Refutation of logical theory based on compatible consequence in set theory

Abstract: We evaluate canonically logical compatibility relations (CM) and complements (CR_S), each in three sets of definitions. None is tautologous, so we avoid the subsequent ten relations. This refutes the "the possibility of a notion of compatibility that allows either for glutty or gappy reasoning". (By extension, paraconsistent logic is rendered untenable.) Therefore the bivalent standard notion of formal theory in logic is confirmed as allowing both assertion and denial as equally valid. In fact, this refutation further disallows injection of a bilateralist approach on many dimensions. This also indirectly reiterates that set theory is *not* bivalent, and hence derivations therefrom, such as the instant relations, are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Blasio, C.; Caleiro, C.; Marcos, J. (2019).
What is logical theory? On theories containing assertions and denials.
arxiv.org/pdf/1903.02338.pdf jmarcos@dimap.ufrn.br

Remark 0: The reproduction below captures use of italic and single quote within the expository narrative only.

1 Capturing the notion of logical consequence

Let L be a non-empty set of *sentences*. We will assume that the judgments of *assertion* and *denial* are primitive in our metalanguage, and in what follows we will intuitively think of the *consecution* (Δ_1, Δ_0) as a meta-logical expression concerning the 'compatibility' of certain judgments, namely, the assertion of all sentences in $\Delta_1 \subseteq L$ and the denial of all sentences in $\Delta_0 \subseteq L$. Building on that idea, a (*canonical logical*) *compatibility relation* (*on* L) will be here defined as any relation ▸ on ℘(L) × ℘(L) satisfying, for every Π, Π', Σ, Σ', Δ ⊆L:

LET p, q, r, s, t, u, v: Π, Π', Σ, Σ', L, Δ, Δ';
▸ Imply; ▹ Not Imply (the complement of Imply).

(CM0) if $\Pi' \cup \Pi \blacktriangleright \Sigma \cup \Sigma'$, then $\Pi \blacktriangleright \Sigma$

```
~(#t<#u)>(((#q+#p)>(#r+#s))>(#q>#r)) ;
                                        TTTT TTTT TTCC TTTT(1),
                                        TTTT TTTT TTTT TTTT(1),
                                        TTTT TTTT TTCC TTTT(3),
                                        TTTT TTTT TTTT TTTT(1),
                                        TTTT TTTT TTCC TTTT(2)
```
(CM0.2)

(CM1) if $\Pi \blacktriangleright \Sigma$, then $\Pi \cap \Sigma = \varnothing$

```
~(#t<#u)>(((#p>#r)>(#p&#r))=(z@z)) ;
                                        TCTC TCTC TCTC TCTC(1)
                                        TTTT TTTT TTTT TTTT(1),
                                        TCTC TCTC TCTC TCTC(3)
                                        TTTT TTTT TTTT TTTT(1),
                                        TCTC TCTC TCTC TCTC(2)
```
(CM1.2)

(CM2) if $\Pi \blacktriangleright \Sigma$, then there is some $\Delta' \subseteq \Delta$ such that $\Delta' \cup \Pi \blacktriangleright \Sigma \cup (\Delta \setminus \Delta')$

```
~(#t<#u)>
(((#p>#r)>%(~(#u<#v)=(z=z)))>((#v+#p)>(#r+(#u\#v)))) ;
                                        TTTT TTTT TTTT TTTT(6),
                                        CCCC TTTT CCCC TTTT(2)
```
(CM2.2)

The reading of (CM0) is immediate: in any state of affairs in which a certain set of sentences $\Delta_1 = \Pi' \cup \Pi$ is asserted while a certain set of sentences $\Delta_0 = \Sigma \cup \Sigma'$ is denied, one may in particular say that all subsets of Δ_1 are asserted and that all subsets of Δ_0 are denied. Furthermore, on the one hand, taking $\Pi = \Sigma = \{A\}$, property (CM1) says that the sentence A may not be simultaneously asserted and denied; on the other hand, taking $\Delta = \{A\}$, property (CM2) says that the sentence A must be either asserted or denied (in a context where the sentences in Π are asserted and those in Σ are denied). One might say thus that (CM1) provides a meta-logical formulation of the 'Principle of Non-Contradiction', and disallows for *glutty* states of affairs in which a sentence is simultaneously asserted and denied: In any given (consistent) state of affairs, asserting a given sentence A should not be compatible with denying it. Dually, one might say that (CM2) provides a meta-logical formulation of the 'Principle of Excluded Middle', and disallows for *gappy* states of affairs in which a sentence is neither asserted nor denied: In no state of affairs can a sentence A fail to be either asserted or denied.

The complement $\triangleright$ of a compatibility relation $\blacktriangleright$ on $\wp(L) \times \wp(L)$ will here be called an S-*consequence relation* (*on* L). It should be clear that it satisfies the following properties, for every $\Pi, \Pi', \Sigma, \Sigma', \Delta \subseteq L$:

(CR_S0) if $\Pi \triangleright \Sigma$, then $\Pi' \cup \Pi \triangleright \Sigma \cup \Sigma'$

```
~(#t<#u)>((#p<#r)>((#q&#p)<(#r+#s))) ;
```

```
TCTT TTTT TCTC TTTT(1),
TTTT TTTT TTTT TTTT(1),
TCTT TTTT TCTC TTTT(3),
TTTT TTTT TTTT TTTT(1),
TCTT TTTT TCTC TTTT(2)
```
($CR_S0.2$)

(CR_S1) if $\Pi \cap \Sigma \neq \emptyset$, then $\Pi \triangleright \Sigma$

~(#t<#u)>(((#p&#r)=(z@z))>(#p<#r)) ;

```
FNFN FNFN FNFN FNFN(1),
NNNN NNNN NNNN NNNN(1),
FNFN FNFN FNFN FNFN(3),
NNNN NNNN NNNN NNNN(1),
FNFN FNFN FNFN FNFN(2)
```
($CR_S1.2$)

(CR_S2) if $\Delta' \cup \Pi \triangleright \Sigma \cup (\Delta \setminus \Delta')$ for every $\Delta' \subseteq \Delta$, then $\Pi \triangleright \Sigma$

~(#t<#u)>
(#(~(#t<#v)=(z=z))>(((#v+#p)<(#r+(#u\#v)))>(#p<#r))) ;

```
TTTT TTTT TTTT TTTT(6),
CTCT TTTT CTCT TTTT(2)
```
($CR_S2.2$)

None of the equations is tautologous, with conclusions in the abstract.