hbar^2} \Leftrightarrow a = \frac{\hbar}{\mathrm{m} \cdot c}$$

The Compton wavelength λ_C is the circumference of the rotation: $\lambda_C = 2\pi \cdot a = h/mc$. Now, if c is equal to the (tangential) velocity of the pointlike charge, then it's easy to see that the Compton radius must diminish if the linear velocity (v) gets larger. We get the following formula for the wavelength λ in Figure 1:

$$\lambda = v \cdot \mathrm{T} = \frac{v}{f} = v \cdot \frac{\mathrm{h}}{\mathrm{E}} = v \cdot \frac{\mathrm{h}}{\mathrm{m}c^2} = \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \beta \cdot \lambda_C$$

Hence, λ is a *fraction* of the Compton wavelength $\lambda_C = 2\pi \cdot a$, and the fraction is the relative velocity $\beta = v/c$. As the linear velocity increases (so that's just the classical velocity of our particle), then it's energy

[1] This diagram is based on an illustration of a circularly polarized wave, which looks exactly the same. We just added the λ wavelength. It is a diagram from Wikimedia Commons. While it is in the public domain, we still want to acknowledge the author here: https://commons.wikimedia.org/wiki/User:Dave3457.

[2] This is a result that Schrödinger had already obtained from an analysis of Dirac's wave equation of the electron. He referred to it as a *Zitterbewegung*. Prof. Dr. David Hestenes is to be credited with reviving this interpretation of what an electron might actually *be*. I should note, however, that Prof. Dr. David Hestenes – and other *zbw* theorists – may not agree with my shortcut to the results of the model.

and, therefore, its equivalent mass is going to *increase*. In fact, using natural units (c = 1 and $\hbar$ = 1), we get the simplest of simple formulas for the Compton radius:

$$a = \frac{1}{\mathrm{m}}$$

Interpreting the *de Broglie* wavelength

The λ wavelength is *not* the *de Broglie* wavelength λ_L = h/p.[3] So what is it? We have *three* wavelengths now: the *Compton* wavelength λ_C (which is a circumference, actually), that weird horizontal distance λ, and the *de Broglie* wavelength λ_L. Can we make sense of that? We can. Let us first re-write the *de Broglie* wavelength:

$$\lambda_L = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}v} = \frac{\mathrm{h}c^2}{\mathrm{E}v} = \frac{\mathrm{h}c}{\mathrm{E}\beta} = \frac{\mathrm{h}}{c} \cdot \frac{1}{\mathrm{m}\cdot\beta} = \frac{\mathrm{h}}{\mathrm{m}_0 c} \cdot \frac{1}{\gamma\beta}$$

What is this? Let's analyze it mathematically. What happens to the *de Broglie* wavelength as m and *v* both increase because our electron picks up some momentum p = m·*v*? Its wavelength must actually *decrease* as its (linear) momentum goes from zero to some much larger value – possibly infinity as *v* goes to *c* – but *how exactly*? The 1/γβ factor gives us the answer. That factor comes down from infinity (+∞) to zero as *v* goes from 0 to *c* or – what amounts to the same – if the relative velocity β = *v*/*c* goes from 0 to 1. The graphs below show how that works. The 1/γ factor is the circular arc that we're used to, while the 1/β function is just the regular inverse function (*y* = 1/*x*) over the domain β = *v*/*c*, which goes from 0 to 1 as *v* goes from 0 to *c*. Their product gives us the green curve which – as mentioned – comes down from +∞ to 0. [Take your time to carefully look at the formulas and the curves so you can digest this.]

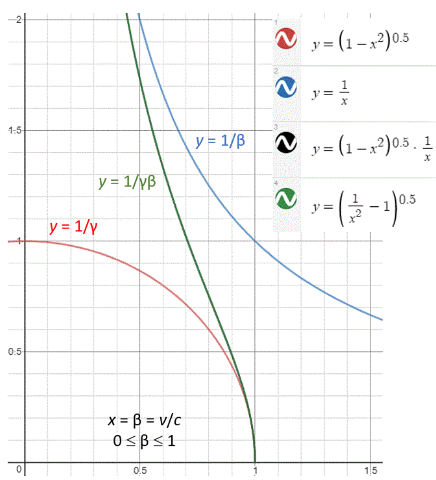

Figure 2: The 1/γ, 1/β and 1/γβ graphs[4]

Now, we re-wrote the formula for *de Broglie* wavelength λ_L as the *product* of the 1/γβ factor and the *Compton* wavelength for *v* = 0:

[3] The use of L as a subscript is a bit random but think of it as the L of *L*ouis de Broglie.

[4] We used the free desmos.com graphing tool for these and other graphs.

$$\lambda_L = \frac{h}{m_0 c} \cdot \frac{1}{\gamma\beta} = \frac{1}{\beta} \cdot \frac{h}{mc}$$

Hence, the *de Broglie* wavelength goes from +∞ to 0. We may wonder: when is it equal to λ_C = h/m*c*? Let's calculate that:

$$\lambda_L = \frac{h}{p} = \frac{h}{mc} \cdot \frac{1}{\beta} = \lambda_C = \frac{h}{mc} \Leftrightarrow \beta = 1 \Leftrightarrow v = c$$

This is a rather weird result, isn't it? But it is what it is. Let's bring the third wavelength in: the $\lambda = \beta \cdot \lambda_C$ wavelength—which is that length between the crests or troughs of the wave.[5] We get the following two rather remarkable results:

$$\lambda_L \cdot \lambda = \lambda_L \cdot \beta \cdot \lambda_C = \frac{1}{\beta} \cdot \frac{h}{mc} \cdot \beta \cdot \frac{h}{mc} = \lambda_C^2$$

$$\frac{\lambda}{\lambda_L} = \frac{\beta \cdot \lambda_C}{\lambda} = \frac{p}{h} \cdot \frac{v}{c} \cdot \frac{h}{mc} = \frac{mv^2}{mc^2} = \beta^2$$

The product of the $\lambda = \beta \cdot \lambda_C$ wavelength and *de Broglie* wavelength is the square of the Compton wavelength, and their ratio is the square of the relative velocity β = *v*/*c*. – *always!* – and their ratio is equal to 1 – *always!* These two results are rather remarkable too but, despite their simplicity and apparent beauty, you might be struggling for an easy geometric interpretation. I was struggling for it too, but then I thought the use of *natural units* might help. Equating *c* to 1 would give us natural distance and time units, and equating *h* to 1 would give us a natural force unit—and, because of Newton's law, a natural mass unit as well. Why? Because Newton's ***F*** = m·***a*** equation is relativistically correct: a force is that what gives some mass acceleration. Conversely, mass can be defined of the inertia to a change of its state of motion—because any change in motion involves a force and some acceleration: m = **F**/***a***. If we re-define our distance, time and force units by equating *c* and *h* to 1, then the Compton wavelength (remember: it's a circumference, really) and the mass of our electron will have a simple inversely proportional relation[6]:

$$\lambda_C = \frac{1}{\gamma m_0} = \frac{1}{m}$$

We get equally simple formulas for the *de Broglie* wavelength and our λ wavelength:

$$\lambda_L = \frac{1}{\beta\gamma m_0} = \frac{1}{\beta m}$$

$$\lambda = \beta \cdot \lambda_C = \frac{\beta}{\gamma m_0} = \frac{\beta}{m}$$

This is quite deep: we have three *lengths* here – defining all of the geometry of the model – and they all depend on two factors only: the *rest* mass of our object and its (relative) velocity. Can we take this

[5] We should emphasize, once again, that our two-dimensional wave has no real crests or troughs: λ is just the distance between two points whose argument is the same—except for a phase factor equal to *n*·2π (*n* = 1, 2,…).
[6] In case you wonder why we get the same 1/m result for *a* as for λ_C, note that it depends on what we consider to be a natural unit: we got the *a* = 1/m equation by equating ħ to 1, as opposed to h.

discussion any further? Perhaps, because what we have found may or may not be related to the idea that we're going to develop in the next section. However, before we move on to the next, let us quickly note the three equations – or lengths – are *not* mutually independent. They are related through that equation we found above:

$$\lambda_{\mathrm{L}} \cdot \lambda = \lambda_C^2 = \frac{1}{\mathrm{m}^2}$$

We'll let you play with that. To help you with that, you may start by noting that the $\lambda_L\lambda = 1/m^2$ reminds us of a property of an ellipse. Look at the illustration below.[7] The length of the chord – perpendicular to the major axis of an ellipse is referred to as the *latus rectum*. One half of that length is the actual *radius of curvature* of the osculating circles at the endpoints of the major axis.[8] We then have the usual distances along the major and minor axis (*a* and *b*). Now, one can show that the following formula has to be true:

$$a \cdot p = b^2$$

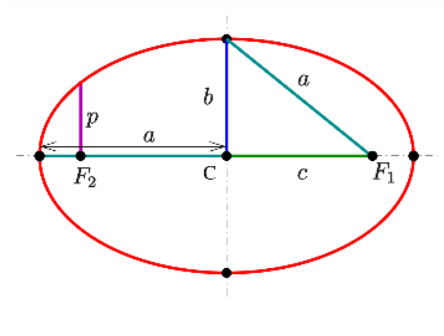

Figure 3: The *latus rectum* formula: $a \cdot p = b^2$

You probably wonder: why would this be relevant? It introduces an asymmetry in what we may loosely refer to as the *shape* of an electron. We get such asymmetry from other models – notably Dirac-Kerr-Newman models of the electron – and it should explain the anomalous magnetic moment without having to resort to weird calculations using Feynman diagrams and renormalization techniques. In short, we think the analysis above gives you a *classical* electron model which may explain all of quantum mechanics in a *classical* way. We've elaborated on that in our book[9] and, hence, will not repeat ourselves here.

[7] Source: Wikimedia Commons (By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=57428275).

[8] The endpoints are also known as the *vertices* of the ellipse. As for the concept of an osculating circles, that's the circle which, among all tangent circles at the given point, which approaches the curve most tightly. It was named *circulus osculans* – which is Latin for 'kissing circle' – by Gottfried Wilhelm Leibniz. You know him, right? Apart from being a polymath and a philosopher, he was also a great mathematician. In fact, he was the one who invented differential and integral calculus.

[9] See: Jean Louis Van Belle, *The Emperor Has No Clothes: A Classical Interpretation of Quantum Mechanics*, 7 March 2019, http://vixra.org/abs/1901.0105.

Introducing uncertainty

What about uncertainty? If there is uncertainty about the velocity, then there is uncertainty about the energy and, therefore, about the (equivalent) mass of our electron. Hence, the radius of the oscillation may also increase or decrease. The two *complementary* expressions of Heisenberg's Uncertainty Principle ($\Delta p \cdot \Delta x = h$ and $\Delta E \cdot \Delta t = h$) can, therefore, easily be related one to another.[10] In fact, *all* uncertainties can easily be related through this very simple geometric model. But so what's *h*? What *is* that quantum of action? It's just the product of the force that keeps that charge in its orbit, the circumference ($\lambda_C = 2\pi \cdot a$) and the cycle time $T = 1/f$. Energy is force over a distance, so we can write this:

$$\mathrm{F} \cdot \lambda_C \cdot \mathrm{T} = \mathrm{E} \cdot \frac{1}{f} = \mathrm{E} \cdot \frac{h}{\mathrm{E}} = h$$

This gives us the formula underpinning the energy-time expression the Uncertainty Principle:

$$\mathrm{E} \cdot \mathrm{T} = \mathrm{h} \Longrightarrow \Delta\mathrm{E} \cdot \Delta\mathrm{T} = \mathrm{h}$$

You may wonder what this really means, but we will give a bit of an interpretation with some diagrams for the position-momentum expression in the next section, so you may want to hold your horses for a while. Let us first see if we can get that position-momentum expression from our geometric interpretation. A force times some time will give us the *physical* dimension of (linear) momentum. Of course, at this point we're not quite sure if we're going to get the *classical* $p = m \cdot v$ momentum, so let us just be careful and use some subscript. We'll use p_c. Why the *c*? I don't know: classical, Compton, *c*,… It is just a placeholder for the time being. Let's see what we get[11]:

$$\mathrm{F} \cdot \lambda_C \cdot \mathrm{T} = \mathrm{p}_c \cdot \lambda_C = \mathrm{F} \cdot \mathrm{T} \cdot \lambda_C = \frac{\mathrm{E}}{\lambda_C} \cdot \frac{h}{\mathrm{E}} \cdot \lambda_C = h$$

So now we can write:

$$\mathrm{p}_c \cdot \lambda_C = \mathrm{h} \Rightarrow \Delta\mathrm{p}_c \cdot \Delta\lambda_C = \mathrm{h}$$

You think this looks good? I don't think so. Why not? Because our pc is *not* the classical $p = mv$ momentum, and that Compton wavelength is not the right length either. Why not? Well… We want to relate this to the linear motion. In other words, we want an equation involving the *classical* velocity. We don't have that here. Just write it all out:

$$\mathrm{p}_c = \frac{\mathrm{E}}{\lambda_C} \cdot \frac{h}{\mathrm{E}} = \frac{h}{\lambda_C} = \frac{h \cdot \mathrm{m} \cdot c}{h} = \mathrm{m} \cdot c$$

What kind of momentum is this? I'll let you think about it. I want to see something else, so let me try something here. If I multiply p_c by $\beta = v/c$, I do get the classical momentum $p = m \cdot v$. Let's see what we get if we write this:

[10] You will often see ħ/2 or ħ on the right-hand side, as opposed to h. However, that just depends on what formula you're going to use to describe the *spread* in your variables. The expressions I use are the ones Dirac and Heisenberg used.

[11] Energy is a force over a distance and, hence, we substitute F for E/λ_C here.

$$\mathrm{p}_c \cdot \lambda_C = \mathrm{h} \Leftrightarrow \mathrm{p}_c \cdot \beta \cdot \lambda_C \cdot \frac{1}{\beta} = \mathrm{h} \Leftrightarrow \mathrm{p} \cdot \lambda_C \cdot \frac{1}{\beta} = \mathrm{p} \cdot \lambda_\mathrm{L} = h$$

That's better! We get the second *de Broglie* equation. In fact, it's *the* de Broglie equation – because the first *de Broglie* equation is just the Planck-Einstein relation, so *de Broglie* didn't contribute much to that.

So now we get what we had secretly hoped to come out of this rather weird discussion on the geometry of an electron:

$$\mathrm{p} \cdot \lambda_\mathrm{L} = h \Rightarrow \Delta\mathrm{p} \cdot \Delta\lambda_\mathrm{L} = h$$

This, then, should guide our interpretation of the $\Delta\mathrm{p}\cdot\Delta x = h$ expression of Heisenberg's Uncertainty Principle. You'll say: sure, we knew that already, didn't we? No. We didn't. The Δx is not an uncertainty in the *position*. It's an uncertainty in the *de Broglie* wavelength, and it's rather hard to relate to that to the position, as we'll explain in the next sections.

Planck's quantum of action as a vector

I have a not-so-secret agenda: I want to think of h as a vector, so I want to write it as **h** (in **bold face**). I feel that should not sound too outrageous because the physical dimension of Planck's quantum of action is that of angular momentum, and angular moment is a vector, so why not think of h as a vector too?

Of course, you may say angular momentum is an *axial* vector and, therefore, perhaps not all that real. We'd like to counter that objection by the following rather subjective statement: an axial vector does *not* reverse its sign when the coordinate system is changed to a new system by a reflection in the origin. In light of those weird 720-degree symmetries that pop up when analyzing spin-1/2 particles, I'd say that property makes an axial vector *very* real – perhaps even more *real* than the polar vectors that enter as factors into the vector cross-product. However, we don't want to engage in philosophy here so let us play some more and see what might make sense.

Modeling uncertainty

Before thinking of Planck's quantum of action as a vector quantity, we should, perhaps, first try to write the $\Delta\mathrm{p}\cdot\Delta x = h$ expression of the Uncertainty Principle as a vector (dot) product:

$$\mathbf{\Delta p}\cdot\mathbf{\Delta x} = \mathrm{h}$$

We use an equality sign here because we think there must be certainty in the uncertainty: something must explain the probabilities.[12] Hence, we write the right-hand side of the equation – currently, that is – as h. No uncertainty there. All of the uncertainty is in the left-hand side. So how should we think of it? **Δp** is the uncertainty in **p**. Both **Δ** as well as **p** are written in **bold type**. We think of it as an entity: a vector in its own. If there is uncertainty in the momentum **p**, the uncertainty may be in the *magnitude* of **p**, but it may also be in its *direction*. Hence, **Δp** can be any of the vectors depicted below – or anything in-between.

[12] If you have read any of my papers, you'll know I am just faithfully following Einstein's basic intuitions. This is *not* the Copenhagen interpretation of quantum mechanics. In fact, the only reason why you want to read this paper is that it's not.

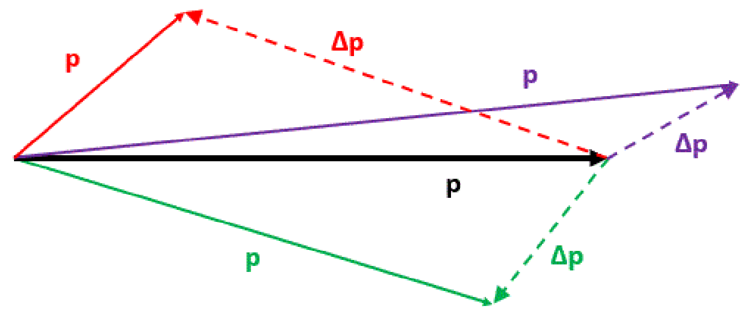

Figure 4: Uncertainty in the linear momentum

Note that the various Δp vectors here can point in *any* direction, and they also can have *any* magnitude. That is a bit weird because – as you can see from the illustration – the momentum vectors themselves – the **p** vectors, that is – are generally in the same direction: we do think that, if a particle has *some* momentum, it will in *some* direction. That's just logic.

But now we really run into trouble when interpreting Δ*x* as some uncertainty in the position. The Δp·Δ*x* = *h* relation gives us an *inverse* proportionality between Δp and Δ*x*, and that doesn't make sense. The uncertainty in the momentum will *amplify* the uncertainty in the position. I should find a better illustration but the one below should already give you an idea. If we have some uncertainty ε in the direction of the momentum of some object (the red billiard ball below, that is), then that uncertainty will be amplified over a longer distance.

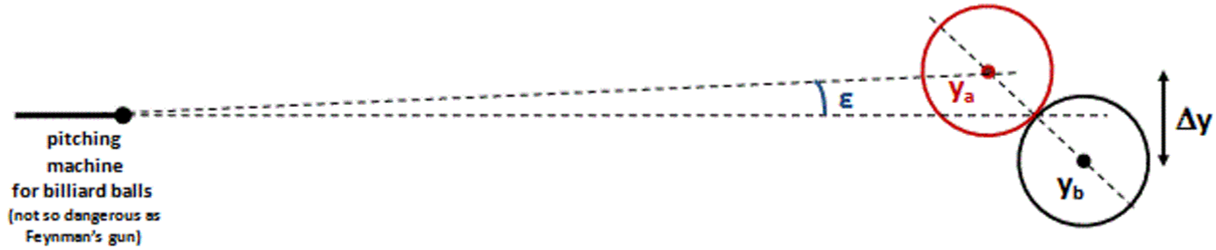

Figure 5: Uncertainty in (linear) momentum gets amplified over distance

Why is that so? The momentum, velocity and position vectors (**p**, ***v*** and ***x***) all have the same direction. Indeed, **p** and ***v*** only differ because of the mass factor: **p** = m·***v***. And ***v*** and ***x*** are, in turn, related by the simplest of equations: the **x** = ***v***·*t.* Hence, their direction is the same – by *definition*: an object with some velocity ***v*** (and some mass m) will be at position ***x*** = ***v***·*t* at any point in time *t*. That is just the way we make sense of the world. Hence, that weird inverse proportionality in the Δp·Δ*x* = *h* expression cannot be explained if we interpret Δ*x* as the *spread* in the position: they should be proportional[13], rather than inversely proportional (as illustrated above and below).

[13] The proportionality relation is given by simple trigonometry: if the angle ε measures the spread, and *c* is the hypotenuse, while *a* and *b* are the adjacent and opposite side of the triangle, then b will be equal to *b* = *c*·*sin*ε.

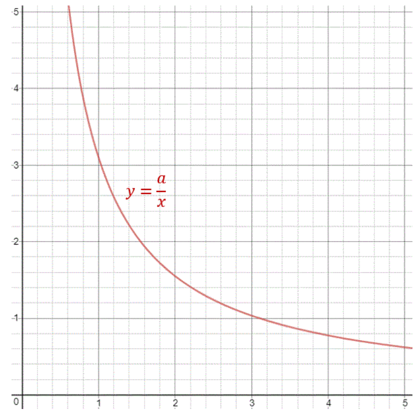

Figure 6: Inverse proportionality

In short, the *second* expression of the Uncertainty Principle should, effectively, be written in terms of the uncertainty in the (classical) momentum and the uncertainty in the (*de Broglie*) wavelength of our electron:

$$\Delta p \cdot \Delta\lambda_L = h$$

Planck's quantum of action as a vector (2)

Let me get back to my not-so-secret agenda: what would it mean to write *h* as a vector? Well… It would write it as ***h***, just like we would write **L** for angular momentum. Look at the wobbling angular momentum of the spinning top below, and then think of the plane of rotation of our pointlike charge in Figure 1).

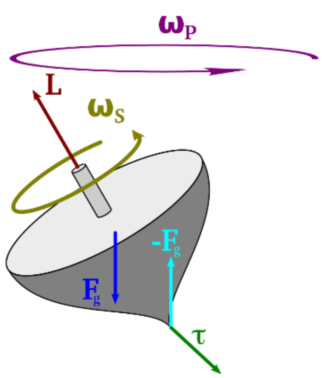

Figure 7: A wobbling plane of rotation[14]

So how would we write the Uncertainty Principle then? Simple, we just write **h** and **p** as vectors. So we have the **p**·λ = **h** expression out of the zbw geometry (with λ the *de Broglie* wavelength[15]). So then we can introduce the idea of a non-precise magnitude, or a non-precise direction and write:

[14] The illustration is taken from Wikimedia Commons under the CC-BY-SA 2.5 license so I need to acknowledge the author: Xavier Snelgrove (https://commons.wikimedia.org/wiki/User:Wxs).
[15] I dropped the subscript here. You may want to add it again if you're getting confused.

$$\Delta\mathbf{p}\cdot\Delta\lambda = \mathbf{h}$$

So I have **p** and **h** as vectors then: **h** wobbles as the electron gains linear momentum, so **p** wobbles too.

[…]

So… Well… We're there. There is only one question left to answer: the *precession* of the spinning top in the illustration above is regular. The movement is regular, and we can associate it with some *precise* precession frequency. Can we do that for an electron? In other words, we have uncertainty – and we can model it – but do we introduce some uncertainty in our model?

My instinctive answer to this is Einstein's: no. The velocity of light is the velocity of light, and the classical velocity of our electron is the classical velocity of our electron. In other words, if we assume the rotational motion is *defined* by *c* and the Compton radius which, in turn, is defined by the *linear* velocity *v*, then there's no room for uncertainty. The uncertainty is the wobbling of ***h***. Nothing more. Nothing less. But I am happy to get other opinions. They'd be philosophical rather than scientific – as philosophical as Einstein's intuition – but that shouldn't prevent us from thinking creatively.

I would like to add one final point. Could we possibly verify this interpretation of Heisenberg's Uncertainty Principle? I haven't given this all that much thought, but I think we should be able to do so. Indeed, if my memory is correct, then the (tiny) spread around the two spots where the electrons hit the detector in a Stern-Gerlach experiment are explained by the (tiny) spread in their (classical) velocity. So, yes, I think that might prove the point.[16]

Uncertainty in particle interactions

What is presented above does *not* say anything about interactions. Needless to say, if we are going to probe the position of some particle, we'll need to use some other particle or – more likely – light. That will disturb the original state of motion the particle. For example, when a photon is used to probe an electron, the electron may pick up some kinetic energy and/or change its direction of motion.

As such, we may think of the uncertainty being transferred or shared as a result of an *interaction* between the particle that's being probed and the particle that probes it. Of course, if the particle that's being probed is sufficiently massive, then the uncertainty won't matter all that much.

A rather obvious but interesting question is whether or not a photon could possibly interact with another photon: can they collide, so to speak? They can: light-by-light scattering has first been observed in the ATLAS detector of CERN's Large Hadron Collider (LHC) in 2016, it seems.[17]

[16] Note that actual Stern-Gerlach experiments are usually done with atoms or ions rather than electrons. I am actually not aware of any Stern-Gerlach experiment with electrons, and would be interested to hear about them.
[17] See: https://en.wikipedia.org/wiki/Two-photon_physics.