Solution proof of Bellman's Lost in the forest problem for triangles

Abstract: From the area and dimensions of an outer triangle, the height point of an inner triangle implies the minimum distance to the outer triangle. This proves the solution of Bellman's Lost in the forest problem for triangles. By extension, it is the general solution proof for other figures.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ⤚ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Williams, S.W. (2000). Million buck problems.
math.buffalo.edu/~sww/0papers/million.buck.problems.mi.pdf sww@buffalo.edu [bounced]

> 12. **Lost in a Forest Problem:** In 1956 R. Bellman asked the following question: Suppose that I am lost without a compass in a forest whose shape and dimensions are precisely known to me. How can I escape in the shortest possible time? Limit answers to this question for certain two-dimensional forests; planar regions. ... For many plane regions the answer is known: circular disks, regular even sided polygonal regions, half-plane regions (with known initial distance), equilateral triangular regions. However, for some regions, for regular odd-sided polygonal regions in general and triangular regions in particular, only approximates to the answer are known.
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> R. Bellman, *Minimization problem*. Bull. Amer. Math. Soc. **62** (1956) 270.
> J. R. Isbell, *An optimal search pattern*. Naval Res. Logist. Quart. **4** (1957) 357-359.
> Web survey and reference article: http://www.mathsoft.com/asolve/forest/forest.html
> [The link above as published maps to ptc.com which apparently hijacked that link.]

We proceed to define a triangle area for A = base * height / 2. (1.1.0)

LET
p left-side base to height-r point,
q right-side base from height-r point,
r height-r point,
s height-s point,
t left-side base to height-s point,
u right-side base from height-s point.

t>p: s = s+(r-s)/2

```
                r
                |
                |      s
                |      |
._______p_______.______|_______q________________.
._t____________________.___u____________________.
```

t>p: s = s+(r-s)/2

```
                r
                |
                |
                |                     s
._______p_______.______________q______|_________.
._t___________________________________.___u_____.
```

t<p: t = t-(p-t)/2

```
                r
                |
            s   |
            |   |
._______p___|___.______________q________________.
._t_________.______________u____________________.
```

t<p: t = t-(p-t)/2

```
                r
                |
                |
     s          |
.____|__p_______.______________q________________.
._t__._____________________u____________________.
```

We map Eq. 1.1.1 with height-r point as the larger area triangle equivalent to the two smaller area triangles. (1.1.1)

((p+q)&(r\(%z<#z)))=((p&(r\(%z<#z)))+(q&(r\(%z<#z)))) ;

`TTTT TTTT TTTT TTTT(128)` (1.1.2)

The height-s triangle inside the outer triangle, maps respectively into two smaller triangles. (1.2.1)

((t+u)&(s\(%z<#z)))=((t&(s\(%z<#z)))+(u&(s\(%z<#z)))) ;

`TTTT TTTT TTTT TTTT(128)` (1.2.2)

The bases of height-r and height-s triangles are equivalent, so we define the base of the larger height-r triangle in terms of the base for the smaller height-s triangle, and vice versa. The facts that height-s is less than or equal to height-r, and that base t is lesser than or equal to base p+q are mapped as the antecedent below. (1.3.1)

(~(r<s)&~((p+q)<t))>(((p+q)=(t+u))>(((p=((t+u)-q))&
(q=((t+u)-p)))=((t=((p+q)-u))&(u=((p+q)-t))))) ; TTTT TTTT TTTT TTTT(128) (1.3.2)

Assuming Eq. 1.3.2, for the inside height-s triangle components, its triangle with the smaller area implies the shortest path to the outside height-r triangle. The shortest path to the edge of the height-r triangle is then the smaller value of s or t, with the direction as vertical for s or horizontal for t. (1.4.1)

((~(r<s)&~((p+q)<t))&
(((p+q)=(t+u))>(((p=((t+u)-q))&(q=((t+u)-p)))=((t=((p+q)-u))&(u=((p+q)-t))))))>
(((t<p)>(t=((u-p)\(%z<#z))))+(~(p<t)>(s=(s+((r-s)\(%z<#z)))))) ;
TTTT TTTT TTTT TTTT(128) (1.4.2)

Bell's forest problem is solved for triangles with the Eq. 1.4.2 as tautologous.