Refutation of the generating positive cone in ordered Banach space

Abstract: From the background definitions in the ordered Banach space, we evaluate equations to produce the term named positive generating cone. It is *not* tautologous, hence refuting the model.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ⇾; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ⇽, ≲; = Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠; % possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L; (z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero; (%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2; ~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Dahlqvist, F.; Kozen, D. (2019).
Semantics of higher-order probabilistic programs with conditioning.
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1) Regular Ordered Banach spaces: An *ordered vector space* V is a vector space together with a partial order ≤ which is compatible with the linear structure in the sense that

for all u, v, w ∈V, λ ∈R^+
u ≤ v ⇒u + w≤v + w and u ≤ v ⇒λu ≤ λv (2.1.1)

```
(((#u&(#v&#w))<p)&(q<~(p&p)))>
((~(#v<#u)>(#u+(~(#v<#w)+#w)))&(~(#v<#u)>~((#q&#u)<(#p&#v)))) ;
                                        TTTT TTTT TTTT TTTT
```
(2.1.2)

A vector v in an ordered vector space V is called *positive* if v ≥ 0 and the collection of all positive vectors is called the *positive cone* of V and denoted V^+. The positive cone is said to be *generating* if V = V^+−V^+, that is to say if every vector can be expressed as the difference of two positive vectors.

V^+=v ≥ 0, V=V^+ - V^+ : (2.3.1)

```
p=((~((p@p)>v))-(~((p@p)>v))) ;          FTFT FTFT FTFT FTFT
```
(2.3.2)

Remark 2.3.1: Eq. 2.3.1 follows from 2.1.1 for V. (2.4.1)

((((#u&(#v&#w))<p)&(q<~(p&p)))>
((~(#v<#u)>(#u+(~(#v<#w)+#w)))&
(~(#v<#u)>~((#q&#u)<(#p&#v)))))>
(p=(~((p@p)>v)-~((p@p)>v))) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F** (2.4.2)

Eq. 2.4.2 is *not* tautologous. Hence, the positive generating cone is refuted in the ordered Banach space.