Toward new thought for the unified theory of electromagnetic field and gravitational field (Ⅱ)

(theory of classical field in KR space)

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Abstract: In this paper, we analyzed the difficulties of Maxwell's electromagnetic theory and Einstein's gravitational theory in detail, built a unified theory of field including consistent nonlinear electromagnetic theory and gravitational theory on the basis of new starting postulates, and extended these results into quantum electrodynamics. In this process, we accepted a new geometrical space in which metric tensor and all main physical functions becomes implicit function, called "KR space" conforming to our starting postulates, and normalization of implicit functions in order to connect all physical functions defined in this space with real world. Here, we naturally unraveled problem of radiation reaction, a historical difficult problem of Maxwell-Lorentz theory, established new quantum electrodynamics without renormalization procedure and also predicted some new theoretical consequences which could not find in traditional theories.

Keyword：Unified theory of field, Gravitational field, Electromagnetic field, KR space, Breaking of gauge symmetry, Quantum electrodynamics,

Contents

3. Physical Analysis for Main Functions Characterizing Particle-Field

In this chapter, we introduce KR space, a non-Euclidean space and, based upon it, define starting postulate 5 postponed in sect. 4 and then further give the physical analysis of main functions.

So, for evolution of our theory, why do we have to be forced to receive so complicated and intricate non-Euclidean KR space? The whole history of physics proved that space and time were a form of existence of matter and geometry of space-time was determined by essential content of matter. Actually as well known in history of physics, in Newton's classical mechanics were introduced the relativity principle of Galileo and three-dimensional spatial geometry. But later, the starting idea of Einstein, especially the idea of invariance of light velocity, led to a revolution in our understanding of matter and then birth of four-dimensional space-time. And in 1916, from the principle of equivalence of Einstein's GR was introduced the non-Euclidean four-dimensional Riemann space as a realistic space. Likewise, the new starting idea which the total energy of particle-its field should be m_0c^2 and the principle of correspondence give naturally birth to KR space and the Lagrangian integral formula is defined in it.

Sect. 9 KR space

We know that Euclidean space is corresponded to any point (or infinitesimal space region) of Riemann space $g'_{\mu\nu}(x)$ and this correspondence is expressed as follows:

$$\delta_{\mu\nu} = g'_{\mu\nu}(x = x_0 = \text{constant}) \qquad (9-1)$$

where $g'_{\mu\nu}$ is Riemann metric tensor and $\delta_{\mu\nu}$ is Euclidean metric tensor. On the other hand, Riemann space is corresponded to any point of Finsler space $g_{\mu\nu}(x, \dot{x})$.

$$g'_{\mu\nu}(x) = g_{\mu\nu}(x, \dot{x} = \dot{x}_0 = constant) \qquad (9-2)$$

where $g_{\mu\nu}$ is Finsler metric tensor and $g'_{\mu\nu}$ Riemann metric tensor. In a word, Riemannian space includes Euclidean space and Finsler space includes Riemann space as a special form.

But non-Euclidean space discussed in this paper is neither Riemann space nor Finsler space. The Lagrangian and equations of particle and field seen and derived before were defined in a new non-Euclidian space. We rewrite the metric tensor considered in formula (6-1)

$$g_{\lambda\sigma} = \delta_{\lambda\sigma}\left(1 + 2\alpha K_\mu u^\mu\right) \qquad (9-3)$$

that is, $g_{\lambda\sigma} = g_{\lambda\sigma}(x_i, u_i)$ and the field function, K_λ, of formula (9-3) to be studied sect. 11 and sect. 16 has the following form:

$$K_\lambda \sim \frac{\underline{K_\lambda}}{\left(1 + 2\alpha K_\mu u^\mu\right)} = \underline{K_\lambda} g^{\mu\nu}\delta_{\mu\nu} \qquad (9-4)$$

where $\delta_{\mu\nu}$ is Minkowski metric tensor, $\underline{K_\lambda}$ the field function obtained in Minkowski space (one in Maxwell's covariant theory) and in case of electromagnetic field becomes $\underline{K_\lambda} = e\dot{x}_\lambda / r$. And in formula (9-4), $g^{\mu\nu}$ has the form

$$g^{\mu\nu} = \delta^{\mu\nu}\frac{1}{1 + 2\alpha K_\mu u^\mu}$$

On the other hand, in formula (9-3) we have

$$u^i = \frac{dx}{cdt(g_{lk}\dot{x}^l\dot{x}^k)^{1/2}} = u^i(\dot{x}, g) \qquad (9-5)$$

$$K_i = K_i(x, g)$$

Consequently, from formula (9-3), (9-4), (9-5) follows

$$g_{\lambda\sigma} = g_{\lambda\sigma}(x_i, \dot{x}_i, g) \qquad (9-6)$$

(※function g included in the right-hand side of formula (9-5) and (9-6) reflects the implicit character of function)

As shown in formula (9-6), space metric has a new character dependent on metric itself in addition to coordinates and velocity. In the formula (9-6) of metric tensor, because K_λ and u_λ is functions dependent on metric, g, implicit function of metric has the characteristic of absolute implicit function which cannot be, by any means, transformed into explicit function. In this connection, we define a new non-Euclidean space called "KR space". We obtain Finsler metric tensor $g_{\lambda\sigma}(x,\dot{x})$, fixing metric of KR space, $g_{\lambda\sigma}(x,\dot{x},g_{ik})$, by a Euclidean constant metric, η_{ik}

$$g_{\lambda\sigma}(x,\dot{x},g_{ik}=\eta_{ik})=g_{\lambda\sigma}(x,\dot{x}) \qquad (9-7)$$

Consequently, KR space is meant by the space whose metric function, as function of metric itself, owns implicit character of function and that Finsler space is corresponded to every points $(x,\dot{x})$ of KR space. Namely, when $g_{ik}=\eta_{ik}$, formula (9-6) naturally results in

$$g_{\lambda\sigma}=\delta_{\lambda\sigma}\left(1+2\alpha\underline{K_i}\underline{u^i}\right) \qquad (9-8)$$

$$\underline{u_i}=u_i(\dot{x}), \quad \underline{K_i}=K_i(x)$$

where $\underline{K_i}$ and $\underline{u_i}$ are four dimensional functions in Maxwell's theory, defined in Minkowski space.

- **The normalization of metric**

Because of the character of absolute implicit function of metric in KR space, it is impossible to say its realistic meaning and essence of metric, necessary for measurement. To do this, we introduce an idea of normalization of metric. g_{ik} can be fixed by arbitrary constant metric in every point $(x,\dot{x})$ of KR space, and so Finsler spaces corresponded to $(x,\dot{x})$ is also different.

We define the *normalization of metric* or *normalized metric* as obtaining the metric of Finsler space, fixing g_{ik} by Minkowski metric δ_{ik} in any point $(x,\dot{x},g)$ of KR space.

From this definition, we have

$$\bar{g}_{\lambda\sigma}=g_{\lambda\sigma}(x,\dot{x},g_{ik}=\delta_{ik})=g_{\lambda\sigma}(x,\dot{x}) \qquad (9-9)$$

where $\bar{g}_{\lambda\sigma}$ is the normalized metric. The metric obtained from formula (9-9) is obviously Finsler metric and has the realistic physical meaning in every point $(x_i,\dot{x}_i)$. (KR space has not been thoroughly studied mathematically and so in this paper we also did not offer the rigorous mathematical definition and analysis of KR space. Therefore, we think that there can or will be many faults in our description for this space) The normalization of metric is the key to arguing the normalization of implicit function in the next sections.

Sect. 10 The normalization of absolute implicit function and its physical meaning

As already seen, functions characterizing field and particle in KR space, from the character of implicit function of metric, also become implicit ones. The implicit function defined in KR space has the character that cannot transform into explicit function by any manner of means. These implicit functions are sometimes called *absolute implicit functions*. The absolute implicit function has the following general form

$$F=F\big(x,F(x,g),f(x,\dot{x},g)\big), \qquad g=g(x,\dot{x},F) \qquad (10-1)$$

where $f(x,\dot{x},g)$ is all functions dependent on metric g. For example, $F(x,g)$ may be either electromagnetic field potential, $A_\lambda(x,g)$, or gravitational field potential, $G_\lambda(x,g)$, and $f(x,\dot{x},g)$ be four-dimensional velocity in KR space, $u_\lambda(x,\dot{x},g)$.

Generally, if coordinates of space-time is given, the value of a function is uniquely defined. But in KR space, owing to the implicit character of the function, even though coordinates are given, the value of the function is not uniquely determined and so it is impossible to consider the real physical meaning of the function. In this regards, we will first see how to define implicit functions and give real physical meaning to implicit function in order to get measurable physical quantities.

The real physical meaning of implicit function, F, is determined by normalized function, $\bar{F}$. We, again, represent the starting postulate 5, postponed in sect. 4; Postulate 5: In unified theory of fields, every physical quantity is expressed as implicit function (correctly speaking, absolute implicit functions) and the functions that have real physical meaning are determined as explicit ones uniquely corresponded to Finsler space according to rule of normalization.

The normalization of implicit function is, in essential, related to normalization of metric tensor. It is referred to the fact that a function includes metric as a variable and metric also includes the function itself as a variable. From this fact follows the implicit character of function.

Now, we define rules of normalization.

1) As for implicit function, F, characterizing fields or interaction between particle and field in KR space, we fix g_{ik} which becomes variable of $F(x,g)$ and $f(x,\dot{x},g)$ by Minkowski metric $\delta_{\mu\nu}$, and obtain $\bar{F}$ corresponded to Finsler space called "normalized function", which is defined as "normalization of implicit function", F.

$$\bar{F} = F\big(x, F(x, g|_\delta), f(x, \dot{x}, g|_\delta)\big) = F(x, \dot{x}) \qquad (10-2)$$

This rule is key to removing infiniteness of physical quantities and getting finite quantities

The normalization of four dimensional velocity is defined as follows: As for implicit function, u, which denotes four-dimensional velocity in KR space, we fix g_{ik} which be variable of u by Minkowski metric $\delta_{\mu\nu}$ and obtain $\bar{u}$ corresponded to Finsler space called *normalized four-dimensional velocity*, which is defined as normalization of implicit function, u, according to rule of normalization 1, namely,

$$\bar{u}_\lambda = u_\lambda(\dot{x}, g|_\delta) = u_\lambda(\dot{x}) \qquad (10-3)$$

$$\bar{g}_{\mu\nu} = \delta_{\mu\nu}\left(1 + 2\alpha\bar{K}_\lambda u^\lambda(\dot{x}, g|_\delta)\right) = \delta_{\mu\nu}\left(1 + 2\alpha\bar{K}_\lambda u^\lambda(\dot{x})\right) \qquad (10-4)$$

where the field function, $\bar{K}_\lambda$, already normalized according to rule 1 was used as it is.

2) For a function $\varepsilon(x,F)$, the normalized function $\bar{\varepsilon}$ is as follows:

$$\bar{\varepsilon} = \bar{\varepsilon}(x, \bar{F}) \qquad (10-5)$$

where $\bar{F}$ is normalized function of F.

3) In case where a function ε consists of series of $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n$, normalization of ε is the same as the sum of normalization of individual terms, i.e.

$$\varepsilon = \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + \cdots + \bar{\varepsilon}_n \qquad (10-6)$$

This idea and rules of normalization become one of important starting points and are essential to establish unified theory of fields. If so, what are true essence and significance of the introduction of implicit function and its normalization? Why should physical quantities be expressed as implicit function and what is its true meaning? If one first says conclusion, what physical quantity is expressed as implicit function is because of feedback of particle-field. Until now, in history of physics has been considered only that a particle, through field, acts on another particle or is acted on by the field created by other particle. But as well known, a particle acts on itself through field produced by the particle. That is to say, particle-field has the character of feedback.

Particle $\rightarrow$ field (field is created by a particle)

Field $\rightarrow$ particle (the field acts on the particle)

The result becomes $F = F\big(x, F(x,g), f(x,\dot{x},g)\big)$, But in case of introducing this feedback to former theory (Maxwell-Lorentz covariant theory), it was well known that unavoidable divergence occurs. As we shall see later, this problem of divergence in KR space is very easily solved, referring to the fact that in KR space, the character of feedback of particle-field is reflected in metric and functions characterizing physical quantities, and so physical quantities come to be expressed as implicit function.

If so, why does feedback in theory of field appear at all? The modern "systems theory" has already clarified that feedback is an important aspect of the nature of matter. As far as external action is applied to matter, some effect surely occurs. Here external action means factors giving birth to effect. For example, in physics external action is meant by external force and in biology it is the change of external environment.

According to systems theory, the effect occurred is applied again to external action giving birth to it. Furthermore, feedback brings about reaction, which is rooted in self-maintenance or self-resistance character of matter which is going to diminish effect by external action as possible as it can. For example, in case of Newtonian mechanics reaction occurs owing to inertia of an object and in case of electrodynamics there exists radiation damping by feedback of radiation wave. Further in case of biology, whenever temperature of surrounding increases, in organism arises endothermic reaction and whenever temperature of surrounding decreases, occurs exothermic reaction to maintain constant temperature of organism. Consequently, feedback is based on the nature of matter and implicit function is mathematical manifestation of feedback. From this, manifestation of implicit function is not artificially introduced but comes naturally from the nature of matter. This is a very important conclusion. On the other hand, matter has wave-particle dualism. In quantum mechanics, one introduced wave model and wave function, and defined real physical meaning of wave function as $|\psi|^2$ in order to describe characteristic of wave of matter. In a similar way to this, in our theory, in order to remove infinite quantities and clarify characteristic of particle without any contradiction, implicit function (inevitable result of feedback character in model of point particle) was introduced and real physical meaning of implicit function defined as the normalized implicit function $\bar{F}$. The description of wave property and particle property has interesting symmetry (table 1). Thus, in order to describe mathematically wave property of matter, just as introduction of complex wave function is indispensable, so for the consistent description of particle property should be introduced implicit function and for clarification of its physical meaning should be defined normalization of implicit function. As we will see in next sections, finiteness of physical quantities and unification of electromagnetic-gravitational field are obtained based upon this idea of normalization.

Table 1

	wave property	**particle property**
model	wave (realistically no exist)	point (realistically no exist)
mathematical description	complex wave function (unmeasurable quantity)	implicit function in KR space (unmeasurable quantity)
physical meaning	defined as $|\psi|^2$	defined as normalized function

4. Theory of Electromagnetic Field in KR Space (Nonlinear Theory of Electromagnetic Field)

In Physics, Maxwell's theory has been recognized as the complete and unique theory for electromagnetic field. The Maxwell's theory was never established so deductively like Einstein's GR. It is well known that Maxwell, by generalizing and systemizing experimental laws discovered at those days, established the well-defined and beautiful theory for electromagnetic field. Of course, it is too far gone to view that Maxwell's theory was built only by inductive method. In fact, Maxwell, generalizing Ampere law so that the law of charge conservation is in agreement with a generalized form, discovered displacement current and established an equation system of field with four-dimensional covariant forms and then gave the wonderful prediction for generation of electromagnetic wave. This course for building of theory can be viewed to be based on inductive method. But, mainly clinging to inductive method, Maxwell did not complete the theory so that, on the basis of such main principles of physics as law of energy-momentum conservation, the total energy of particle and field possesses finite value.

The experimental laws which underlie Maxwell's theory clearly were discovered and argued within weak field. The weak field is meant by the field which interactional energy of particles is much less than rest energy of a particle m_0c^2. In this case $e^2/m_0c^2 \ll 1$ is satisfied and is dealt with interaction between particle and field in distance father than electron radius $e^2/m_0c^2 \approx 10^{-1}\ cm$. Simply put, in Maxwell's theory was considered the field made by a macro-charge and electric current. But there is no special reason that experimental laws discovered within weak field should always be in agreement with those in strong field. Actually in scientific view, it is reasonable that new characteristics which cannot be found in weak field is manifested in strong field. But, it is impossible that one carries out all experiments in the whole area comprising not only weak field but also strong field. So, that one uses inductive method as well as deductive method based on main principles of physics to establish more generalized and consistent theory is essential for building of theory. In this regard, main difficulties of Maxwell's theory are rooted in concluding that the experimental facts obtained in weak field are universal truths in the whole region comprising weak field and strong field and making no advance into establishment of more generalized and consistent theory.

Sect. 11 The equation of electrostatic field and variation of Coulomb's law

From equation (6-3) and (6-4) can be easily obtained equation of electrostatic field

$$\nabla^2\varphi = -4\pi\frac{e}{1+2\alpha\varphi u^0}\delta(\mathbf{r}-\mathbf{r}_0) \qquad (11-1)$$

where $\alpha = e/m_0c^2$. This equation, from the character of electrostatic field, satisfies the condition $d\bar{e}/dt = 0$ and from this equation we can obtain the solution of potential of electrostatic field. Supposing that right-hand side of this equation, as density of effective charge, is known at spatial coordinate, $\mathbf{r}_0$, we can find the solution of the above equation by the well-known method

$$\varphi = \int\frac{e}{r(1+2\alpha\varphi u^0)}\delta(\boldsymbol{r}-\boldsymbol{r}_0)dV = \frac{e}{r(1+2\alpha\varphi(\boldsymbol{r}_0)u^0)} + C \qquad (11-2)$$

where $\varphi(\mathbf{r}_0)$ is the potential at spatial coordinate, $\mathbf{r}_0$, where charge is placed. $\varphi(\mathbf{r}_0)$ consists of two components, i.e. $\varphi^{ex}(\mathbf{r}_0)$ (the potential in external field) and $\varphi^{in}(\mathbf{r}_0)$ (the potential in self-field)

$$\varphi(\boldsymbol{r}_0) = \varphi^{ex}(\boldsymbol{r}_0, g) + \varphi^{in}(\boldsymbol{r}_0, g) \qquad (11-3)$$

From formula (11-3), (11-2) for $\varphi(\mathbf{r}_0, g)$, we get

$$\varphi(r, \mathbf{r}_0) = \frac{e}{r\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0, g)+\varphi^{in}(\mathbf{r}_0, g)\right)u^0\right)\right]} + C \qquad (11-4)$$

where potential in external field $\varphi^{ex}(\mathbf{r}_0)$ is given automatically by the initial condition.

Now, let us find $\varphi^{in}(\mathbf{r}_0)$ from formula (11-4). The field produced by a charge, e, acting on the charge itself can be define as follows.

$$\varphi^{in}(\mathbf{r}_0) = \lim_{r\to 0}\varphi(r, \mathbf{r}_0) = \lim_{r\to 0}\frac{e}{r\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0, g)+\varphi^{in}(\mathbf{r}_0, g)\right)u^0\right)\right]} + C \qquad (11-5)$$

where $\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0, g)+\varphi^{in}(\mathbf{r}_0, g)\right)u^0\right)\right]$ is metric component of KR space. Because of $g = g\left(x, \dot{x}, \varphi^{in}(\mathbf{r}_0, g)u^0\right)$, $\varphi^{in}(\mathbf{r}_0)$ results in absolute implicit function. The normalization of implicit function φ^{in} subject to normalization rules (10-2) and (10-3) yields $\bar{\varphi}^{in}(\mathbf{r}_0) = \varphi^{in}(\mathbf{r}_0, \bar{g})$

$$\bar{\varphi}^{in}(\mathbf{r}_0) = \lim_{r\to 0}\varphi(r, \mathbf{r}_0, \bar{g}) = \lim_{r\to 0}\frac{e}{r\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0, \bar{g})+\varphi^{in}(\mathbf{r}_0, \bar{g})\right)\underline{u}^0\right)\right]} + C$$

$$= \lim_{r\to 0}\frac{e}{r\left[\left(1+2\alpha\left(\underline{\varphi}^{ex}(\mathbf{r}_0)+\underline{\varphi}^{in}(\mathbf{r}_0)\right)\underline{u}^0\right)\right]} + C \qquad (11-6)$$

(See formulas (9-4), (9-5), (9-8)), where $\underline{\varphi}^{in}$ and $\underline{\varphi}^{ex}$ are potentials defined in Minkowski four-dimensional space-time in Maxwell's theory and $\underline{u}^0 = u^0(\dot{x}, g|_\delta) = 1$. Thus, formula (11-6) gives

$$\bar{\varphi}^{in}(\mathbf{r}_0) = \lim_{r\to 0} \frac{e}{r\left[1 + 2\alpha\left(\frac{e}{r} + \underline{\varphi}^{ex}(\mathbf{r}_0)\right)\right]} + C = \frac{m_0 c^2}{2e} + C$$

$$(11-7)$$

On the other hand, according to the starting postulate 1, the Lagrangian for the motion of free-charge should be the same as one in special theory of relativity (SR). Thus, formula (11-7) gives

$$\bar{\varphi}^{in}(\mathbf{r}_0) = \frac{m_0 c^2}{2e} + C = 0$$

$$C = -\frac{m_0 c^2}{2e} \qquad (11-8)$$

Therefore, electrostatic potential can be written as follows.

$$\begin{cases} \varphi = \dfrac{e}{r\left[\left(1 + 2\alpha\left(\bar{\varphi}^{ex}(\mathbf{r}_0) + \bar{\varphi}^{in}(\mathbf{r}_0)\right)\right)\right]} + C \\ \bar{\varphi}^{in}(\mathbf{r}_0) = 0 \\ C = 0 \end{cases} \qquad (11-9)$$

In formula (11-9), C constant, because field vanishes at infinity far from source charge, should become zero. It is remarkable that, in our theory, a constant of electrostatic potential is not such arbitrary value as in Maxwell's theory but one determined uniquely under some physical condition.

The formula (11-9) yields important conclusions.

1. As easily understandable in formula (11-9), the electrostatic potential depends on external field φ^{ex}, *and so our theory does not agree with principle of superposition; that is, the field is not equal to the arithmetical sum of fields produced by individual charges.*

2. In formula (11-9), as C *constant is defined uniquely, one of difficulties of Maxwell's theory seen in sect. 2, inconsistency which the energy of system loses the physical meaning is very easily solved.*

3. From formula (11-9) follows the conclusion according to which electron and positron cannot approach infinitely near.

The interactional energy of electron-positron can be written as follows.

$$E = -\frac{e^2}{r\left(1 - 2\frac{e^2}{m_0 c^2}\cdot\frac{1}{r}\right)} \qquad (11-10)$$

In case where electron and positron approach the region of electron radius $r = 2e^2/m_0c^2 \approx 10^{-13}cm$, interactional energy, E diverges. In order that interactional energy possesses finite value, e^2 in denominator should vanish. From this is drawn a new conclusion that electron radius $2e^2/m_0c^2$ is the critical distance which electron and positron can approach and if electron and positron approaches this distance, annihilation of electron and positron and production of new particles should follow. Figuratively speaking, this is such a similar situation as the relation between mass and velocity in SR; when matter approaches the velocity of light, rest mass should become zero and then photon with rest mass of zero is predicted.

4. The potential formula (11-9) help us resolve the problem of divergence of the total energy of particle-field (it will be seen in sect. 13).

Sect. 12 The equation of electromagnetic field and finiteness of radiation damping

Here, we drive the equation of electromagnetic field and resolve the problem for infiniteness of radiation damping potential, i.e. a fatal flaw of Maxwell's theory. We rewrite equation of field (6-3) [1]

$$\frac{\partial F_{ik}}{\partial x^k} = -\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (12-1)$$

The substitution of F_{ik} expressed by potentials into equation (12-1) gives

$$\frac{\partial^2 A_k}{\partial x^i \partial x^k} - \frac{\partial^2 A_i}{\partial x^k \partial x^k} = -\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (12-2)$$

Taking four dimensional divergences on both sides of equation (12-2), the left-hand side from definition of F_{ik} becomes zero

$$\nabla_i\left(\frac{\partial F_{ik}}{\partial x^k}\right) = 0 \qquad (12-3)$$

On the other hand, the right-hand side becomes zero, provided that the approximate condition $d\bar{e}/dt \approx 0$ considered in sect. 6 holds. So, equation (12-2) is satisfied.

According to theory for equation of partial differentiation well known in mathematics, as for partial differentiation equations with 4 unknowns, if there exists one identity between unknowns, independent unknowns are 3 and then one equation can be arbitrarily chosen. In this regards, an additional condition is given as follows:

$$\frac{\partial A_i}{\partial x^i} = 0 \qquad (12-4)$$

In Maxwell's theory, in case where equation (12-4) is satisfied, one can take a gauge transformation $A_i' = A_i + \frac{\partial f}{\partial x^i}$, where f obeys the following wave equation.

$$\Delta f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0 \qquad (12-5)$$

But as Gauge invariance is not satisfied in our theory, the condition like equation (12-5) cannot hold. In relation with the argument mentioned above, some books described as if the Lorentz condition (12-4) was drawn from gauge invariance. But, this is wrong. For example, Einstein's GR, from nonlinear character of equations, does not obey Gauge invariance, but by the theory of partial differentiation equation is obtained an additional condition similar to equation (12-4). In Riemannian space, the equation of gravitational field is written as follows.

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi k}{c^4}T_{ik} \qquad (12-6)$$

Taking four-dimensional divergences on both of sides, the left-hand side and right-hand side become zero

$$\nabla_i\left(R_{ik} - \frac{1}{2}g_{ik}R\right) = 0 \qquad (12-7)$$

Thus, among 10 unknowns, independent unknowns are 6, remained 4 unknowns can be arbitrarily chosen. So we can give an additional condition

$$g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0 \qquad (12-8)$$

where $\Gamma^{\lambda}_{\mu\nu}$ is Christoffel symbol, which consists of the sum of partial differentiation with respect to $g_{\mu\nu}$.

From the weak field approximation, $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$ ($h_{\mu\nu}$ is potential of weak field) and $\psi_i^k = h_i^k - \frac{1}{2}\delta_i^k h$ are allowed, and formula (12-8) leads to

$$\frac{\partial \psi_i^k}{\partial x^k} = 0 \qquad (12-9)$$

The equation (12-9) is similar to equation (12-4) in Maxwell's theory. In GR, using the equation (12-9), one can obtain the linear partial differential equation similar to that in Maxwell's theory and derive a formula for radiation of gravitational wave from this equation. All of these show that equation (12-4) in our theory and an additional condition (12-9) in GR follow from character of field equation itself, not from gauge principle.

Now, let us return to the main subject and if one substitutes equation (12-4) into equation (12-2), then field equation (12-2) becomes

$$\Box A_i = -\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (12-10)$$

If we finds the solution of the equation (12-10) according to the well-known method of solution, we have

$$\varphi = \frac{\bar{e}(t-R/c)}{R} + \varphi_0 \ , \qquad \mathbf{A} = \frac{1}{c}\cdot\frac{\bar{e}\mathbf{V}(t-R/c)}{R} + \mathbf{A}_0 \qquad (12-11)$$

where φ_0 and $\mathbf{A}_0$ mean external field and are not arbitrary. In formula (12-11) effective charge, source of field, as shown in formula (5-9), depends on the interaction Lagrangian of particle and field, $(A_\lambda u^\lambda)$ and then stands against principle of superposition. This fact is a key to solve the problem of radiation damping, recognized as knotty point, "greatest crisis in Maxwell's theory" [2].

In order to get potentials of radiation damping produced by a moving charge, the following approximation is used.

1) Suppose that the distance between a moving particle and a system of charges which create external field acting on it is much farther than the radius of electron $r_0 = e^2/m_0c^2$. In this case $\frac{e}{m_0c^2}A_\lambda u^\lambda \ll 1$ holds (see formula (5-5)). The approximate formula (5-4) is obtained from condition $\frac{e}{m_0c^2}A_\lambda u^\lambda \ll 1$

$$dS \approx -m_0c^2\int dt\left(1-\frac{V^2}{c^2}\right) - \frac{e}{c}\int A_\lambda dx^\lambda \qquad (12-12)$$

where $A_\lambda = A_\lambda^{in} + A_\lambda^{ex}$ and A_λ^{in}, as a finite quantity, is supposed to be very small quantity (order $1/c^3$).

2) Suppose the equation of field produced by a moving particle satisfies enough the condition $|d\bar{e}/dt| \ll 1$ or formula (6-6) (see reference [1]). In this case, we can write equation (12-10) by the vector potential and scalar potential of the field, respectively

$$\Box \boldsymbol{A} = -\frac{4\pi}{c}\cdot\frac{e\mathbf{V}}{1+2\alpha\left(A_\lambda^{in}+A_\lambda^{ex}\right)u^\lambda}\delta(\mathbf{r}-\mathbf{r}_0)$$

$$\Box\varphi = -4\pi\cdot\frac{e}{1+2\alpha\left(A_\lambda^{in}+A_\lambda^{ex}\right)u^\lambda}\delta(\mathbf{r}-\mathbf{r}_0)$$

$$(12-13)$$

From the well-known method of solution are obtained the following solutions

$$\mathbf{A} = -\frac{1}{c}\cdot\frac{e\mathbf{V}\left(t-\frac{R}{c}\right)}{R\left[1+2\alpha\left(A_{\lambda(1)}^{in}+A_\lambda^{ex}\right)u^\lambda\right]} + \mathbf{C}_1 \qquad (12-14)$$

$$\varphi = \frac{e\left(t-\frac{R}{c}\right)}{R\left[1+2\alpha\left(A_{\lambda(1)}^{in}+A_\lambda^{ex}\right)u^\lambda\right]} + C_2 \qquad (12-15)$$

where A_λ^{ex} is the potential of external field and $A_{\lambda(1)}^{in}$ term of first order of expansion of field potential acting on particle itself in powers of R/c. Terms more than second order have more than $1/c^4$ which in our consideration was ignored.

Under the condition $V \ll c$, expansion of potential gives

$$\varphi = b\frac{e}{R} - \frac{1}{c}\cdot\frac{\partial e}{\partial t} + b\frac{e}{2c^2}\cdot\frac{\partial^2 R}{\partial t^2} - b\frac{e}{6c^3}\cdot\frac{\partial^3 R^2}{\partial t^3} = \varphi_{(1)} + \varphi_{(2)} + \varphi_{(3)} + \varphi_{(4)} \qquad (12-16)$$

$$\mathbf{A} = b\frac{1}{c}\cdot\frac{e\mathbf{V}}{R} - b\frac{e}{c^2}\cdot\frac{\partial \mathbf{V}}{\partial t} = \mathbf{A}_{(1)} + \mathbf{A}_{(2)} \qquad (12-17)$$

$$b = \frac{1}{1 + 2\alpha\left(A^{in}_{\lambda(1)} + A^{ex}_{\lambda}\right)u^{\lambda}}$$

where $\varphi_{(1)}$, $\varphi_{(2)}$, $\varphi_{(3)}$ and $\varphi_{(4)}$ are terms of first order, second order , third order and fourth order of a power series of φ in R/c and $\mathbf{A}_{(1)}$ and $\mathbf{A}_{(2)}$ are terms of first order and second order of a power series of $\mathbf{A}$ in R/c.

Next, if one lets R (radius of interaction) go to zero to get potentials of radiation damping, φ^{in} and $\mathbf{A}^{in}$ in formula (12-16) and (12-17) are found from

$$\varphi^{in} = \lim_{R\to 0}\varphi, \qquad \mathbf{A}^{in} = \lim_{R\to 0}\mathbf{A} \qquad (12-18)$$

where φ^{in} and $\mathbf{A}^{in}$, as $\varphi^{in} = \varphi^{in}(x,g)$ and $\mathbf{A}^{in} = \mathbf{A}^{in}(x,g)$, are implicit functions.

Now, if one, according to normalization rule 4 defined in sect. 10, normalizes φ^{in} and $\mathbf{A}^{in}$, the results are

$$\bar{\varphi}^{in} = \bar{\varphi}^{in}_{(1)} + \bar{\varphi}^{in}_{(2)} + \bar{\varphi}^{in}_{(3)} + \bar{\varphi}^{in}_{(4)} \qquad (12-19)$$

$$\bar{\boldsymbol{A}}^{in} = \bar{\boldsymbol{A}}^{in}_{(1)} + \bar{\boldsymbol{A}}^{in}_{(2)} \qquad (12-20)$$

With reference of potential formula (11-5), according to normalization rule 1, let us normalize $\varphi^{in}_{(1)}$ and $\mathbf{A}^{in}_{(1)}$ (divergent quantities). The normalized functions result in $\bar{\varphi}^{in}_{(1)} = \varphi(x,\bar{g})$, $\bar{\mathbf{A}}^{in}_{(1)} = \mathbf{A}(x,\bar{g})$ and $\bar{g}$ is the normalized metric tensor. Consequently, first order terms, $\varphi^{in}_{(1)}$ and $\mathbf{A}^{in}_{(1)}$ become

$$\bar{\varphi}^{in}_{(1)} = \lim_{R\to 0}\frac{e}{R\left[1 + 2\alpha\left(\frac{e}{R} - \frac{e}{R}\cdot\frac{V^2}{c^2}\right) + 2\alpha A^{ex}_{\lambda}u^{\lambda}\right]} + C_0 = \frac{1}{2\alpha(1-\beta^2)} + C_0 \qquad (12-21)$$

$$\bar{\mathbf{A}}^{in}_{(1)} = \lim_{R\to 0}\frac{1}{c}\cdot\frac{e\mathbf{V}}{R\left[1 + 2\alpha\left(\frac{e}{R} - \frac{e}{R}\cdot\frac{V^2}{c^2}\right) + 2\alpha A^{ex}_{\lambda}u^{\lambda}\right]} + \mathbf{C}_1 = \frac{1}{c}\cdot\frac{\mathbf{V}}{2\alpha(1-\beta^2)} + \mathbf{C}_1 \qquad (12-22)$$

(see formula (9-8), (9-9)). Now, if we finds C_0 and $\mathbf{C}_1$ so that $\frac{1}{c}\bar{A}^{in}_{\lambda(1)}\dot{x}^{\lambda} = 0$, we have

$$\frac{1}{c}\bar{A}^{in}_{\lambda(1)}\dot{x}^{\lambda} = \varphi^{in}_{(1)} - \frac{1}{c}\mathbf{A}^{in}_{(1)}\mathbf{V} = \frac{(1-\beta^2)}{2\alpha(1-\beta^2)} + C_0 - \frac{\mathbf{V}}{c}\mathbf{C}_1 = 0$$

If one puts in $\mathbf{C}_1 = \mathbf{0}$ and $C_0 = -1/2\alpha$, formula (12-22) has

$$\frac{1}{c}\bar{A}^{in}_{\lambda(1)}\dot{x}^{\lambda} = 0 \qquad (12-23)$$

In the Lagrangian integral formula (12-1) for the motion of a charge, the term of first order of interaction with self-field, i.e. $\frac{e}{c}\bar{A}^{in}_{\lambda(1)}dx^{\lambda}$, vanishes. Thus, $\left(\bar{\varphi}^{in}_{(1)}, \bar{\mathbf{A}}^{in}_{(1)}\right)$ have not influence on the potential of radiation damping. $\bar{\varphi}^{in}_{(2)}$ vanishes because e in $\bar{\varphi}^{in}_{(2)}$ term is constant and then the differentiation of time is zero. Next, in $\bar{\varphi}^{in}_{(3)}$ term, according to normalization rule 3, $A^{in}_{\lambda(1)}$ in denominator of b should be replaced by already normalized potential $\bar{A}^{in}_{\lambda(1)}$ and $\bar{A}^{in}_{\lambda(1)}\underline{u}^{\lambda}$ becomes zero in terms of formula (12-23). Accordingly, $\bar{\mathbf{E}}^{in}_{(3)}$ expressed by $\bar{\mathbf{E}}^{in}_{(3)} = \mathrm{grad}\bar{\varphi}^{in}_{(3)}$ becomes

$$\bar{\mathbf{E}}^{in}_{(3)} = \lim_{R\to 0} \mathrm{grad}\bar{\varphi}^{in}_{(3)} = b\frac{e}{2c^2}\dot{\mathbf{n}} \quad (12-24)$$

with

$$\dot{\mathbf{n}} = \frac{\partial}{\partial t}\frac{\mathbf{R}}{R}$$

where b has 1 from $\frac{e}{c}\bar{A}^{in}_{\lambda(1)}dx^{\lambda} = 0$ and $\mathbf{n}$ is unit vector of radius vector $\mathbf{R}$ from the charge to the given field point. Supposing time change of the unit vector is very slow, so we can put in $\bar{\mathbf{E}}^{in}_{(3)} = \mathbf{0}$. Finally, $\bar{\varphi}^{in}_{(4)}$ does not go to zero. In Maxwell's theory, $\varphi_{(4)}$, by gauge transformation, leads to zero. But in our theory as gauge principle is not allowed, $\varphi_{(4)}$ cannot be transformed to zero. b in $\bar{\varphi}^{in}_{(4)}$ term is equal to 1 in same way as in case of $\bar{\varphi}^{in}_{(3)}$, that is why, $\bar{\varphi}^{in}_{(4)}$ is the same as fourth term of power series in Maxwell's theory (The dependence of $\bar{\varphi}^{in}_{(4)}$ on external field is ignored in our consideration, because it includes high order term more than $1/c^4$).

Now, let us find electric force $\mathbf{E}^{in}_{(4)}$ from $\bar{\varphi}^{in}_{(4)}$

$$\mathbf{E}^{in}_{(4)} = -\mathrm{grad}\bar{\varphi}^{in}_{(4)} = \frac{e}{6c^3}\cdot\frac{\partial^3}{\partial t^3}(\nabla R^2) = \frac{e}{3c^3}\cdot\frac{\partial^3\mathbf{R}}{\partial t^3}$$

$$\mathbf{R} = \mathbf{R_0} - \mathbf{r}, \quad \dot{\mathbf{R}} = -\dot{\mathbf{r}} = -\mathbf{V}, \quad \ddot{\mathbf{R}} = -\dot{\mathbf{V}}, \quad \dddot{\mathbf{R}} = -\ddot{\mathbf{V}}$$

where $\mathbf{R_0}$ is the distance from the reference point to the given field point and $\mathbf{r}$ from the reference point to the charge. Therefore, the result is

$$\mathbf{E}^{in}_{(4)} = -\frac{e}{3c^3}\ddot{\mathbf{V}} \quad (12-25)$$

Next, let us get $\mathbf{E}^{in}_{(2)}$ from $\bar{\mathbf{A}}^{in}_{(2)}$. As the method is formally the same as finding $\mathbf{E}^{in}_{(4)}$, we write only result

$$\mathbf{E}^{in}_{(2)} = -\frac{1}{c}\frac{\partial\bar{\mathbf{A}}^{in}_{(2)}}{\partial t} = \frac{1}{c^3}e\ddot{\mathbf{V}} \quad (12-26)$$

where normalization of $\bar{\mathbf{A}}^{in}_{(2)}$ was done in same way as in $\bar{\varphi}^{in}_{(2)}$. Thus, the force of radiation damping can be written as follows.

$$\mathbf{F}_R = e\mathbf{E}^{in}_{(4)} + e\mathbf{E}^{in}_{(2)} = \frac{2}{3c^3}e\ddot{\mathbf{V}} \quad (12-27)$$

The result is similar to that of Maxwell's theory. But there is the essential difference in content. First, in our theory, infinite divergence vanishes naturally and without any contradiction. Second, in our theory was discussed radiation damping effect without gauge transformation, from the fact that gauge principle is not valid and gave the formula consistent to experiment of radiation damping.

We used an approximate method for description of sect. 11 and sect. 12. As for this, there will be, we are sure, readers who express dissatisfaction. In fact, the left-hand side of field equation is the same as in Maxwell's theory. We are sure that a completed nonlinear equation of field will be, in the future, obtained. But even though new solution for nonlinear equation is found, it has not influence on our description for natural elimination of infinite quantities with method of normalization. It is due to the fact that as far as in right -hand side of equation there exists current of effective charges, functions of potential field are always expressed as implicit function and our method according to which eliminates divergence using rules of normalization cannot be altered but applied as it is. On the other hand, in case of expanding potentials as a power series, under some condition $(V/c \approx 1)$, divergence is always manifested in term of first order of expansion. This shows that even if completed potential equation and solution is, in the future, founded, correction newly added to former solution will be reflected in higher order term, not first order term and then not give rise to divergence. Thus in our argument, as divergence manifested in first order term is naturally removed by normalization of implicit function, this method will be still valid for completed nonlinear equation to be founded in the future.

Sect. 13 Breaking of gauge symmetry and its physical meaning, equivalence of inertial mass and total energy of particle-field.

(1) Breaking of gauge symmetry and its physical meaning,

In this sect., first of all, we intend to argue some problems related to breaking of gauge symmetry. As a result necessarily following from Lagrangian integral (5-1) (see reference [1]), our electrodynamics with non-linearity does not agree with gauge symmetry. Actually, modern theory of fields (except gravitation) is based upon gauge symmetry. Hence, breaking of gauge symmetry brings about great impact to not only classical theory of field but also quantum theory of field. But, careful study on physical meaning which gauge symmetry breaking involves provides the key to getting deeper and comprehensive, further new understanding of physical interaction and matter.

Now, let us grasp physical essence of gauge symmetry. In Maxwell's theory, the Lagrangian for the motion of a charge is invariant under gauge transformation. That is, when

$$A'_\lambda = A_\lambda - \frac{\partial f}{\partial x^\lambda}, \qquad (13-1)$$

a new additional term appears in Lagrangian integral formula.

$$\frac{e}{c}\cdot\frac{\partial f}{\partial x^\lambda}dx^\lambda = d\left(\frac{e}{c}f\right) \qquad (13-2)$$

As this term is perfect differentiation, it does not bring about any change in the equation of the motion of a charge. This result is drawn from the fact which charge is always a constant and the Lagrangian for interaction is linear. But, it is obvious that if a charge varies in time or the Lagrangian for interaction is nonlinear (for example, $\alpha\left(A_\lambda u^\lambda\right)^{1/2}$), formula (13-2) cannot hold. On the other hand, the field equation in Maxwell theory, as a linear partial differential equation satisfying gauge symmetry, obey the principle of linear superposition.

Summarizing arguments mentioned above, we arrive at the following conclusions.

First, charge, as a constant at any point of space and time, is always a conserved quantity. Accordingly, charge, during the motion, should not change and moreover be neither annihilated nor created.

Second, with principle of linear superposition, the field produced by total charge is the same as the sum of fields created by individual charges which comprise the system of charges. In this case, fields are independent and do not interfere each other, so we have

$$A^\lambda = A^\lambda_{(1)} + A^\lambda_{(2)} + \cdots + A^\lambda_{(n)} \qquad (13-3)$$

Third, the conservation of total charge of a system is the same as sum of conservations of individual charges. If a system of charges has discrete distribution, we have

$$\frac{\partial}{\partial t}\int \rho dV = \frac{\partial}{\partial t}\int\left(\sum_{i=1}^{n} e_i\delta(\mathbf{r}-\mathbf{r}_i)\right)dV$$

$$\frac{\partial}{\partial t}\int\left(\sum_{i=1}^{n} e_i\delta(\mathbf{r}-\mathbf{r}_i)\right)dV + \int\left(\sum_{i=1}^{n} e_i V_i\delta(\mathbf{r}-\mathbf{r}_i)\right)dS = 0 \qquad (13-4)$$

On the other hand, according to principle of linear superposition, formula (13-4) arrives at

$$\int\left(\frac{\partial e_1\delta(\mathbf{r}-\mathbf{r}_1)}{\partial t} + dive_1V_1\delta(\mathbf{r}-\mathbf{r}_1)\right)dV + \cdots$$

$$+\int\left(\frac{\partial e_n\delta(\mathbf{r}-\mathbf{r}_n)}{\partial t}+dive_nV_n\delta(\mathbf{r}-\mathbf{r}_n)\right)dV=0 \qquad 13-5)$$

$$\begin{cases}\dfrac{\partial e_1\delta(\mathbf{r}-\mathbf{r}_1)}{\partial t}+\mathrm{div}e_1V_1\delta(\mathbf{r}-\mathbf{r}_1)=0\\ \cdots \qquad \cdots \qquad \cdots\\ \dfrac{\partial e_n\delta(\mathbf{r}-\mathbf{r}_n)}{\partial t}+\mathrm{div}e_nV_n\delta(\mathbf{r}-\mathbf{r}_n)=0\end{cases} \qquad (13-6)$$

These results follows from gauge principle and principle of linear superposition, but do not agree with experimental data in micro-world. Considering physical interaction accompanying creation and annihilation of particles, not individual charge but only total charge of a system is conserved. This shows that gauge principle and principle of linear superposition in Maxwell's theory stand entirely against experimental facts within very close distance between particles (accompanying creation and annihilation of particles).

In our theory, with the breaking of gauge principle, the problem mentioned above is easily solved. Here, effective charge which refers to charge in Maxwell's theory is not a constant but the function dependent on field and the interaction of charge and field is non-linear (Lagrangian for field to be found in the future should also be non-linear). (See sect. 5 of referfence [1]).

From this, we arrive at following conclusions.

First, as effective charge is function of space and time, there exists a critical distance (about 10^{-13}cm, in case of annihilation of electron-positron) within which annihilation of particles and creation of new particles appears. Unlike Maxwell's theory, our theory gives an explanation for annihilation of particles despite non-quantum theory (See sect. 11, sect. 21). Thus, charge is no more a quantity conserved individually.

Second, as field equation is non-linear, field is not arithmetical sum of fields produced by individual charges and it can be studied and considered only as field created by the total charge of a system. In this case, it is self-evident that the field equation of a system of charges cannot be separated into field equations of individual charges. The form of field equation can be written as follows.

$$(nonlinear\ equation\ of\ field)_\lambda\sim\sum_{i=1}^{n}\bar{e}_{(i)}V_{(i)}^{\lambda}\,\delta(\mathbf{r}-\mathbf{r}_i) \qquad (13-7)$$

Third, the conservation of total charge of a system does not mean conservations of individual charges which constitute the system and then in a close system which accompanies creation and annihilation of particles, the total charge of a system is only conserved. Furthermore, this shows that non-linear character of interaction and breaking of gauge symmetry is a basic factor which underlies creation and annihilation of particles.

The principle of linear superposition is a basic principle that underlies gauge symmetry. This principle, in a word, is used to "atomism", founded by Carteret and Newton in 16~17 century, according to which the whole is the sum of its constituents and understanding of the parts leads to understanding of the whole. But this idea, with appearance of modern systems theory, got serious criticism. According to systems theory, all substances and phenomena can be considered as a system in which elements (constituents of a system) are combined by structure or relation. This idea has been introduced to many individual sciences and interdisciplinary sciences. In this regard, breaking of principle of linear-superposition in our theory shows again validity of idea of systems theory. As for final conclusion related to the breaking of principle of linear-superposition and gauge symmetry, there is nothing to be surprise or afraid. This will be an advance in clarifying the essence of material world.

(2) Equivalence of inertial mass and total energy of particle-field.

First of all, let us calculate energy of field. From formula (7-4), the energy-momentum tensor of electromagnetic field is

$$T_i^k = -\frac{1}{4\pi}\left(F_{i\lambda}F^{k\lambda} - \frac{1}{4}F_{lm}F^{lm}\delta_i^k\right) \qquad (13-8)$$

From formula (13-8), the energy density of electrostatic field is

$$T_0^0 = -\frac{1}{8\pi}E_iE^i \qquad (13-9)$$

where i is spatial component. From formula (7-5) is obtained

$$E^i = -\frac{1}{\sqrt{-g}}E_i \qquad (13-10)$$

and then

$$T_0^0 = \frac{1}{8\pi}E_iE_i\frac{1}{\sqrt{-g}}$$

Hence, the energy of electrostatic field is

$$U = \int T_0^0\sqrt{-g}\,dV = \frac{1}{8\pi}\int \frac{1}{\sqrt{-g}}(E_i)^2\sqrt{-g}dV = -\frac{1}{8\pi}\int E_i\frac{\partial\varphi}{\partial x^i}dV$$

$$= -\frac{1}{8\pi}\int \partial_i(E_i\varphi)dV + \frac{1}{8\pi}\int \varphi\partial_iE_i$$

According to Gauss's theorem, the first integral is equal to the integral of $E_i\varphi$ over the surface bounding the volume of integration, but since the integral is taken over all space and the field is zero at infinity, this integral vanishes. Substituting $\partial_iE_i = 4\pi\int \bar{e}\delta(\mathbf{r}-\mathbf{r}_0)dV$ into the second integral, we find the following expression for the energy of a system of charges

$$U = \frac{1}{2}\int \varphi\bar{e}\delta(\mathbf{r}-\mathbf{r}_0)dV = \frac{1}{2}\bar{e}\varphi^{in} \qquad (13-11)$$

According to normalization rule 3, if φ^{in} is replaced by the normalized function $\bar{\varphi}^{in}$ which $\bar{\varphi}^{in} = 0$ (see formula (11-7), (11-8)). The formula (13-11) arrives at

$$\bar{U} = \frac{1}{2}\bar{e}\bar{\varphi}^{in} = \frac{1}{2}e\bar{\varphi}^{in} = 0 \qquad (13-12)$$

In formula (13-11) normalized effective charge becomes a constant charge. Namely, when one normalizes effective charge, $\bar{e}$, in $\bar{e} = eg^{\mu\nu}\delta_{\mu\nu}$, because $g^{\mu\nu}$ is replaced by $\delta^{\mu\nu}$, normalized effective charge becomes a constant charge.

Next, let us calculate the energy of a particle. Considering that the energy-momentum tensor of a particle is

$$T_0^0 = \frac{m_0}{\sqrt{-g}}u_0\delta(\mathbf{r}-\mathbf{r}_0)$$

the energy of a particle is

$$T = \int \frac{m_0}{\sqrt{-g}}u_0\delta(\mathbf{r}-\mathbf{r_0})\sqrt{-g}dV = m_0c^2u_0 \qquad (13-13)$$

where u_0 is implicit function.

Normalization of $\bar{u}_0$ subject to normalization rule (10-4) yields

$$\bar{u}_0 = \bar{g}_{0\lambda}\bar{u}^\lambda = \delta_{0\lambda}\left(1+2\alpha\bar{\varphi}^{in}\right)\bar{u}^\lambda = \frac{\left(1+2\alpha\bar{\varphi}^{in}\right)^{\frac{1}{2}}}{c\sqrt{1-\beta^2}} = \frac{1}{c\sqrt{1-\beta^2}} \qquad (13-23)$$

where $\bar{\varphi}^{in} = 0$ were used. (see formula (10-4)). So, the energy of particle is

$$T = \frac{m_0 c^2}{(1-\beta^2)^{\frac{1}{2}}} \qquad (13-15)$$

Consequently, total energy of particle and field produced by it is

$$E = T + U = \frac{m_0 c^2}{(1-\beta^2)^{\frac{1}{2}}} = mc^2 \qquad (13-16)$$

The formula (13-16) is, formally, the same as energy formula of particle in SR but different entirely in essential content. In SR, mc^2 is energy confined to particle only but mc^2 in formula (13-16) is energy which includes not only energy of particle but also energy of electrostatic field. This is the inevitable conclusion following from starting postulate 3 (sect. 4 of reference [1]). Therefore, in case of discussing energy of field in our theory, the energy of static self-field (diverging in former theory) becomes naturally zero and accordingly only interactional energy with external field has real meaning. If one generalizes argument mentioned above to a system of multi-particles, infinite terms arising from self-field vanish naturally and only terms of interactional energy dependent on mutual distribution of particles remains.

5. Theory of Gravitational Field in KR Space

The Einstein's general theory of relativity (GR) established in 1916 has been recognized as one and only theory of gravitation. The validity of this theory was confirmed by marvelous experiments for its main theoretical results, i.e. three effects (deflection of light rays in the sun's gravitational field, shift of Mercury's perihelion, red shift of spectrum of light). But later among many scientists were presented doubts about whether the theoretical answers to these effects of gravitational field can be decisive ground on which verifies validity of GR. It is referred to facts that, except Einstein's GR, there are various kinds of theories with well-ordered logical system which gives theoretical answer to three effects. In the context, many scientists emphasize that theoretical description of three effects cannot be the touchstone or decisive factor which verifies uniqueness and rightness of GR and accordingly present experiments for gravitation are not enough to confirm the truth of GR.

We, refering to views presented by many scientists, put forward the following solution measures for finding out uniqueness and rightness of theory of gravitational field.

The first, with further improvement of modern experimental apparatus, is to select the best theory which explains wonderfully approximation of higher order about three effects among theories known until now.

The second is to discover new experiments, especially effects concerning strong field and give theoretical answer to it.

The third is to introduce more necessary and enough, physical and logical requirements to theory of gravitational field so that it leads to a consistant theory.

In fact, what realizes the solution measures of first and second are nearly impossible at present. They are related to facts that observable gravitational phenomena, until now, manifested in weak, static field and new improvement of experiment apparatus is also difficult to be realized in near future. On this context, third solution measure should be focused on study.

As well known, GR was build by deductive method. It is because observable gravitational effects are phenomena manifested in weak, static field and not sufficient so as to get necessary and enough data for generalized theory. At present also were not discovered very rich experimental data necessary and enough for building of theory. But if one put forward a new idea of the unification of electromagnetic field and gravitational field and introduce it to the theory of gravitation, this becomes presentation of new unprecedented requirement for building of the consistent theory of gravitation and will give a great shock to completion of theory of gravitation. First of all, experimental verification for the unification of electromagnetic field and gravitational field will lead to confirm, at the new angle, justness of the theory of gravitation. As a simple example, the experiment for the annihilation of particle and antiparticle argued in sect. 1 needs establishment of the unified conservation formula of gravitation-electromagnetic field. If theory of gravitation in the form of combination of electromagnetic field, makes it possible to provide a unified conservation formula of energy-momentum and describes all experiments well, it will be a great advance for confirmation of validity of theory of gravitation.

Next, establishment of the unification theory of gravitation and electromagnetic field, by indirect method, will open up new way to verify theory of gravitation. The theory of electromagnetic field gave, unlike theory of gravitation, wonderful answers to so many phenomena in not only static field but also non-static field and micro-world. That is why, we, regarding electrodynamics as a pivotal thing, comes to combine theory of gravitation with it and from this, function of gravitational field results in four dimensional vector, but not tensor. If the new theory of gravitation is established, as an integrated field, without any contradiction and without being separated from the theory of electromagnetic field which was verified under different and various situations, it leads to verify by indirect method that theory of gravitation has universal meaning in not only static field of macro-world but also area of micro-world.

Sect. 14 Motion of object in a spherically symmetric gravitational field and shift of Mercury perihelion

Referring to sect. 5 and sect. 6 (see reference [1]), we find Hamilton-Jacobi equation widely used in consideration of the motion of an object. The starting formula is

$$u_k u^k = 1 \qquad (14-1)$$

Using $P_k = \partial S/\partial x^k$, we find the following equation

$$g^{\alpha\beta}\left(\frac{\partial S}{\partial x^\alpha} - \widehat{m}_{(g)}\widehat{G}_\alpha\right)\left(\frac{\partial S}{\partial x^\beta} - \widehat{m}_{(g)}\widehat{G}_\beta\right) - m_0^2 c^2 = 0 \qquad (14-2)$$

where $\widehat{m}_{(g)}$ is the effective gravitational mass and $\widehat{G}_\alpha$ gravitational potential and

$$g^{\alpha\beta} = \delta^{\alpha\beta}\frac{1}{1 + 2\widehat{\alpha}_{(m)}\widehat{G}_\lambda u^\lambda}$$

If one, in equation (14-2), replaces $\mathbf{P}$ with $\partial S/\partial \mathbf{r}$ and E with $-\partial S/\partial t$ and then formula (14-2) arrives at

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \widehat{m}_{(g)}\widehat{G}_0\right)^2 - \left(\mathrm{grad}S - \frac{1}{c}\widehat{m}_{(g)}\widehat{G}\right)^2\right] - m_0^2 c^2\left(1 + 2\widehat{\alpha}_{(m)}\widehat{G}_\lambda u^\lambda\right) = 0 \qquad (14-3)$$

This formula is essential to discuss the motion of an object in gravitational field including the shift of Mercury's perihelion.

Now, let us consider the shift of Mercury's perihelion. Because equation (14-3) has the characteristic of spherical symmetry, it is easy to treat it in spherical coordinates. Then equation (14-3) leads to

$$\left[\frac{1}{c^2}\left(-E_0 + \widehat{m}_{(g)}\widehat{\varphi}\right)^2 - \left(\frac{\partial S_r}{\partial r}\right)^2 - \frac{M^2}{r^2}\right] - m_0^2 c^2\left(1 + 2\widehat{\alpha}_{(m)}\widehat{\varphi}u^0\right) = 0 \qquad (14-4)$$

where $E_0 = -\partial S/\partial t$ was used. A rearrangement of equation (14-4) yields.

$$\left(\frac{\partial S_r}{\partial r}\right)^2 = \frac{\left(E_0 - \widehat{m}_{(g)}\widehat{\varphi}\right)^2}{c^2} - \frac{M^2}{r^2} - m_0^2 c^2\left(1 + 2\widehat{\alpha}_{(m)}\widehat{\varphi}u^0\right)$$

$$S_r = \int dr\left[\frac{\left(E_0 - \widehat{m}_{(g)}\widehat{\varphi}\right)^2}{c^2} - \frac{M^2}{r^2} - m_0^2 c^2\left(1 + 2\widehat{\alpha}_{(m)}\widehat{\varphi}u^0\right)\right]^{\frac{1}{2}} \qquad (14-5)$$

The path of motion is determined from $\partial S/\partial M = \mathrm{constant}$. We consider, in detail, terms of equation (14-5)

$$E_0 - \widehat{m}_{(g)}\widehat{\varphi} = m_0 c^2 u_0$$

$$E = m_0 c^2 u_0 = \frac{m_0 c^2 \left(1 + 2\hat{\alpha}_{(m)} \hat{\varphi} u^0\right)^{\frac{1}{2}}}{\sqrt{1-\beta^2}} = \frac{\overline{m}_0 c^2}{(1-\beta^2)^{\frac{1}{2}}} \qquad (14-6)$$

where $\overline{m}_0$ is effective inertial mass. In the field produced by rest object, m', we have

$$1 + 2\hat{\alpha}_{(m)} \hat{\varphi} u^0 \approx 1 - 2\frac{k_{(g)}}{c^2} \cdot \frac{m'}{r\left(1 - 2k_{(g)} \frac{m'}{c^2 r}\right)} \cdot \frac{1}{\left[\left(1 - 2k_{(g)} \frac{m'}{c^2 r}\right)(1-\beta^2)\right]^{1/2}} \qquad (14-7)$$

where $k_{(g)}$ is the *gravitational constant*. Introducing $r_0 = 2\frac{k_{(g)}}{c^2} m'$ called the *gravitation radius* and allowing, in nonrelativistic approximation, $(1-\beta^2)^{-1/2} \approx 1 + \frac{1}{2}\beta^2$, formula (14-7) becomes

$$1 + 2\hat{\alpha}_{(m)} \hat{\varphi} u^0 \approx 1 - \frac{r_0}{r\left(1 - \frac{r_0}{r}\right)} \cdot \frac{1}{\left(1 - \frac{r_0}{r}\right)^{\frac{1}{2}}} + 0(V^2/c^4)$$

$$\approx 1 - \frac{r_0}{r\left(1 - \frac{r_0}{r}\right)^{\frac{3}{2}}} \approx 1 - \frac{r_0}{r}\left(1 + \frac{3}{2} \cdot \frac{r_0}{r}\right) \qquad (14-8)$$

where $\hat{\alpha}_{(m)} \hat{\varphi} = \frac{\hat{m}}{mc^2} \cdot k_{(g)} \frac{\hat{m}'}{r} = -\frac{1}{c} k_{(g)} \frac{\hat{m}'}{r}$ in isotopic vector space was used. On the other hand, in formula (14-6) is $E = \frac{\overline{m}_0 c^2}{(1-\beta^2)^{1/2}} \approx \overline{m}_0 c^2 + \frac{1}{2}\overline{m}_0 v^2 \approx \overline{m}_0 c^2 + \frac{1}{2} m_0 v^2$ and then putting $T' = \frac{1}{2} m_0 v^2$, the result is

$$E = T' + \overline{m}_0 c^2 \qquad (14-9)$$

Using formula (14-8) and (14-9) is obtained formula for S_r

$$S_r = \int \left\{\frac{E^2}{c^2} - \frac{M^2}{r^2} - m_0^2 c^2 \left[1 - \frac{r_0}{r} - \frac{3}{2}\left(\frac{r_0}{r}\right)\right]\right\}^{\frac{1}{2}} =$$

$$= \int \left\{\frac{1}{c^2}(T'^2 + 2T'\overline{m}_0 c^2 + \overline{m}_0^2 c^4) - \frac{M^2}{r^2} - m_0^2 c^2 + m_0^2 c^2 \frac{r_0}{r} + m_0^2 c^2 \frac{3}{2}\left(\frac{r_0}{r}\right)^2\right\}^{\frac{1}{2}} dr$$

$$(14-10)$$

Now, using $\overline{m}_0^2 c^2 = m_0^2 \left(1 - \frac{r_0}{r}\right) c^2 = m_0^2 c^2 - m_0^2 c^2 \frac{r_0}{r}$, $\overline{m}_0 \approx m_0 \left(1 - \frac{r_0}{2r}\right)$, formula (14-10) leads to

$$S_r = \int \left[\left(\frac{T'^2}{c^2} + 2T'\overline{m}_0\right) - \frac{1}{r^2}\left(M^2 - \frac{3}{2} m_0^2 c^2 r_0^2\right)\right]^{\frac{1}{2}} dr$$

$$= \int \left[\left(2T' m_0 + \frac{T'^2}{c^2} - m_0 T' \frac{r_0}{r}\right) - \frac{1}{r^2}\left(M^2 - \frac{3}{2} m_0^2 c^2 r_0^2\right)\right]^{\frac{1}{2}} dr \qquad (14-11)$$

Let us transform formula (14-11) as follows:

$$T' = E' - U, \quad E' = T' + U$$

And then arrangement of formula (14-11) gives

$$S_r = \int \left[\left(2E'\overline{m}_0 + \frac{E'^2}{c^2} \right) - \frac{1}{r} \left(2m_0^2 m' k_{(g)} - \frac{1}{2} k_{(g)} \frac{m_0^2 m'}{r} r_0 \right) - \frac{1}{r^2} \left(M^2 - \frac{3}{2} m_0^2 c^2 r_0^2 \right) \right]^{\frac{1}{2}} dr \qquad (14-12)$$

Comparing formula (14-12) with formula of GR, second term in second bracket is different. But this difference is not essential. As already well known, in case of discussing the motion of an object in spherical symmetry field, corrections in first two brackets contribute only to the relation between energy and momentum and the change of parameters of Newton's trajectory (ellipse), which has no significance in consideration. Therefore, even though correction of second bracket are different from that of GR, it does not spoil the validity of our theory. The most important thing is that the term related to Mercury's perihelion in our theory is the same as that in GR which has been already experimentally verified.

Sect. 15 The equation of propagation of light, red shift of spectrum of light and deflection of light

It is well known that there exists formal similarity between Hamilton-Jacobi equation and equation of propagation of light. In our theory, it is also supposed that this similarity holds as it is. In Hamilton-Jacobi equation, the total momentum of a particle, four-dimensional momentum of a particle and the total energy of field of conservative force can be written as follows, respectively:

$$P_\alpha = \frac{\partial S}{\partial x_\alpha}, \qquad m_0 c u_\alpha = \frac{\partial S}{\partial x^\alpha} - \widehat{m}_{(g)} \widehat{G}_\alpha, \qquad P_0 = \frac{\partial S}{\partial t} \qquad (15-1)$$

Referring to formula (5-22) (see reference [1]), we make the equation of propagation of light

$$g^{\alpha\beta} \left(\frac{\partial \Psi}{\partial x^\alpha} - h_\alpha \right) \left(\frac{\partial \Psi}{\partial x^\beta} - h_\beta \right) = 0 \qquad (15-2)$$

where Ψ is *eikonal* and h_α is the term concerning interaction between light and gravitational field (This term is similiar to term of interaction of particle and field). Comparing equation (15-2) with eikonal equation considered in GR, the term of interaction between light and gravitational field is added. But what describe h_α in the strict mathematical viewpoint is a difficult problem. Moreover, on the left-hand side of equation (15-2), as $V/c = 1$ in denominator of u^0 appears, $g^{\alpha\beta}$ goes to zero and then equation (15-2) leads to indeterminate form (0/0). Accordingly, as for tangible calculation should be paid deep attention so that this indeterminate form is solved. Therefore, we, for simplification of consideration, intend to discuss equation (15-2) established in static gravitational field. In this case, $\omega_0 = -\partial\Psi/\partial t$ is proper frequency which is not changed at any point of space-time and $\partial\Psi/\partial x^i = \mathbf{n}_i \omega_0 / c$ (i denotes spatial component) are components of wave vector. From this is obtained the simpler form of equation (15-2). In arbitrary point of space time, actually the changed frequency is

$$\omega = \frac{\partial \Psi}{\partial \tau} \quad (\tau; \text{proper time}) \qquad (15-3)$$

This change is caused by interaction of light and gravitational field. So we have

$$\frac{\partial \Psi}{\partial t} = \frac{\partial \Psi}{\partial \tau} + c h_0 \qquad (15-4)$$

In static gravitational field, gravitational vector potential is $G_i = 0$ and therefore,

$$h_i = 0 \qquad (15-5)$$

Using formula (15-4) and (15-5), equation (15-2) arrives at

$$g^{00}\left(\frac{\partial\Psi}{\partial\tau}\right)^2+g^{ij}\frac{\partial\Psi}{\partial x^i}\frac{\partial\Psi}{\partial x^j}=0 \qquad (15-6)$$

or

$$\left(1+2\hat{\alpha}_{(m)}\hat{G}_0u^0\right)\left[\left(\frac{\partial\Psi}{\partial\tau}\right)^2-\frac{\partial\Psi}{\partial x^i}\frac{\partial\Psi}{\partial x^i}\right]=0 \qquad (15-7)$$

where x^i, x^j is spatial components of x^α.

(1) The red shift of spectrum of light

In static gravitational field, the following formula is obtained

$$\omega_0=-\frac{\partial\Psi}{\partial t}=-c\frac{\partial\Psi}{\partial x^0} \qquad (15-8)$$

From the character of static field, as formula (15-8) does not include explicitly world time, x_0, frequency ω_0, remains invariant during propagation of light. On the other hand,

$$\omega=-\frac{\partial\Psi}{\partial\tau} \qquad (15-9)$$

is changed at any point of space. In order to see change of frequency, we can consider two infinitely near events taking place in a point of space. In this case, the line element between two events observed, because events occur in a point of space, allowing $(dx_i)^2=dl^2=0$, can be written as follows

$$ds^2=c^2d\tau^2=g_{00}(dx^0)^2 \qquad (15-10)$$

where $g_{00}=1+2\alpha_{(m)}\varphi u^0$. From this, we have

$$d\tau^2=\frac{1}{c^2}\left(1+2\hat{\alpha}_{(m)}\frac{\hat{\varphi}}{c}\frac{dx^0}{d\tau}\right)(dx^0)^2$$

$$d\tau=\frac{1}{c}\left(1+2\hat{\alpha}_{(m)}\frac{\hat{\varphi}}{c}\frac{dx^0}{d\tau}\right)^{\frac{1}{2}}(dx^0) \qquad (15-11)$$

Consequently $d\tau$ is implicit function in KR space. Normalizing $d\tau$ by normalization rule 1, the result is

$$\overline{d\tau}=\overline{d\tau}(x,\dot{x},\bar{g})=\overline{d\tau}(x,\dot{x})=\frac{1}{c}\left(1+2\hat{\alpha}_{(m)}\hat{\varphi}\right)^{\frac{1}{2}}dx^0=\frac{1}{c}\left(1+2k(g)\frac{\hat{m}}{mc^2}\cdot\frac{\hat{m}'}{r}\right)^{\frac{1}{2}}dx^0=$$

$$=\frac{1}{c}\left(1-\frac{r_0}{r}\right)^{\frac{1}{2}}dx^0 \qquad (15-12)$$

where

$$\left.\frac{1}{c}\frac{dx^0}{d\tau}\right|_{g_{00}=\delta_{00}}=\left.\frac{1}{\sqrt{g_{00}}}\right|_{g_{00}=\delta_{00}}=1$$

was used in the normalization of $d\tau$ and m is, in case of light, mass formally introduced and $r_0=2k(g)\frac{m'}{c^2}$ the gravitational radius.

Also, for ω we have

$$\omega=-\frac{\partial\psi}{\partial\bar{\tau}}=-\frac{\partial\psi}{\partial x^0}\frac{\partial x^0}{\partial\bar{\tau}}=\frac{c}{(1-r_0/r)^{\frac{1}{2}}}\frac{\partial\psi}{\partial x^0}$$

When $r_0\ll r$,

$$\omega=\omega_0\left(1+\frac{1}{2}\cdot\frac{r_0}{r}\right) \qquad (15-13)$$

is found. The formula (15-13) is the same as formula concerning red shift of spectrum of light in GR.

(2) The deflection of light from rectilinear path

Let us consider formula (14-5) for S_r in detail. Denominator of u^0 in third term inside root of formula (14-5), i.e. $m_0^2c^2\left(1+2\hat{\alpha}_{(m)}\hat{\varphi}u^0\right)$, includes $(1-V^2/c^2)^{1/2}$ and in case of light, u^0 would diverge. But as $m_0=0$, $m_0^2c^2u^0$ has indeterminate form of 0/0. Hence, a path of light rays cannot be easily obtained like in GR. In this regard, first of all, we transform formula (14-5) for the motion of a particle to an approximate form and solve the problem of indeterminate form of $m_0^2c^2u^0$, and then, by using similarity of the motions of particle and light, obtain the path of a light ray. To do this, we transform u^0 as follows (in this case the normalized u^0 is used)

$$u^0=\frac{1}{\sqrt{1-\beta^2}\sqrt{1-\frac{r_0}{r}}}=\frac{1}{1-\beta^2}\cdot\frac{(1-\beta^2)^{\frac{1}{2}}}{\left(1-\frac{r_0}{r}\right)^{\frac{1}{2}}} \qquad (15-14)$$

where $b=(1-\beta^2)^{\frac{1}{2}}/\left(1-\frac{r_0}{r}\right)^{\frac{1}{2}}$ has a very interesting character. In case of $V\approx c,\ r\approx r_0$, that is to say, when a particle moves in the vicinity of r_0 and so the velocity of the particle approaches the light velocity, $b\approx 1$ holds. On the other hand, when a particle moves far away from r_0 (i.e. $r_0\ll r$) and the velocity of the particle is much smaller than the velocity of light (i.e. $V\ll c$), $b\approx 1$ holds. Therefore, the result is

$$b=(1+\alpha_0),\qquad \alpha_0\ll 1 \qquad (15-15)$$

Assuming that $\frac{V}{c}\ll 1,\ \frac{r_0}{r}\ll 1$ and expanding b as a power series, we get

$$b\approx\left(1-\frac{1}{2}\beta^2\right)\left(1+\frac{1}{2}\frac{r_0}{r}\right)\approx\left[1+\left(\frac{r_0}{r}-\frac{1}{2}\beta^2\right)\right] \qquad (15-16)$$

Considering formula (15-15) and (15-16), formula (15-14) leads to

$$u^0=\frac{1}{1-\beta^2}(1+\alpha_0),\qquad \alpha_0\ll 1 \qquad (15-17)$$

and then we have

$$m_0^2c^22\hat{\alpha}_{(m)}\hat{\varphi}u^0=-m_0^2c^2\frac{r_0}{r}u^0=-m_0^2c^2\frac{r_0}{r}\cdot\frac{1}{1-\beta^2}(1+\alpha_0)$$

$$=-\left(\frac{m_0c^2}{\sqrt{1-\beta^2}}\right)^2\frac{1}{c^2}\cdot\frac{r_0}{r}(1+\alpha_0)=-\left(\frac{E}{c}\right)^2\frac{r_0}{r}(1+\alpha_0)\approx-\left(\frac{E}{c}\right)^2\frac{r_0}{r} \qquad (15-18)$$

where

$$E=\frac{m_0c^2}{\sqrt{1-\beta^2}}$$

Substituting formula (15-18) into formula (14-5) for S_r, we obtain

$$S_r=\int\left[\frac{E^2}{c^2}-\frac{M^2}{r^2}+\frac{r_0}{r}\left(\frac{E}{c}\right)^2\right]^{1/2}dr \qquad (15-19)$$

If one substitutes $\omega=-\partial\psi/\partial\tau$ for E and, allowing for formula (15-13) and $m_0=0$, introduces a new constant $\rho=cM/\omega_0$, the result is

$$\left(\frac{E}{c}\right)^2\rightarrow\left(\frac{\omega_0}{c}\right)^2\left(1+\frac{r_0}{r}\right)$$

$$\psi_r=\frac{\omega_0}{c}\int\left[\left(1+\frac{r_0}{r}\right)-\frac{\rho^2}{r}+\frac{r_0}{r}\left(1+\frac{r_0}{r}\right)\right]^{\frac{1}{2}}dr \qquad (15-20)$$

Ignoring term $\left(\frac{r_0}{r}\right)^2$ in formula (15-20), we have

$$\psi_r = \frac{\omega_0}{c}\int\left(1 + 2\frac{r_0}{r} - \frac{\rho^2}{r}\right)^{1/2} dr \qquad (15-21)$$

This formula agrees with the formula for path of light ray in GR. Formula (15-21) yields formula for light deflection which has been already experimentally verified.

Sect. 16 Equivalence of inertial mass and total energy of particle-gravitational field

This topic is formally the same as in theory of electromagnetic field (see sect. 13.2). Taking the following transformation $e \to m,\ A_i \to G_i,\ F_{ik} \to R_{ik}$ and considering $\hat{m}k_{(g)}\frac{\hat{m}}{r} = k_{(g)}\frac{m^2}{r}$, we can get the formula for the equivalence of inertial mass and total energy of particle-field

$$E = T + U = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = mc^2 \qquad (16-1)$$

Our theory is of significance in relation to the fact that conservation formula of total energy-momentum of particle and gravitational field is derived on the basis of Noether's theorem and then the difficulties seen in sect. 3 are unraveled.

References

[1] Ho Dong Jo, Chol Song Kim "Toward New Thought for the Unified Theory of Electromagnetic and Gravitationl Field (I) " viXra: 1902.0399 (2019)

[2] A. O. Barut, "Electrodynamics and Classical Theory of Fields & Particles" Dover, Chapter 5, (1980)