Some Problems About The Electron-Pair Theory in Superconductor

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Abstract-We review the superconductor theory based on the electron pair first. Then several viewpoints are discussed and concluded that such electron pair is not stable in the superconducting state. The speed of each electron in the electron pair is about $2.02\text{x}10^6$ *m/s* in Al. However, the longitudinal and transverse speeds of the crystal waves in Al is merely $6.47\text{x}10^3$ *m/s* and $3.40\text{x}10^3$ *m/s* in [100] direction, respectively. It is almost impossible that the mediated phonon can real-time transfer momentum and energy between two so high-speed and inverse-momentum electrons in the superconducting state. The more possible process is that each electron can absorb other phonons propagating from any place in the crystal. The best condition for the electron pair is total zero momentum but how to make such electron pair stably conduct electric current is a problem because one of them is accelerated and the other is decelerated applied the external electric field. In conclusion, the electron pair is not physical but just the quasi-physical process and the mediated phonon is a virtual one for calculating the second-order perturbation.

Keywords: electron pair, superconductor, mediated phonon

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1. INTRODUCTION

Superconductor is a very special material. Since the first superconductor, Hg, was found with the critical temperature T_c of 4.153 K in 1911 [1], it has attracted a lot of researcher to study the physical properties and its performances. In the superconducting state, Meissner effect exhibits the magnetic field B=0 inside a bulk superconductor [2-4]. According to magnetization, the superconductor can be divided into two kinds of type I and type II [2-4]. Nowadays, some high-temperature superconductors have already been investigated, like $YBa_2Cu_3O_7$ with T_c=92 K [4,5] and $Tl_2Ba_2Ca_2Cu_3O_{10}$ with T_c=125 K [3,4,6]. It shows the progress in superconductor and its applications have been widely extended to photonics and proposed recently [7-18]. How to explain its special physical phenomenon is still a developed region even it has passed one century.

2. REVIEW FOR THE ELECTRON-PAIR THEORY

The traditional theory considers the entropy of the superconductor contributed from the normal electrons which are excited and separated from the superconducting electron pairs. However, these excited normally electrons should cause some resistance as the electrons in the common metals because of inelastic scatterings and energy dissipation. We might ask why the resistance is almost zero in the superconductor experiments? When we study the electron-pair theory, it naturally implies the normal electron current also existing in the superconductor. In the following, the electron-pair theory is briefly reviewed [5,6,19]. The energy $E_{\vec{k}}$ of single electron with wavevector $\vec{k}$ in the excited state is

$$E_{\vec{k}} = \sqrt{\varepsilon_{\vec{k}}^2 + \Delta^2} \tag{1}$$

where Δ is the binding energy of an electron pair and the energy $\varepsilon_{\vec{k}}$ of quasi-electrons related to the Fermi energy ε_F is

$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m_e^*} - \varepsilon_F, \tag{2}$$

where m_e^* is the effective electron mass and $\hbar$ is equal to $h/2\pi$ with h the Planck's constant. Its density of stat $N_s(E_{\vec{k}})$ is

$$N_s(E_{\vec{k}}) = N(E_{\vec{k}})\frac{d\varepsilon_{\vec{k}}}{dE_{\vec{k}}} = N(E_{\vec{k}})\frac{E_{\vec{k}}}{\sqrt{E_{\vec{k}}^2 - \Delta^2}} \tag{3}$$

when $|E_{\vec{k}}| > \Delta$, and

$$N_S(E_{\vec{k}}) = 0 \tag{4}$$

when $|E_{\vec{k}}| < \Delta$. So the normally electron current in superconductor $\vec{J}_{sn}$ is

$$\vec{J}_{sn} = -\int_{\Delta}^{\infty} eN_S(E_{\vec{k}})\, f(E_{\vec{k}})\vec{v}_{\vec{k}} dE_{\vec{k}} \tag{5}$$

where $f(E_{\vec{k}})$ is the Fermi-Dirac distribution, $-e$ is the electron charge, and $\vec{v}_{\vec{k}}$ is the electron velocity at wavevector $\vec{k}$. It is similar to the electric current $\vec{J}$ under the applied electric field $\vec{E}$ in the traditional metal which has the form

$$\vec{J} = -n\frac{e^{*2}}{m_e^*}\tau\vec{E}, \tag{6}$$

where n is the average charge density, e^* is the effective charge, , and τ is the average scattering time. The resistance R has relationship with τ, that is

$$R \propto \frac{1}{\tau}. \tag{7}$$

It means that if the experiments show almost zero resistance, the normally excited electron would have very large τ like a quasi-superconducting electron. It further makes us to think about the role of the electron pair. Do we really need the concept of the electron pair? In the following, we focus on this problem.

In solid state physics, the bulk material consists of a lot of atoms and electrons where the conducting electron is treated as a quasi-electron with effective charge and mass due to the shielded Coulomb effect and the periodical structure in crystal. The Hamiltonian for this conducting electron system with the locally moved atoms is [5,19,20]

$$H = H_{\substack{free \\ electron}} + H_{\substack{Coulomb \\ interaction}} + H_{\substack{free \\ phonon}} + H_{\substack{electron-phonon, \\ interaction}} \tag{8}$$

where

$$H_{\substack{free \\ electron}} = \sum_{\vec{k},s} \varepsilon_{\vec{k},f}\, c^{+}_{\vec{k},s} c_{\vec{k},s}, \tag{9}$$

$$H_{\substack{Coulomb \\ interaction}} = \frac{1}{2} \sum_{\substack{\vec{k},\vec{k}',\vec{k}'' \\ s,s'}} V_{k''}\, c^{+}_{\vec{k}'-\vec{k}'',s'} c^{+}_{\vec{k}+\vec{k}'',s} c_{\vec{k},s} c_{\vec{k}',s'}, \tag{10}$$

$$H_{\substack{free \\ phonon}} = \sum_{\vec{q}} \hbar\omega_{\vec{q}} \left(a^{+}_{\vec{q}} a_{\vec{q}} + \frac{1}{2} \right), \tag{11}$$

$$H_{\substack{electron-phonon \\ interaction}} = \sum_{\substack{\vec{k},\vec{q} \\ s}} g_q \left(a_{\vec{q}} + a^{+}_{-\vec{q}} \right) c^{+}_{\vec{k}+\vec{q},s} c_{\vec{k},s}, \tag{12}$$

where the free electron energy is

$$\varepsilon_{\vec{k},f} = \frac{\hbar^2 k^2}{2m}, \tag{13}$$

and the electron-phonon coupling is

$$g_q = \frac{4\pi e^2}{q} \sqrt{\frac{N\hbar}{2\omega_q M}}. \tag{14}$$

In above equations, $a^{+}_{\vec{q}}$ and $a_{\vec{q}}$ are the creation and annihilation operators for phonon, $c^{+}_{\vec{k}}$ and $c_{\vec{k}}$ are the creation and annihilation operators for electron, s is the electron spin, $\vec{q}$ is the wavevector for phonon, $\omega_{\vec{q}}$ is the phonon frequency, M is the mass of an ion, and N is the number of Wigner-Seitz cells in a crystal. Eqs. (9) to (12) describe a process that an electron is scattered from the initial state $\vec{k}$ to the final state $\vec{k}'$ through the emission or absorption of a phonon with the wavevector $\vec{q}$ and the frequency $\omega_{\vec{q}}$.

3. DISCUSSIONS OF THE ELECTRON-PAIR THEORY

Let's review the main concept of the electron-pair theory. It consists of two electrons in the range about $k_B T_c$ above the Fermi level ε_F where k_B and T_c are the Boltzman constant and the superconducting transition temperature, respectively. These two electrons combine with each other due to the exchange of phonon and have total energy slightly lower than the sum of energy of the two free electrons. The phenomenon is considered physically that one electron is attracted near the positive ions causing the deformation of the lattice and the creation of a phonon. Then another electron is affected by this deformation and absorbs a phonon. This phenomenon is originally so-called the electron-phonon interaction. It is a second-order perturbation and doesn't appear explicitly in the Hamiltonian. However, through complicated canonical transformation [20], the original one can transfer to another form as

$$H_{transfer} = H_{\substack{quasi\\electron}} + H_{\substack{shielded\\Coulomb\\interaction}} + H_{\substack{dressed\\phonon}} + H_{\substack{electron-phonon-electron\\interaction}}, \quad (15)$$

where

$$H_{\substack{quasi\\electron}} = \sum_{\vec{k},s} \varepsilon_{\vec{k}} c^{+}_{\vec{k},s} c_{\vec{k},s}, \quad (16)$$

$$H_{\substack{dressed\\phonon}} = \sum_{\vec{q}} \hbar\omega_{\vec{q}} \left(a^{+}_{\vec{q}} a_{\vec{q}} + \frac{1}{2}\right), \quad (17)$$

$$H_{\substack{shielded\ Coulomb\\interaction+\\electron-phonon-electron\\interaction}} = \frac{1}{2}\sum_{\substack{\vec{k},\vec{k}',\vec{q}\\s,s'}} V_{\vec{k},\vec{q}}\, c^{+}_{\vec{k}'-\vec{q},s'} c^{+}_{\vec{k}+\vec{q},s} c_{\vec{k},s} c_{\vec{k}',s'}, \quad (18)$$

$$V_{\vec{k},\vec{q}} \equiv V_{\vec{k}+\vec{q},\vec{k}'-\vec{q},\vec{k},\vec{k}'} = \frac{4\pi e^2}{q^2+\lambda^2} + \frac{2\hbar\omega_{\vec{q}}\left|M_q\right|^2}{\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}\right)^2 - \left(\bar{h}\omega_{\vec{q}}\right)^2} \quad (19)$$

and

$$\lambda = \sqrt{\frac{6\pi n e^2}{\varepsilon_F}} \quad (20)$$

is the shielded parameter for the shielded Coulomb interaction where n is the electron density. In Eq. (16), $\varepsilon_{\vec{k}}$ is the quasi electron energy in Hartree-Fock approximation and defined in Eq. (2). The M_q is proportional to the shield electron-phonon interaction and $\omega_{\vec{q}}$ is the normalized phonon frequency. Such phonon causes weak attraction and this kind of electron pair is in the bound state. The best condition for the electron pair is that their momentum $\vec{k}$ and spin s are equal and antiparallel, and the two electrons move in the opposite directions. From these descriptions, we might ask whether it is a physical picture or just a quasi-physical one?

For example, aluminum (Al) has the superconducting transition temperature of 1.2 K [5] and its corresponding Fermi energy ε_F is 11.63 eV [1] of which the corresponding speed v_F for the bare electron is about

$$v/c = \sqrt{2\varepsilon_F/511000eV} = \sqrt{23.26/511000} \sim 1/148.0, \tag{21}$$

where c is the speed of light in the free space and the bare electron mass m_e is 0.511MeV/c^2 or 9.1x10^{-31} kg. It means that the speed of electron is very high and about 2.02x10^6 m/s or (1/148) c [1]. However, the propagation of the lattice wave, relying on the vibration of ions, is not as so high as the electron speed. The mediated phonon between two electrons is the acoustic phonon and the crystal wave can be approximated to the continuously elastic wave. For Al, the elastic stiffness constant C_{11} is 1.143x10^{12} dyne/cm^2 and the density ρ is 2.733 g/cm^3 at 0 K [1]. Considering the wave propagating in [100] directions, the speed of the longitudinal wave v_{LA} is

$$v_{LA} = \sqrt{\frac{C_{11}}{\rho}} = 6.47 \times 10^3 \ m/s. \tag{22}$$

This speed is about three hundred times slower than that of Fermi electron. Then we calculate the speed of the transverse wave v_{TA} is

$$v_{TA} = \sqrt{\frac{C_{44}}{\rho}} = 3.40 \times 10^3 \ m/s, \tag{23}$$

The speed of Fermi electron is about 600 times faster than v_{TA}.

When we consider the electron-pair picture, it necessarily estimates the average distance between two electrons. The electrons of each electron pair are within the range about k_BT above the Fermi energy and the density of these electrons is estimated

$$nk_BT_c/\varepsilon_F \sim 10^{29} \cdot 10^{-5}\ \mathrm{m}^{-3} = 10^{24}\ \mathrm{m}^{-3} \tag{24}$$

or 10^6/μm^3 for Al [1]. It means that this kind of electron above ε_F roughly occupies a cubic with each side about 100 Å so the average distance between two electrons is about 100 Å or roughly 30~40 atoms. Such two-electron pair is not very close, and it has to cross several ten atoms to transfer the changes of the momentum and energy from one electron to the other. When the superconducting electrons are described by the nearly-free-electron models and the mediated phonon takes responsibility for the transfers of momentum and energy, it immediately causes a further problem why and how such the electron pair can be so very stable in the superconducting state? How is it possible for the mediated phonon real-time transferring momentum and energy from one electron to the other in the electron pair?

4. RESULTS AND DISCUSSION

If one electron can absorb a phonon with the changes of momentum and energy from the other, these ions have to response very quickly. Actually, it is almost impossible that two electrons can have enough time to exchange a phonon between them when both electrons have so high speed and the average distance is about 100 Å. The electron much more possibly absorbs additional momentum and energy from other electron-electron or electron-ion scattering events. In fact, this electron-pair theory only focuses on the superconducting contributions from all electron pairs but ignores the effects from other electron-phonon interactions. The phonon is not only created between two electrons in one electron pair, but also from other electron pairs and those thermal vibrations of ions. The phonon is the quantization of the lattice wave and this wave is the collective excitation. It means that the wave can propagate through the crystal and each electron can absorb or scatter phonons originally from any other place. However, such phonons in the electron-pair theory cannot propagate globally and is limited to exist locally because their existence is only in charge of transferring momentum and energy between two electrons. It seems unreasonable because each electron can have probability to scatter or absorb phonons from any places in the superconductor. It is an open system in the superconductor. This electron-phonon interaction in the electron pair necessarily finishes in a very short time as mentioned above. However, the crystal waves propagate much slower than the Fermi electrons and they cannot real-time response the transfers of momentum and energy between two high-speed electrons, so the stable electron-pair picture doesn't exist physically.

We further use some pictures to illustrate our viewpoints. The original concept for the electron pair is described in Fig. 1(a) where one electron emits a phonon with momentum $\vec{q}$ and this electron momentum is changed from $-\vec{k}$ to $-\vec{k}-\vec{q}$, and the other electron absorbs this phonon and its momentum is changed from $\vec{k}$ to $\vec{k}+\vec{q}$. However, this process is non-conservation of energy and the mediated phonon is virtual. Actually, the really possible processes in the real world are shown in Figs. 1 (b) and (c) as described in Eq. (12). One electron emits a real phonon and changes its momentum from $-\vec{k}$ to $-\vec{k}-\vec{q}$ but the other electron doesn't be affected by this phonon and keeps its momentum at $\vec{k}$. This possible process is shown in Fig. 1(b). Another possible process shown in Fig. 1(c) is that the momentum of two electrons is changed due to the absorptions of two different phonons. One electron emits a real phonon resulting in its momentum changed from $-\vec{k}$ to $-\vec{k}-\vec{q}$, and the other one absorbs another real phonon and its momentum is changed from $\vec{k}$ to $\vec{k}+\vec{q}'$.

We can use another way to discuss this dynamical problem. Although the wave function of each electron in the electron pair is treated the same as free electron, it is actually the Bloch wave in the crystal and the free-electron wave function is just an approximation. According to the Pauli's principle, when each quantum state only depends on $\vec{k}$ and *s*, it can be occupied by two electrons. At the best condition each electron pair has total zero momentum and spin initially. Once these two electrons absorb additional phonons and have numerically unequal momentum, then each electron has to search other electrons with inverse momentum and spin in order to meet the best condition in superconductor. The sample often has dimension in *cm*. The two electrons might separate in an order of *cm*. It also needs a certain time to search other electron by a mediated phonon and reach the best condition. This mediated phonon doesn't know where to meet the other electron. However, it would cause the superconducting state disappear locally. To avoid this situation, the possible way is that two new combined electrons in the new electron pair have numerically unequal momentum and then the most electron pairs don't hold the best condition. We consider a situation that all electron pairs initially exist in the best condition, then the system will quickly enter into a chaotic state after a lot of scattering events happen. Finally, it is very hard to find an electron pair possessing two electrons just with inverse momentum and spin. The best condition of the electron pair is very hard to hold for all time in the superconductor when the electron-electron and electron-phonon interactions continuously take place anywhere.

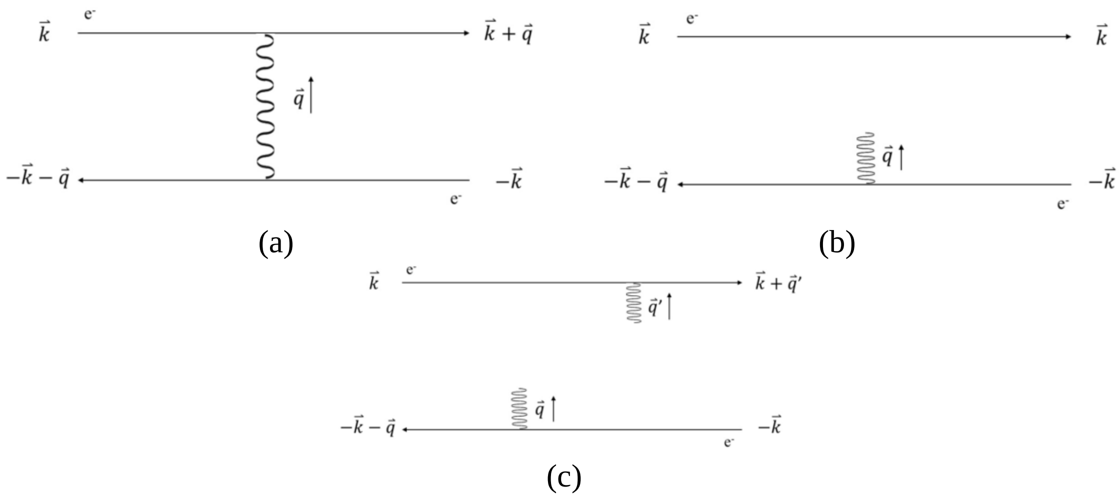

Fig. 1 (a) An original concept for two electrons 'bounded' by a phonon. However, this process is non-conservation of energy accompanied with a virtually mediated phonon. Actually, the speed of two electrons is too high to real-time transfer a real phonon between them. (b) This really possible process describes that one electron emits one phonon but the other electron doesn't absorb it. (c) Another really possible process for two electrons. Here one electron emits one phonon and the other electron absorbs another one. These two real phonons are different.

Since the electron pair is impossibly stable in superconductor, why the resistance is almost zero? Actually, each electron in Fig.1(b) or 1(c) encounters a lot of events about emitting and absorbing phonons, but each event causes the energy change ΔE very small and we can treat those events quasi-elastic scatterings. Finally, the average energy change for each conducting electron in the superconductor is almost zero, that is,

$$\langle \Delta E \rangle \approx 0, \tag{25}$$

so the conducting electron moves without energy loss in the superconductor.

The other question worthy to discuss is about the superconducting current applied the external electric field. Considering the dynamical process, the superconducting electrons are accelerated by the external electric field and no resistance exists. As discussions above, these two electrons in the same electron pair move in opposite directions and easily leave away each other. They are much hard at the best condition and the distance between two electrons quickly increase due to acceleration. We might ask how such electron pairs can stably conduct electron current? Furthermore, one of them would almost stop even move reversely because of deceleration.

5. CONCLUSIONS

In summary, the electron pair including two inverse momentum and antiparallel spin electrons cannot exist very stably in the superconductor. The exchange of a phonon between two electrons is not a real-time process and the speed of crystal waves is much slower than the speed of electrons above ε_F. Continuous long-distance exchange is impossible to hold for a long time because it has to cross several ten atoms and this process necessarily finishes in a very short time. The mediated phonon can propagate anywhere, not only exist between two electrons. Actually, two electrons can have much large probability to absorb other phonons. The phonon is the collective excitation and there are a lot of phonons with different frequencies coexisting in the system. One electron can change the energy and momentum from emitting or absorbing phonons no matter where these phonons originally come from in the superconductor.

Even under the applied electric field, the dynamic process still takes place continuously, and two electrons leave away from each other because one of them is accelerated and the other is deaccelerated. The best condition for an electron pair is the two electrons moving in the opposite directions. However, such electron pair cannot conduct the electric current completely and smoothly.

Acknowledgement

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