The electron as a harmonic quantum-mechanical oscillator

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Abstract : The particular flavor of the *Zitterbewegung* interpretation that we have developed in previous paper assumes the electron mass is the equivalent energy of a harmonic oscillation in a plane. We developed the metaphor of a *perpetuum mobile* driven by two springs that work in tandem—in a 90-degree angle and with the same phase difference. This paper explores the *limitations* of that metaphor.

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The *Zitterbewegung* model of an electron

The two illustrations below recap the basics of our particular *flavor* of the *Zitterbewegung* model of an electron.

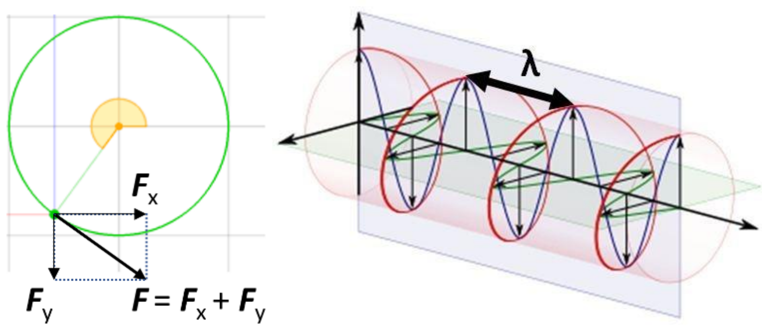


Figure 1: The *Zitterbewegung* model of an electron

We refer to it as a quantum-mechanical oscillator because we get the Compton *radius* of an electron from equating the E = m·c^2 and E = m·a^2·ω^2 equations. For the E and m to be the same in both equations, we must equate *c* to *a*·ω, which suggested an interpretation of *c* as a tangential velocity. We effectively think of the green dot as a *pointlike charge* – the elementary charge – orbiting around some center. This model of an electron combines Wheeler's notion of 'mass without mass' with the idea of a pointlike charge: the mass of the electron is the equivalent oscillation.

Why is this an oscillation? The force grabs onto a massless pointlike charge and must, therefore, be electromagnetic in its nature. However, to keep the pointlike charge orbiting at velocity *c*, it must continually change direction. There is, therefore, an equivalent acceleration—not because the *magnitude* of the velocity is changing (*v* = *c*, always) but because of its ever-changing *direction*.

A rotation corresponds to a cycle whose cycle time will be equal to T = 1/*f* = 2π·*a*/*c* = λ_C/*c* [One should not confuse the *Compton* wavelength λ_C with the λ wavelength in the illustration, which we will explain in a moment. Also, the subscript in λ_C (C) stands for *Compton*, not for the speed of light. It is tempting to invent new notations, but we will not do so.]

We then boldly assumed Planck's quantum is the quantum of (physical) action in this elementary cycle. Physical action is the product of a force (F), a distance (Δs) and some time (Δt): *S* = F·Δs·Δt. The idea of associating Planck's quantum with an elementary cycle implies we think of h as the following product:

$$h = F \cdot \lambda_C \cdot T = E \cdot T$$

Why the $F \cdot \lambda_C = E$ substitution? Energy can be written as a force over a distance, so we just assume the energy, force and distance in the two formulas are just the same. We are talking about the same object: the pointlike charge, in this case. [The reader may feel I am a bit pedantic but it is important to be clear about assumptions and what leads to what here.] The point is, we get the Planck-Einstein relation out of this:

$$h = F \cdot \lambda_C \cdot T = E \cdot T = \frac{E}{f} \Leftrightarrow E = h \cdot f = \hbar \cdot \omega$$

Again, the reader may feel I am pedantic but let us summarize what we did: we equated the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equations because we're talking *the* energy of some object here—an electron, to be precise. This equation implies that we should interpret the $c = a \cdot \omega$ as a tangential velocity. If we do this, we get the Planck-Einstein relation: $E = \hbar \cdot \omega$. We can now calculate the radius a:

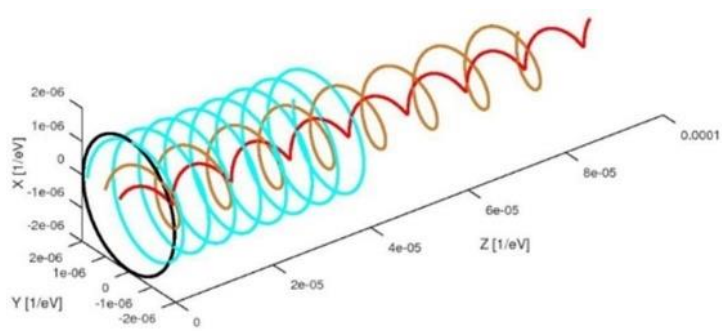
$$E = ma^2\omega^2 = ma^2\frac{E^2}{\hbar^2} \Leftrightarrow \hbar^2 = ma^2E = ma^2mc^2 = m^2a^2c^2$$

$$\Leftrightarrow a = \frac{\hbar}{mc} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12} \text{ m}$$

That is the Compton radius. That's what we wanted to find. We think this is the shortest *route*.

The *de Broglie* wavelength

If the tangential velocity remains equal to c, and the pointlike charge has to cover some horizontal or *linear* distance as well – as it does in the illustration on the right-hand side above, and in the illustration below (for which credit goes to an Italian group of *zbw* theorists[1]) – then the circumference of its rotational motion *must* decrease so it can cover the extra distance.



Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

[1] Vassallo, G., Di Tommaso, A. O., and Celani, F, *The Zitterbewegung interpretation of quantum mechanics as theoretical framework for ultra-dense deuterium and low energy nuclear reactions*, in: Journal of Condensed Matter Nuclear Science, 2017, Vol 24, pp. 32-41. Don't worry about the rather weird distance scale (1×10^{-6} eV^{-1}). Time and distance can be expressed in *inverse* energy units when using so-called *natural units* ($c = \hbar = 1$). We are not very fond of this because we think it does *not* necessarily clarify or simplify relations. Just note that 1×10^{-9} eV^{-1} = 1 GeV^{-1} $\approx 0.1975\times10^{-15}$ m. As you can see, the *zbw* radius is of the order of 2×10^{-6} eV^{-1} in the diagram, so that's about 0.4×10^{-12} m, which is what we calculated: $a \approx 0.386\times10^{-12}$ m.

Figure 2: The Compton radius must decrease with increasing velocity

Can the linear velocity go to *c*? In the limit, yes. This is very interesting, because we can see that the circumference of the oscillation becomes a wavelength in the process! We may come back to this but, as for now, let us analyze it the way we should analyze it, and that's by using our formulas. Let us first think about our formula for the *zbw* radius *a*:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi}$$

The λ_C is the *Compton* wavelength, so that's the circumference of the circular motion.[2] How can it decrease? The mass in the $a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}$ m formula was the rest mass m_0. If the electron moves, it will have some kinetic energy, which we must add to the *rest energy*. Hence, the mass m in the denominator (m*c*) increases and, because ħ and c are physical constants, *a* must decrease. How does that work with the frequency? The frequency is proportional to the energy (E = ħ·ω = h·*f* = h/T) so the frequency – in whatever way you want to measure it – will *increase*. Hence, the *cycle* time T must *decrease*. We write:

$$\theta = \omega t = \frac{\mathrm{E}}{\hbar} t = \frac{\gamma \mathrm{E}_0}{\hbar} t = 2\pi \cdot \frac{\mathrm{t}}{\mathrm{T}}$$

So our Archimedes' screw gets stretched, so to speak. Let us think about what happens here. We got the following formula for this λ wavelength, which is like the distance between two crests or two troughs of the wave[3]:

$$\lambda = v \cdot \mathrm{T} = \frac{v}{f} = v \cdot \frac{\mathrm{h}}{\mathrm{E}} = v \cdot \frac{\mathrm{h}}{\mathrm{m}c^2} = \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \beta \cdot \lambda_\mathrm{C}$$

This wavelength is *not* the *de Broglie* wavelength λ_L = h/p.[4] So what is it? We have *three* wavelengths now: the *Compton* wavelength λ_C (which is a circumference, actually), that weird horizontal distance λ, and the *de Broglie* wavelength λ_L. Can we make sense of that? We can. Let us first re-write the *de Broglie* wavelength:

$$\lambda_\mathrm{L} = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}v} = \frac{\mathrm{h}c^2}{\mathrm{E}v} = \frac{\mathrm{h}c}{\mathrm{E}\beta} = \frac{\mathrm{h}}{c} \cdot \frac{1}{\mathrm{m} \cdot \beta} = \frac{\mathrm{h}}{\mathrm{m}_0 c} \cdot \frac{1}{\gamma\beta}$$

What is this? Let's analyze it mathematically. What happens to the *de Broglie* wavelength as m and *v* both increase because our electron picks up some momentum p = m·*v*? Its wavelength must actually *decrease* as its (linear) momentum goes from zero to some much larger value – possibly infinity as *v* goes to *c* – but *how exactly*? The 1/γβ factor gives us the answer. That factor comes down from infinity (+∞) to zero as *v* goes from 0 to *c* or – what amounts to the same – if the relative velocity β = *v*/*c* goes from 0 to 1. The graphs below show how that works. The 1/γ factor is the circular arc that we're used to, while the 1/β function is just the regular inverse function (*y* = 1/*x*) over the domain β = *v*/*c*, which goes from 0 to 1 as *v* goes from 0 to *c*. Their product gives us the green curve which – as mentioned – comes

[2] Hence, the *C* subscript stands for the *C* of Compton, not for the speed of light (*c*).
[3] Because it is a wave in two dimensions, we cannot really say there are crests or troughs, but the terminology might help you with the interpretation of the geometry here.
[4] The use of L as a subscript is a bit random but think of it as the L of *L*ouis de Broglie.

down from +∞ to 0. [Take your time to carefully look at the formulas and the curves so you can digest this.]

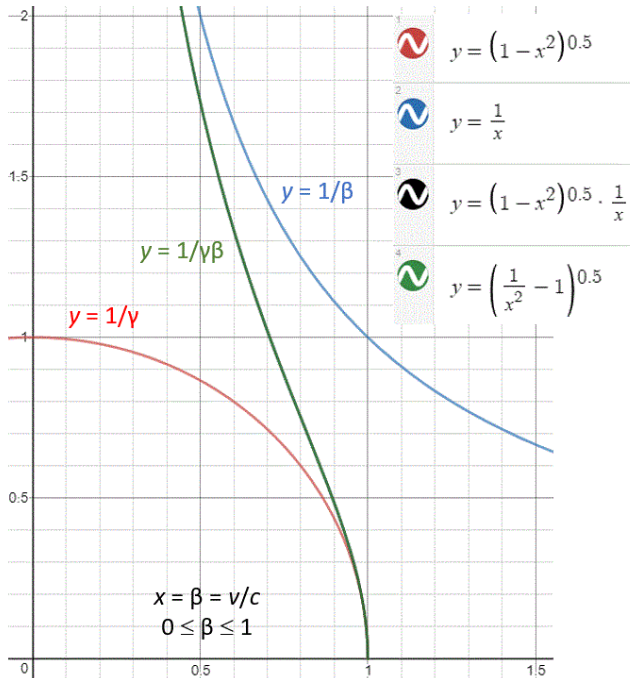


Figure 3: The 1/γ, 1/β and 1/γβ graphs[5]

Now, we re-wrote the formula for *de Broglie* wavelength λ_L as the *product* of the 1/γβ factor and the *Compton* wavelength for $v = 0$:

$$\lambda_{\mathrm{L}} = \frac{\mathrm{h}}{\mathrm{m}_0 c} \cdot \frac{1}{\gamma\beta} = \frac{1}{\beta} \cdot \frac{\mathrm{h}}{\mathrm{m}c}$$

Hence, the *de Broglie* wavelength goes from +∞ to 0. We may wonder: when is it equal to $\lambda_C = h/mc$? Let's calculate that:

$$\lambda_{\mathrm{L}} = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}c} \cdot \frac{1}{\beta} = \lambda_C = \frac{\mathrm{h}}{\mathrm{m}c} \Leftrightarrow \beta = 1 \Leftrightarrow v = c$$

This is a rather weird result, isn't it? But it is what it is. Let's bring the third wavelength in: the $\lambda = \beta\cdot\lambda_C$ wavelength—which is that length between the crests or troughs of the wave.[6] We get the following two rather remarkable results:

$$\lambda_{\mathrm{L}} \cdot \lambda = \lambda_{\mathrm{L}} \cdot \beta \cdot \lambda_{\mathrm{C}} = \frac{1}{\beta} \cdot \frac{\mathrm{h}}{\mathrm{m}c} \cdot \beta \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \lambda_{\mathrm{C}}^2$$

$$\frac{\lambda}{\lambda_{\mathrm{L}}} = \frac{\beta \cdot \lambda_{\mathrm{C}}}{\lambda} = \frac{\mathrm{p}}{\mathrm{h}} \cdot \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \frac{\mathrm{m}v^2}{\mathrm{m}c^2} = \beta^2$$

[5] We used the free desmos.com graphing tool for these and other graphs.

[6] We should emphasize, once again, that our two-dimensional wave has no real crests or troughs: λ is just the distance between two points whose argument is the same—except for a phase factor equal to $n\cdot 2\pi$ (n = 1, 2,…).

The product of the $\lambda = \beta \cdot \lambda_C$ wavelength and *de Broglie* wavelength is the square of the Compton wavelength, and their ratio is the square of the relative velocity $\beta = v/c$. – *always!* – and their ratio is equal to 1 – *always!* These two results are rather remarkable too but, despite their simplicity and apparent beauty, you might be struggling for an easy geometric interpretation. I was struggling for it too, but then I thought the use of *natural units* might help. Equating *c* to 1 would give us natural distance and time units, and equating *h* to 1 would give us a natural force unit—and, because of Newton's law, a natural mass unit as well. Why? Because Newton's $\boldsymbol{F} = \mathrm{m} \cdot \boldsymbol{a}$ equation is relativistically correct: a force is that what gives some mass acceleration. Conversely, mass can be defined of the inertia to a change of its state of motion—because any change in motion involves a force and some acceleration: $\mathrm{m} = \mathbf{F}/\boldsymbol{a}$. If we re-define our distance, time and force units by equating *c* and *h* to 1, then the Compton wavelength (remember: it's a circumference, really) and the mass of our electron will have a simple inversely proportional relation:

$$\lambda_C = \frac{1}{\gamma \mathrm{m}_0} = \frac{1}{\mathrm{m}}$$

We get equally simple formulas for the *de Broglie* wavelength and our λ wavelength:

$$\lambda_\mathrm{L} = \frac{1}{\beta\gamma \mathrm{m}_0} = \frac{1}{\beta \mathrm{m}}$$

$$\lambda = \beta \cdot \lambda_C = \frac{\beta}{\gamma \mathrm{m}_0} = \frac{\beta}{\mathrm{m}}$$

This is quite deep: we have three *lengths* here – defining all of the geometry of the model – and they all depend on two factors only: the *rest* mass of our object and its (relative) velocity. Can we take this discussion any further? Perhaps, because what we have found may or may not be related to the idea that we're going to develop in the next section. However, before we move on to the next, let us quickly note the three equations – or lengths – are *not* mutually independent. They are related through that equation we found above:

$$\lambda_\mathrm{L} \cdot \lambda = \lambda_C^2 = \frac{1}{\mathrm{m}^2}$$

We'll let you play with that. To help you with that, you may start by noting that the $\lambda_\mathrm{L}\lambda = 1/\mathrm{m}^2$ reminds us of a property of an ellipse. Look at the illustration below.[7] The length of the chord – perpendicular to the major axis of an ellipse is referred to as the *latus rectum*. One half of that length is the actual *radius of curvature* of the osculating circles at the endpoints of the major axis.[8] We then have the usual distances along the major and minor axis (*a* and *b*). Now, one can show that the following formula has to be true:

$$a \cdot p = b^2$$

[7] Source: Wikimedia Commons (By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=57428275).

[8] The endpoints are also known as the *vertices* of the ellipse. As for the concept of an osculating circles, that's the circle which, among all tangent circles at the given point, which approaches the curve most tightly. It was named *circulus osculans* – which is Latin for 'kissing circle' – by Gottfried Wilhelm Leibniz. You know him, right? Apart from being a polymath and a philosopher, he was also a great mathematician. In fact, he was the one who invented differential and integral calculus.

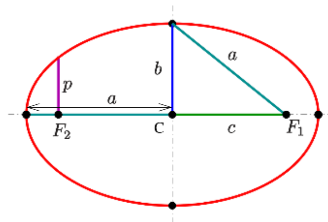


Figure 4: The *latus rectum* formula: $a \cdot p = b^2$

You probably wonder: why would this be relevant? It introduces an asymmetry in what we may loosely refer to as the *shape* of an electron. We get such asymmetry from other models – as we'll explain below – and it should explain the anomalous magnetic moment without having to resort to weird calculations using Feynman diagrams and renormalization techniques. In short, we think the analysis above gives you a *classical* electron model which may explain all of quantum mechanics in a *classical* way.

Classical electron models

Our *Zitterbewegung* model of the electron implies a delightfully simple geometry, but it is *not* a perfect sphere, nor is it a perfect disk. In fact, if anything, we might way our electron occupies a space whose shape is an ellipsoid, as shown below.

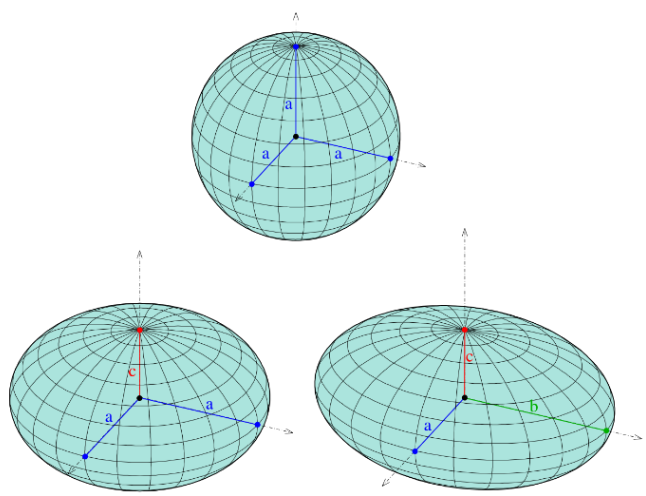


Figure 5: A sphere, a spheroid and an ellipsoid[9]

An ellipsoid is defined by *three* parameters (*a*, *b* and *c* in the illustration above), as opposed to a spheroid, which is defined by *two* parameters only (or, for a perfect sphere, only one parameter: the radius). Of course, these three parameters are not independent: they are mutually related. We can relate them through various equations, but the most obvious way to relate them is the equation for the ellipsoid itself:

[9] Source: Wikimedia Commons, User: Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45585493

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

This is a very straightforward formula. It relates the coordinates to the three axes of the ellipsoid (*a*, *b* and *c*). However, there are other ways of defining the ellipsoid, and the latus rectum formula is one of them. In case you would doubt, I will give you one of the more significant examples of how one get these results from far more advanced models. One is the model of Dr. Alexander Burinskii. [10] We have been in touch with him, and he would probably *not* wish to describe his Dirac-Kerr-Newman model of an electron as a classical electron model but that is what it is for us: a charge with a geometry in three-dimensional space. To be precise, it is a disk-like structure, and its *form factor* – read: the ratio between the radius and thickness of the disk – depends on various assumptions (as illustrated below) but reduces to the ratio between the Compton and Thomson radius of an electron when assuming classical (non-perturbative) theory applies. We quote from Mr. Burinskii's 2016 paper: "It turns out that the flat Compton zone free from gravity may be achieved without modification of the Einstein-Maxwell equations."

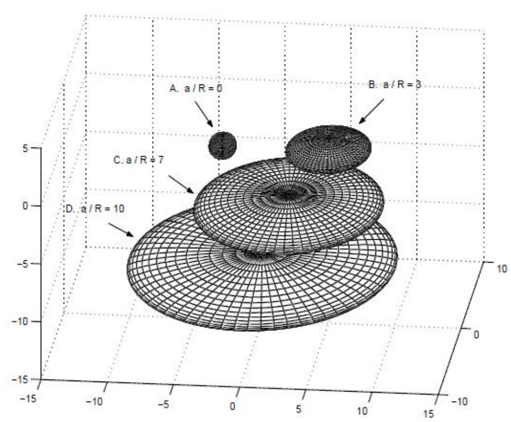


Figure 6: Alexander Burinskii's electron model

Hence, it would seem we get the fine-structure constant as the ratio of the Compton radius – i.e. the radius of the disk *R* – and the classical electron radius – i.e. the thickness of the disk *r* – out of a smart model based on Maxwell's and Einstein's equations, i.e. classical electromagnetism and general relativity theory:

$$\alpha=\frac{r}{R}=\frac{r_e}{r_C}=\frac{\mathrm{e}^2/\mathrm{mc}^2}{\hbar c/\mathrm{mc}^2}=\frac{\mathrm{e}^2}{\hbar c}$$

There is no need for smart quantum mechanics here! These results, therefore, confirm the intuitive but, admittedly, rather primitive *Zitterbewegung* model we introduced in our own papers. To illustrate the point, we can note that we can interpret the fine-structure constant as a dimensional scaling constant.[11]

[10] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008, https://arxiv.org/abs/hep-th/0507109. A more recent article of Dr. Burinskii (*New Path to Unification of Gravity with Particle Physics*, 2016, https://arxiv.org/abs/1701.01025, relates the model to more recent theories – most notably the "supersymmetric Higgs field" and the "Nielsen-Olesen model of dual string based on the Landau-Ginzburg (LG) field model."

[11] See: Jean Louis Van Belle, *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/abs/1812.0273.

The model is wonderful because it combines both the wave- as well as the particle-like character of an electron—or, potentially, of any *charged* elementary particle. In addition, we can develop a similar model for a photon: it's like an electron but without the pointlike charge. 😉 We know that sounds weird but we'll refer the reader to previous publications for more detail.[12] The photon and electron model can be combined and give us a nice classical explanation for electron orbitals. So it is all perfect!

Is it? Maybe. Maybe not. What are we talking about, *really*? I think the issue is nicely summarized in one of Dr. Burinskii's very first communications to me. He wrote the following to me when I first contacted him on the viability on *my* flavor of a *zbw* model of an electron:

> "I know many people who considered the electron as a toroidal photon[13] and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about *Zitterbewegung* [because of ideological reasons[14]], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?"[15]

He noted that this fundamental flaw was (and still is) the main reason why had abandoned the simple *Zitterbewegung* model in favor of the much more sophisticated Kerr-Newman approaches to the (possible) geometry of an electron.

I am reluctant to make the move he made – mainly because I prefer simple math to the rather daunting math involved in Kerr-Newman geometries – and so that is why I am continuing to explore this alternative explanation. However, Dr. Burinskii is right: we need to be more explicit about that *oscillator* model. What is it, *exactly*?

What *is* that oscillation?

We think of an oscillation because the motion implies an oscillating *force*. We used the *metaphor* of a V-2 engine – or, what's equivalent, two springs on a crankshaft, in our previous papers.[16] This metaphor is nice but suffers from the fact that it's a non-relativistic analysis: we treat mass as a constant. Let us quickly go through the basics of it, however.

The energy of any oscillation will always be proportional to its amplitude (let us denote that by *a*). However, we also know that the energy in the oscillation will also be proportional to its frequency (let us denote the frequency by ω). Hence, we will have some proportionality coefficient *k* and we can write this:

$$\mathrm{E} = \mathrm{k}a^2\omega^2$$

[12] See: Jean Louis Van Belle, *The Emperor Has No Clothes: A Classical Interpretation of Quantum Mechanics*, 27 February 2019, http://vixra.org/abs/1901.0105.

[13] This is Dr. Burinskii's terminology: it does refer to the *Zitterbewegung* electron: a pointlike charge with no mass in an oscillatory motion – orbiting at the speed of light around some center.

[14] This refers to perceived censorship from the part of Dr. Burinskii. In fact, some of what he wrote me strongly suggests some of his writings have, effectively, been suppressed because – when everything is said and done – they do fundamentally question – directly or indirectly – some key assumptions of the mainstream interpretation of quantum mechanics.

[15] Email from Dr. Burinskii to the author dated 22 December 2018.

[16] See: Jean Louis Van Belle, *The Wavefunction as an Energy Propagation Mechanism*, 8 June 2018, http://vixra.org/abs/1806.0106.

For one *single* oscillator (think of a spring or a piston in a cylinder compressing and decompressing air) the factor of proportionality is equal to 1/2, so we write:

$$E = \frac{1}{2} m a^2 \omega^2$$

If we combine *two* oscillators in a 90-degree angle – think of two springs or two pistons attached to some crankshaft as illustrated below – then we get some *perpetuum mobile* which stores *twice* that energy.

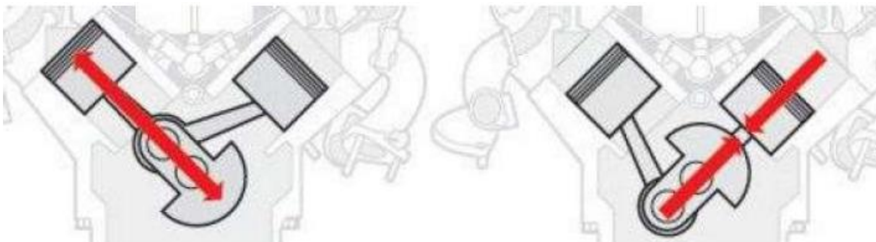


Figure 7: The V-2 metaphor[17]

The analogy can be extended to include two *pairs* of springs or pistons, in which case the springs or pistons in each pair would help drive each other. The point is: we have a great *metaphor* here. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle. While transferring kinetic energy from one piston to the other, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa. More importantly, we can now just *add* the total energy of the two oscillators to get the total energy of the whole system, and so we get the $E = ma^2\omega^2$ formula.[18]

We get the circular motion from adding the sine and cosine and, hence, we can also represent the circular motion by Euler's function, as illustrated below:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\omega \cdot t) + i \cdot a \cdot \sin(\omega \cdot t) = (x, y)$$

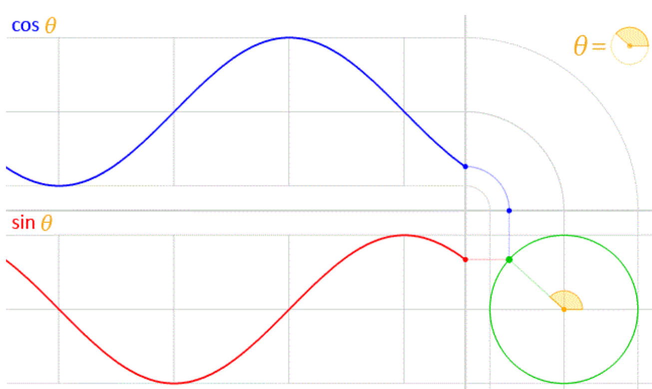


Figure 8: Rotational motion

[17] The illustration is from a January 2011 article in the *Car and Driver* magazine, titled: The Physics of Engine Cylinder-Bank Angles. See: https://www.caranddriver.com/features/a15126436/the-physics-of-engine-cylinder-bank-angles-feature/.
[18] See the above-mentioned paper for more detail.

Think of the green dot going around and around as our pointlike charge, as the argument θ ticks away with time. The original is, in fact, an *animated* GIF that you can easily google[19] and you may want to stare at it for a while so as to appreciate the *dynamics*.

Indeed, the wavefunction consist of a sine and a cosine: the cosine is the real component, and the sine is the imaginary component. We believe they are equally real, and we believe each of the two oscillations carries *half* of the total energy of our particle—if this metaphor would reflect the reality of the electron, that is.

However, as we already mentioned, the metaphor suffers from the fact the analysis is not relativistically correct. On the other hand, we said our pointlike charge has *zero* rest mass, so what does it all mean? It is, effectively, a rather weird business to analyze a frictionless spring with a (rest) mass that is equal to zero. But let us see what we get from analyzing an oscillator using the relativistically correct force law.

If the velocity of our mass on this spring – on the *two* springs, really – becomes a sizable fraction of the speed of light, then we can no longer treat the mass as a constant factor: it will vary with velocity, and its variation is given by the Lorentz factor (γ). The relativistically correct force equation for *one* oscillator is:

$$F = dp/dt = F = -kx \text{ with } p = m_v v = \gamma m_0 v$$

The $m_v = \gamma m_0$ varies with speed because γ varies with speed:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}} = \frac{\mathrm{dt}}{\mathrm{d\tau}}$$

What's the dt/dτ here? Don't worry about it. We actually don't need it for what follows, but we quickly wanted to insert it so as to remind you that we no longer have a unique concept of time: there is the time in our reference frame (t) – aka as the *coordinate time* – and the time in the reference frame of the object itself (τ) – which is known as the *proper time*. But let's get on with that equation above. It is a differential equation (it involves a derivative), but we don't need to solve it. We'll just derive an energy conservation equation from it. We do so by multiplying both sides with v = dx/dt. I am skipping a few steps (we're not going to do *all* of the work for you) but you should be able to verify the following:

$$v\frac{\mathrm{d}(\gamma \mathrm{m}_0 v)}{\mathrm{dt}} = -\mathrm{k}xv \Leftrightarrow \frac{\mathrm{d}(\mathrm{m}c^2)}{\mathrm{dt}} = -\frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2\right] \Leftrightarrow \frac{\mathrm{dE}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2 + \mathrm{m}c^2\right] = 0$$

So what's the energy concept here? We recognize the potential energy: it is the same $kx^2/2$ formula we got for the non-relativistic oscillator. No surprises: potential energy depends on position only, not on *velocity*, and there is nothing relative about position.[20] However, the $(½)m_0v^2$ term that we would get when using the non-relativistic formulation of Newton's Law is now replaced by the $mc^2 = \gamma m_0 c^2$ term. Of course, the sum of these two doesn't change. Hence, we get the total energy for this oscillator from

[19] Source: https://en.wikipedia.org/wiki/Sine#/media/File:Circle_cos_sin.gif. The illustration is public domain content from Wikimedia Commons.

[20] You may want to think about this.

equation x to 0, at which point the *velocity* of our mass will reach its maximum. That maximum is equal the speed of light in our electron model. Hence, we get the $E = m_c \cdot c^2$ formula.

What's m_c? It's the mass on the spring when $v = c$.

But that does not make much sense either, because we get zero (1 – 1 = 0) in the denominator of the Lorentz factor. So we are stuck here too! Our metaphor has obvious limits: it is just like God doesn't want us to know what *really* happens there. So what's the conclusion?

I am not so sure. The metaphor feels right – we have two oscillators working in tandem, *somehow* – but the zeroes or infinities in our simplistic models tell us it's *not* an easy idea to grasp. Hence, we're left with a funny feeling: what's going on here, *really*? The following reflection may help you to work yourself through that question.

Is the speed of light a velocity or a resonant frequency?

That's a good question! We think of it as a velocity. The idea of c being some resonant frequency of the spacetime fabric is tempting but... Well… It's not that easy to interpret it that way. Why not? Think of the following. One of the most obvious implications of Einstein's $E = mc^2$ equation is that the ratio between the energy and the mass of *any* particle is always equal to c^2. We write:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photon}}{m_{photon}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

This should, effectively, remind you of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ formulas of harmonic oscillators – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom.[21] In contrast, $c^2 = E/m$ for *any* particle, *always*. In fact, that's *exactly* the point we are trying to make here: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges here as *the* defining property of spacetime.

I should, perhaps, note that Maxwell's equations tell us exactly the same thing: c is *the* defining property of spacetime! It's the (absolute) propagation speed of an electromagnetic *signal*. As I must assume you have a basic background in physics – and in electromagnetics in particular – you will know Maxwell's theory was relativistically correct decades before Einstein actually invented the notion of what is and isn't relativistically correct. You will know that, in fact, it is fair to say that Einstein was inspired by the implications of Maxwell's equations: Einstein saw they had to be true and that, therefore, *Newtonian* or *Galilean* relativity had to be wrong.

I won't spend too much time on this. Let me just note that it is, in fact, *very* tempting to think of c as some kind of *resonant frequency*. However, the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will refer to as *the rest energy oscillation*. It is that what

[21] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the inertia, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

gives mass to our electron: its *rest* mass is nothing but the equivalent mass of the energy in its two-dimensional oscillation. As such, the only way we can interpret it, is as the velocity of the pointlike charge in its *Zitterbewegung*.

We should also note that we didn't really answer Dr. Burinskii's question here: what keeps the pointlike charge in its circular orbit? Perhaps we just can't solve that question: perhaps we should just accept it as a mystery. What's clear is that our electron occupies some space, and its *shape* changes as it picks up velocity: instead of a disk-like structure, it becomes an ellipsoid and – in the limit, when v approaches c – it just become a line – but with some *torque* on it. I am afraid that's all I can say about it.

Dirac once claimed that, if God exists, he must be a mathematician. If he is, he's surely a smarter mathematician than all of us, because I wonder how he gets away with those zeroes and those infinities. 😊

Jean Louis Van Belle, 1 March 2019