Are Imaginary Numbers Rooted in an Asymmetric Number System? The Alternative is a Symmetric Number System!

Espen Gaarder Haug*and Pankaj Mani†

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Abstract

In this paper, we point out an interesting asymmetry in the rules of fundamental mathematics between positive and negative numbers. Further, we show that there exists an alternative numerical system that is basically identical to today's system, but where positive numbers dominate over negative numbers. This is like a mirror symmetry of the existing number system. The asymmetry in both of these systems leads to imaginary and complex numbers.

We also suggest an alternative number system with perfectly symmetrical rules – that is, where there is no dominance of negative numbers over positive numbers, or vice versa, and where imaginary and complex numbers are no longer needed. This number system seems to be superior to other numerical systems, as it brings simplicity and logic back to areas that have been dominated by complex rules for much of the history of mathematics. We also briefly discuss how the Riemann hypothesis may be linked to the asymmetry in the current number system.

The foundation rules of a number system can, in general, not be proven incorrect or correct inside the number system itself. However, the ultimate goal of a number system is, in our view, to be able to describe nature accurately. The optimal number system should therefore be developed with feedback from nature. If nature, at a very fundamental level, is ruled by symmetry, then a symmetric number system should make it easier to understand nature than a asymmetric number system would. We hypothesize that a symmetric number system may thus be better suited to describing nature. Such a number system should be able to get rid of imaginary numbers in space-time and quantum mechanics, for example, two areas of physics that to this day are clouded in mystery.

Key words: Asymmetry, symmetry, imaginary numbers, quantum physics, Riemann hypothesis. Classification: 11-00 and 03Gxx

*Corresponding author e-mail espenhaug@mac.com, Norwegian University of Life Sciences. Thanks to Victoria Terces for helping me edit this manuscript.

†Independent researcher.

1 Imaginary Numbers: Was their "discovery" a sophisticated innovation to hide the fundamental inconsistency of algebra?

The early days of mathematics were only concerned with the arithmetic of positive numbers. Over time, ancient mathematicians conceived of negative numbers in their mental journey and in confronting such numbers, the rules for mathematical operations were adjusted. As we begin our own exploration of the properties of numbers and number systems, let's return to the foundation of mathematics again. What is positive and what is negative in context of arithmetic? These are basic conventions of directions along a number line, and are attached to some magnitudes like vectors. Positive and negative are relatively well-defined conventions with respect to each other, taking the origin on a basic x-y graph as the reference point, i.e., numeric '0'.

As the study of mathematics progressed through the Middle Ages, more complex operations and concepts, some of which had emerged in earlier periods, were given greater attention. Square roots of negative numbers, for example, appeared in Ars Magna by Girolamo Cardano in 1545, who considered several forms of quadratic equations (e.g., $x^2 + px = q, \quad px - x^2 = q, \quad x^2 = px + q$) simply to avoid using negative numbers. Cardano [1] said,

> *A second type of the false position makes use of roots of negative numbers. I will give an example: If someone says to you, divide 10 into two parts, one of which multiplied into the other shall produce 30 or 40, it is evident that this case or question is impossible. Nevertheless, we shall solve it in this fashion. This, however, is closest to the quantity which is truly imaginary since operations may not be performed with it as with a pure negative number, nor as in other numbers... This subtlety results from arithmetic of which this final point is, as I have said, as subtle as it is useless.*

Rafael Bombelli in his book Algebra (1572), was more comfortable with negative numbers and addressed the rules of handling the signed quantities in the following manner:

- Plus times plus makes plus
- Minus times minus makes plus
- Plus times minus makes minus
- Minus times plus makes minus

Gauss, who gave a proof of the Fundamental Theorem of Algebra (1799), said in 1825 that "*the true metaphysics of $\sqrt{-1}$ is illusive.*", see for example [2]. Yet by 1831, he had overcome his doubts with the application of complex numbers to number theory, which gave a tremendous boost to the acceptance of complex numbers in the mathematical community. Still, the acceptance was not universal. Augustus De Morgan, a famous mathematician and logician wrote in 1831,

> *The imaginary expression $\sqrt{-a}$ and the negative expression $-b$ have this resemblance, that either of them occurring as the solution of a problem indicates some inconsistency or absurdity. As far as real meaning is concerned, both are equally imaginary, since 0 - a is as inconceivable as $\sqrt{-a}$.* [1]

It is clear that historically there has been a lack of universal consensus among mathematically-oriented scientists. In this paper, we will examine this issue from both mathematical and philosophical perspectives. To begin, let us ask, "What does the square root of 4 mean" And in contrast, "What does the square root of (-1) mean at a fundamental level?"

The "square" term in square root was imported from geometry to arithmetic in ancient times. Square root itself means the length of each side of a square whose area might, for example, consist of 4 units. We know that the area of the square is equal to the length of one of the sides squared (and all sides are of equal length), so in this case, the length of each side is 2 units. Ideally, the sign assigned to the 2 here should correspond to the sign of the 4, whether it is positive or negative (we are mapping the square on an x-y graph here). If the area of square is positive with regard to the reference origin, then the length should be positive. Similarly, if the area of square is negative with regard to the reference origin, then the length should be negative. This is the logic from a real world example, i.e., $\sqrt{+\text{Number}} = +\text{Number}$ and $\sqrt{-\text{Number}} = -\text{Number}$. In fact, the sign is not a geometrical figure, so we cannot take the square root of that in a true sense.

So let's take a step back: will multiplying these things always lead to the square? For magnitudes it will be true, but what about signs? As a step towards the answer, we can see that the current system takes $\sqrt{-\text{Number}}$ as being"Imaginary" and hence violating the logic drawn from geometry.

Another basic rule in mathematics is $\sqrt{a}\sqrt{b} = \sqrt{a \times b}$. This is defined for positive numbers, a and b. However, if a and b are -1, for example, then assuming this system is thoroughly consistent, $\sqrt{-1}\sqrt{-1} = \sqrt{-1 \times -1} = \sqrt{1} = 1$. But with the definition of $\sqrt{-1} = i$, this leads to -1, not 1. This simple example shows how the earlier

[1] As cited in Kline, p. 593.

number system was actually incomplete. Then, in order to address this fundamental problem, mathematics was made inconsistent in the effort to make it complete.

Taking another simple example, the algebraic equation $x^2 + 4 = 3$, will lead to $x^2 = 3 - 4 = -1$. This indicates that, given the concept of our defined rules of squaring, multiplication, addition, and subtraction, there is no value of x possible here. So, we must either recheck and modify the incomplete assumptions made in the model/game or we must conclude that no such x exists in this system. Yet this truth has been hidden and forcibly made complete at the cost of introducing inconsistency and asymmetry into the system.

Confronted with this incompleteness in the rules of algebra defined for positive numbers, the long distant generation of mathematicians came up with $\sqrt{-1}$. They created a new convention to define this as an imaginary number (i). It is a nice trick, but it also hides the fundamental and foundational incompleteness in the earlier defined rules of algebra. It was an attempt to create a new system where the relative nature of sign/direction(negative) could be transformed into an absolute number by defining $\sqrt{-1}$ as (i) and building a new field of imaginary numbers.

We might ask, "Does the imaginary number try to model those non-observable aspects of Nature mathematically?" But again, if this could be achieved at all, it would do so at the cost of compatibility between the incompleteness and asymmetry in the existing numerical system and the symmetry at the depth of reality in Nature's own system. One might note that this tension between completeness and incompleteness has been considered by many great mathematical minds; one of the highlights of the past 100 years is seen in the form of the Göedel Incompleteness Theorem (1931), which is considered to be one of the most powerful mathematical results of the 20th century.

It is also worth mentioning there is an asymmetry in exponential rules, where $2^{-1} = \frac{1}{2}$, while $2^1 = 2$ and $-2^{-1} = -\frac{1}{2}$ and $-2^1 = -2$. In other words, the asymmetry is found in several of the basic rules of mathematics.

Returning to our own questions, the imaginary numbers are, in our view, created to make an asymmetrical system consistent. The output from such a number system, including imaginary numbers, can be difficult to interpret in terms of the physical world, especially when we are working at the deepest level of reality – the quantum level where we cannot even make direct observations, e.g., at the Planck scale. This means that such a numerical system can lead to strange and multiple interpretations, just as we see in mathematics and certain areas of physics today. This raises a fundamental question: Is this entire system of arithmetic consistent? Does it fall into line with Nature's symmetries and principles? We think not, but what kind of system might take its place?

2 Perfect Symmetry

In the section above, we have seen how negative numbers dominate over positive numbers in our current number system. We have also shown that there must exist an equivalent number system where positive numbers dominate over negative ones. These number systems are basically identical, or we could say they are mirror images of one other. That is, both of these number systems are asymmetric, and both need the concepts of imaginary and complex numbers in order to handle the square root function for all numbers.

Wouldn't it be nice to have a set of perfectly symmetrical mathematical rules that are identical for negative and positive numbers? This is also possible. We will suggest the following axioms:

1. A negative number multiplied with a negative number always gives a negative number.
2. A positive number multiplied with a positive number always gives a positive number.
3. A positive number multiplied by a negative number, or a negative with a positive number, always gives two solutions, namely a plus and minus solution of the absolute value of the result.
4. a^b is as in the standard number system, but $a^{-b} = -a^b = \pm a^b$ and $-a^{-b} = -(a^b)$.

The fact that we now have two solutions rather than one (under axiom 3) may seem strange at first, but this will help us to eliminate multiple solutions in the answer when it comes to the square root function. The square root of a positive number will now always be positive, and the square root of a negative number will always be negative. Only the square root of a plus/minus number will have a plus/minus solution, and the plus/minus solution means that $\sqrt{\pm 4} = -2 \times 2$, since ± 4 can only be created by multiplying a positive number with a negative number.

Table 1 show the three number systems mentioned here. The left hand column is today's number system, the middle column is the mirror number system of our current system, and the right hand column is the newly suggested perfect symmetric number system. Again, we ask: "What is the rationale behind having negative numbers dominating over positive ones, or positive numbers dominating over negative ones"? Such asymmetry rules do not sound logical or appear to have any fundamental reasoning behind them. The asymmetric rules are a main cause behind more complex mathematics, such as imaginary numbers and complex number theory. Our symmetric number system seems more logical and may open up new possibilities in the field of mathematics, as well as other fields that rely heavily on the theory and practice of math. Obviously, as it is a new number system, there could be challenges that we have not yet understood. However, it is clear that small changes in the fundamental properties and rules of the prevailing number system can have a series of consequences for rules "higher" up in the constructions of math and physics, for example.

Today's number system Asymmetric rules	**Mirror of today's system Asymmetric rules**	**Perfect Symmetry Yin-Yang system**
$+\times+=+$	$+\times+=-$	$+\times+=+$
$-\times-=+$	$-\times-=-$	$-\times-=-$
$+\times-=-$ $-\times+=-$	$+\times-=+$ $-\times+=+$	$+\times-=\pm$ $-\times+=\pm$
$\sqrt{+}=\pm$	$\sqrt{+}=i$	$\sqrt{+}=+$
$\sqrt{-}=i$	$\sqrt{-}=\pm$	$\sqrt{-}=-$
$\sqrt{\pm}=?$ No rule	$\sqrt{\pm}=?$ No rule	$\sqrt{\pm}=-\times+$
Numerical examples		
$2\times 2=4$	$2\times 2=-4$	$2\times 2=4$
$-2\times -2=4$	$-2\times -2=-4$	$-2\times -2=-4$
$2\times -2=-4$ $-2\times 2=-4$	$2\times -2=4$ $-2\times 2=4$	$2\times -2=\pm 4$ $-2\times 2=\pm 4$
$\sqrt{4}=\pm 2$	$\sqrt{4}=2i$	$\sqrt{4}=2$
$\sqrt{-4}=2i$	$\sqrt{-4}=\pm 2$	$\sqrt{-4}=-2$
$\sqrt{\pm 4}=?$	$\sqrt{\pm 4}=?$	$\sqrt{\pm 4}=-2\times 2$ $\sqrt{\pm 4}=2\times -2$
Addition and subtraction identical for all systems, as there is no sign dominance.		
$2+2=4$	$2+2=4$	$2+2=4$
$-2+2=0$ $2-2=0$	$-2+2=0$ $2-2=0$	$-2+2=0$ $2-2=0$
$-2-2=-4$	$-2-2=-4$	$-2-2=-4$

Table 1: This table summarizes three different number systems. The first one is today's number system, where negative numbers dominate over positive. The next one is the mirror image of that system, where positive numbers dominate over negative. The third system is a number system with perfect symmetry, where negative and positive numbers have the same status. Only in the first two asymmetric number systems do we need imaginary and complex numbers.

3 The Riemann Hypothesis and Its Possible Link to Asymmetric Number Systems

One of the most interesting mathematical problems yet to be solved is the Riemann hypothesis. It is one of the seven "Millennium" Problems described by the Clay Mathematics Institute (CMI) and the only problem remaining from David Hilbert's original set of 23 problems, curated and presented in 1900, see [5, 9].

As described by the CMI, the Riemann hypothesis states that "some numbers have the special property that they cannot be expressed as the product of two smaller numbers, e.g., 2, 3, 5, 7, etc. Such numbers are called prime numbers, and they play an important role, both in pure mathematics and its applications. The distribution of such prime numbers among all natural numbers does not follow any regular pattern. However, the German mathematician G.F.B. Riemann (1826 – 1866) observed that the frequency of prime numbers is very closely related to the behavior of an elaborate function, called the Riemann Zeta function.

The Riemann hypothesis asserts that all interesting solutions of the equation $\zeta(s) = 0$ lie on a certain vertical straight line. This has been checked for the first 10,000,000,000,000 solutions. A proof that it is true for every interesting solution would shed light on many of the mysteries surrounding the distribution of prime numbers."[2]

Interestingly, there can be no Riemann hypothesis in our new symmetric number system, as there are no imaginary numbers and complex planes in this system. In other words, the Riemann hypothesis actually seems to be linked to and perhaps may even arise from the asymmetry in today's number system. In addition, we conjecture that the Riemann Zeta function, if developed under the mirror system (where the rules of dominant negative number rules are switched so positive number rules are dominant), then what we could call the mirror Riemann Zeta function should have all of its zeros only at the positive even integers and complex numbers with real part $-\frac{1}{2}$. In other words, we suggest that the Riemann hypothesis is partly rooted in the choice of the asymmetric number system. We have two asymmetric number systems that are the mirror of each other, reflecting the Riemann hypothesis around zero (and it is suggested that all non-trivial solutions are at 1/2 and -1/2). Further, it appears that we have one symmetric number system with no equivalent Riemann

[2]See the general entry for the Riemann hypothesis on the Clay Mathematics Institute website at: http://www.claymath.org/millennium-problems/riemann-hypothesis.

hypothesis. This leads us to think that the Riemann hypothesis is mostly about understanding the complex effects of a fundamental issue that is rooted in asymmetric rules between positive and negative numbers.

4 How to Decide on an Optimal Number System

A number system is similar to a set of rules for a game, like chess, where in a number system the game rules are about numbers themselves. Yet, the rules of a number system cannot, in general, be proven inside that number system. For example, one cannot prove or disprove that $-2 \times 2 = -4$ is incorrect or correct; it is correct inside the rules of the system. However, as we have seen, the asymmetric rules in the standard number system lead to the fact that one has to introduce strange rules such as imaginary numbers. If the rules are defined and the system is just used to solve purely mathematical problems inside the numerical system, there is nothing wrong with this. It is like checking out all of the rules in the system and playing around with them in many types of scenarios allowed inside the system. It is even possible to add new rules along the way, when new challenges inside the system arise, such as finding the square root of minus one.

However, the ultimate goal of mathematics should be the development of a language to better describe and understand Nature. Here, even if the optimal rules cannot be proven right or wrong, they should emerge out of feedback from Nature. For example, if one discovers that the rules of nature seem to be symmetrical, rather than asymmetrical, one should reconsider the approach to mathematical rules that would make them symmetrical and therefore more in harmony with what one is trying to describe. In other words, (and paraphrasing Einstein), one wants the numerical system to be as simple as possible so that it can describe Nature, but not simpler than that. For perspective, note that the fact that we have to rely on imaginary numbers in quantum mechanics seems a bit strange. What would quantum mechanics look like if it was derived from a symmetric number system? To find the optimal number system, we need to evaluate it in relation to nature and see if that system seems better suited than another system with different rules.

In Table 2, we mention a few more possible number systems that are symmetrical. The system in column 1 is a very interesting alternative. Systems 3 and 4 could also be fully usable, but they seem to add complexity in the way of thinking, e.g., why should one times one become minus one when we can have one times one is simply one? We suspect that our original Yin and Yang system, or our Yin and Yang system-2 might be the optimal numerical system to describe Nature, but more research is needed in relation to Nature and its feedback loop to determine if this is the case or not.

Symmetric system 2 **Yin-Yang system 2**	**Symmetric system 3**	**Symmetric system 4**
$+\times+=+$	$+\times+=-$	$+\times+=-$
$-\times-=-$	$-\times-=+$	$-\times-=+$
$+\times-=$Not allowed, needed? $-\times+=$Not allowed, needed?	$+\times-=\pm$ $-\times+=\pm$	$+\times-=$Not allowed, needed? $-\times+=$Not allowed, needed?
$\sqrt{+}=+$	$\sqrt{+}=-$	$\sqrt{+}=-$
$\sqrt{-}=-$	$\sqrt{-}=+$	$\sqrt{-}=+$
Numerical examples		
$2\times2=4$	$2\times2=-4$	$2\times2=-4$
$-2\times-2=-4$	$-2\times-2=4$	$-2\times-2=4$
$2\times-2=$Not allowed $-2\times2=$ Not allowed	$2\times-2=\pm4$ $-2\times2=\pm4$	$2\times-2=$ Not allowed $-2\times2=$ Not allowed
$\sqrt{4}=2$	$\sqrt{4}=-2$	$\sqrt{4}=-2$
$\sqrt{-4}=-2$	$\sqrt{-4}=2$	$\sqrt{-4}=2$
$\sqrt{\pm4}=$ Not allowed	$\sqrt{\pm4}=\pm2$	$\sqrt{\pm4}=$ Not allowed
Addition and subtraction identical for all systems, as there is no sign dominance.		
$2+2=4$	$2+2=4$	$2+2=4$
$-2+2=0$ $2-2=0$	$-2+2=0$ $2-2=0$	$-2+2=0$ $2-2=0$
$-2-2=-4$	$-2-2=-4$	$-2-2=-4$

Table 2: This table summarizes three different number systems. The first one is today's number system, where negative numbers dominate over positive. The next one is the mirror image of that system, where positive numbers dominate over negative. The third system is a number system with perfect symmetry, where negative and positive numbers have the same status. Only in the first two asymmetric number systems do we need imaginary and complex numbers.

[3]

[3] Thanks to Andrea Berdondi for suggesting that one also can make a symmetric number system by having plus times plus is minus and minus times minus is plus. Still, this system appears to be more awkward and less logical than our Yin-Yang number systems.

5 Imaginary Numbers in Physics

In 1908, Minkowski [7] introduced imaginary numbers in his space-time geometry:

> Thus the essence of this postulate can be expressed mathematically very concisely in the mystical formula: $3 \cdot 10^5$ km $= \sqrt{-1}$ seconds.

According to Unruh [11], whether or not Minkowski space-time is compatible with quantum theory is still an open question. Also, in quantum mechanics we have imaginary numbers in the wave equation. The imaginary number $\sqrt{-1} = i$ plays a central part in standard quantum mechanics. We find it in the the matrix mechanics of Born and Jordan, then the Poisson bracket of Dirac and the Schrödinger equation [10], and later the Feynman's path integral, see [3, 4, 8]. It appears that it is indispensable in the formulation of quantum mechanics and its interpretation. Freeman Dyson in a recent lecture stated (see [6])

> *"Surely to the unpreoccupied mind, complex numbers are far from natural or simple and they cannot be suggested by physical observations. Furthermore, the use of complex numbers is in this case not a calculational trick of applied mathematics but comes close to being a necessity in the formulation of the laws of quantum mechanics. ???. . . But then came the surprise. Schrödinger put the square root of minus one into the equation, and suddenly it made sense. Suddenly it became a wave equation instead of a heat conduction equation... And that square root of minus one means that nature works with complex numbers and not with real numbers"*

In our view, we have reasons to think that nature does not work with complex numbers, but that we have ended up using a universally complex asymmetric number system that could be replaced with a symmetric number system that better describes nature. We encourage future research to look onto whether or not a new quantum mechanics rooted in our symmetric yin and yang number system could be developed. In this framework, clearly no imaginary number can exist, and if such a theory then still fits both classic experiments as well as modern quantum mechanics, then it would be preferable, as it would be more logical. After all, what does an imaginary number in an equation representing the real world truly represent? Truly, we would say, nothing but a overly complex model used to describe a simpler and perhaps more symmetrical reality. However, no conclusions should be drawn prematurely before this proposal is carefully investigated and this is likely to take a considerable amount of time.

6 Conclusion

We have pointed out that our modern number system has "strange" asymmetric rules, where negative numbers dominate over positive numbers (when multiplying positive numbers with negative). These asymmetric rules likely came into being because early mathematicians first developed rules for positive numbers and then tried to fit negative numbers into this system. The main focus was to have a practical everyday number system. Later on, it was necessary to develop a rule for the square root of numbers, which required some accommodation in the rules that were used. The asymmetric rules seem to lead to the need for imaginary numbers, for example. Further, we have shown that there exists an identical or mirror number system, where the dominance rules are switched from negative numbers to positive ones. In this case, the imaginary numbers are linked to the square root of one rather than the square root of minus one.

We have also introduced a new perfectly symmetrical number system, where there is no dominance of negative over positive numbers, or positive over negative numbers. In this perfectly symmetrical number system, there is no need for imaginary numbers or complex number theory in this system. We also have indicated that the Riemann hypothesis is likely rooted in the asymmetry of the dominance rules in the existing number system. Also, we hypothesize that a new and likely simpler quantum mechanics could be formulated based on our suggested symmetrical number systems. We have reasons to think nature does not consist of imaginary stuff, but that the asymmetry in the current number systems needs to incorporate imaginary numbers to describe a deeper symmetry at the depth of realty; this naturally requires further investigation before any firm conclusions are made.

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