Refutation of ideals

Abstract: We evaluate the definition of ideals. Two of three parts are *not* tautologous, refuting ideals.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ⇸ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≙, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Uzcategui, C. (2019). Ideals on countable sets: a survey with questions.
arxiv.org/pdf/1902.08677.pdf cuzcatea@saber.uis.edu.co

An ideal I on a set X is a collection of subsets of X such that: (2.0.0)

Remark 2.0.0: The assertion is that each "such that" below implies Eq. 2.0.0 above as:

X∈I. (2.0.1)

LET p, q, r, s: A, B, I, X,

(s<r) ; **FFFF** **FFFF** TTTT **FFFF** (2.0.2)

(i) ∅∈I and X∉I. (2.1.1)

(((p@p)<r)&~(s<r))>(s<r) ; TTTT TTTT TTTT TTTT (2.1.2)

(ii) If A,B∈I, then A∪B∈I. (2.3.1)

(((p&q)<r)>((p+q)<r))>(s<r) ; **FFFF** **FFFF** TTTT **FFFF** (2.2.2)

(iii) If A⊆B and B∈I, then A∈I. (2.3.1)

((~(q<p)&(q<r))>(p<r))>(s<r) ; **FFFF** **FFFF** TTTT **FFFF** (2.3.2)

While Eq. 2.1.2 is tautologous, the other parts in Eq. 2.2.2 and 2.3.2 are *not* tautologous. This refutes the definition of ideals and subsequent assertions.