1. POWER OF THE SET OF PRIME NUMBERS:

As a proof of the claim I have made (postulate), I intend to present to you a way to calculate the power of a set of prime numbers, i.e. an algorithm that allows us to determine the number of primes proper within a given numerical range \((0, x)\). This is the so-called "theorem on prime numbers".

Of course, I will base my proof on the equations derived in my innovative scientific work „Distribution of prime numbers”.

![Fig. 4. Generation of multiples– numbers \(u_r\) for the number \(p_r = 7\). (\(w – nth\) multiples, \(d i m – coefficients from equation (11)).

The "prime number theorem" defining the power of the set \(\pi(n)\) is a serious problem to this day. The value of the result of this equation is burdened with an error, and its size decreases asymptotically when \(n \to +\infty\).

\[
\pi(n) \approx \frac{n}{\log(n)} \tag{13}
\]

To this day, it has not been possible to design a formula that would unambiguously and without error calculate the number of prime numbers in the set of whole numbers. Therefore, my observations can become an innovative way to deal with this issue.

ASSUMPTIONS:

1) It is obvious that prime numbers are searched for odd numbers, the sum of which is not divisible by 3, which is unequivocally looking for them among the numbers that are part of the set \(Z_r\).

2) The number \(x\) is the right edge of the set being examined \((0, x)\), which we will consider in terms of the number of prime numbers proper.
POSTULATE 2:
If the number $x$ is divided by 5 (the first first proper number of the set $Z_r$) we get the right boundary of the set $(0, \frac{x}{5}>$, which will contain numbers $z_r$ affecting the generation of all numbers $u_r$ in the entire collection space $(0, x>$. 

EXAMPLE 1:
1) Let $x$ be 77.
2) If we divide 77 into 5 we get 15,4.
3) We quickly notice that in a set of integers $(0,15>$ there are four numbers $z_r:\{5,7,11,13\} \in (0,15>.$
4) According to the above postulate, these numbers suffice to determine all numbers $u_r$ located in the whole space of the set $(0, x>.$
5) Below I present a list of all combinations of numbers $u_r$ resulting from numbers $\{5,7,11,13\}$ (products of pairs of numbers $z_r$):

$$
\begin{align*}
5 \times 5 &= 25 \\
5 \times 7 &= 35 \\
5 \times 11 &= 55 \\
5 \times 13 &= 65 \\
7 \times 7 &= 49 \\
7 \times 11 &= 77 \\
7 \times 13 &= 91 \\
11 \times 11 &= 121 \\
11 \times 13 &= 143 \\
13 \times 13 &= 169 \\
\end{align*}
$$

It is worth noting that all (in bold) above the results are exactly a two-element combination with repetitions from the $n – element set$, in this case a four-element set. It is important to reject mirror combinations, i.e. $ab$ and $ba$ are treated as twin combinations. The formula for the number of two-element combinations from a four-element set is shown below:

$$
C_n^k = \binom{k + n - 1}{k} = \frac{(k + n - 1)!}{k! (n - 1)!} = \frac{(2 + 4 - 1)}{2! (4 - 1)!} = 10
$$
Let's look ...

In the above-mentioned example, we have affect with the number \( z_r \) small sizes. Questions arise as to how subsections 3 and 7 will be solved in case of much larger numbers \( z_r \).

In this case, I will present you two ways to solve these doubts, and their order will not be accidental, because the below presented mathematical calculator will be a good reference and visualization of theoretical mathematical calculations:

— mathematical calculator (based on the EXCEL program from the MS Office package) using the above-mentioned formulas and theoretical relations;
— theoretical (using mathematical calculations);

MATHEMATICAL CALCULATOR:

In order to solve the problem related to the power of prime numbers, I used the EXCEL program from the MS Office suite, which according to my person is a great and extremely versatile tool useful not only in the world of mathematics.

In the further part of my research I intend to introduce to you step by step the algorithm used to determine the power of the set of natural numbers:

STEP 1.

Let us assume that for the experiment \( x = 1411 \).

STEP 2.

Let's build a numerical table whose elements \( u_r \) are multiples of all numbers \( z_r \) determined in postulate No. 2. This table will therefore have sizes (Figure 5):

a. height: 93
   ✓ the number of rows in the numeric table should be consistent with the number of numbers \( z_r \) falling within the range \( (0, \frac{x}{5}) \), it means \( (0; 282,6) \),

a. width: 12
   ✓ the number of columns in the numeric table should be consistent with the number of numbers \( z_r \) falling within the range \( (0, \sqrt{x}) \), it means \( (0; 37,6) \),
### Tabular of numbers \( r \) generated from numbers \( e \)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 25 | 49 | 121 | 169 | 289 | 361 | 529 | 625 | 841 | 961 | 1225 | 1369 | | | | | | |
| 2 | 3 | 77 | 243 | 329 | 563 | 641 | 989 | 1021 | 1561 | 1681 | 2521 | 2704 | | | | | | |
| 3 | 55 | 91 | 187 | 247 | 391 | 475 | 737 | 875 | 1309 | 1429 | 2069 | 2269 | | | | | | |
| 4 | 45 | 99 | 229 | 263 | 455 | 481 | 781 | 859 | 1329 | 1411 | 2061 | 2159 | | | | | | |
| 5 | 125 | 209 | 317 | 393 | 631 | 689 | 1051 | 1137 | 1717 | 1801 | 2717 | 2809 | | | | | | |
| 6 | 9 | 161 | 322 | 389 | 659 | 721 | 1161 | 1257 | 1897 | 1993 | 2983 | 3089 | | | | | | |
| 7 | 49 | 97 | 203 | 247 | 413 | 449 | 751 | 857 | 1317 | 1401 | 2161 | 2253 | | | | | | |
| 8 | 181 | 301 | 481 | 509 | 809 | 839 | 1319 | 1349 | 2069 | 2101 | 2981 | 3019 | | | | | | |
| 9 | 289 | 484 | 737 | 769 | 1193 | 1229 | 1889 | 1925 | 2969 | 3005 | 4049 | 4081 | | | | | | |
| 10 | 625 | 1021 | 1561 | 1601 | 2521 | 2561 | 3625 | 3665 | 5365 | 5405 | 7205 | 7245 | | | | | | |
| 11 | 1921 | 2917 | 4141 | 4281 | 6325 | 6465 | 9209 | 9339 | 13539 | 13679 | 18729 | 18869 | | | | | | |
| 12 | 4025 | 6041 | 8161 | 8301 | 13501 | 13641 | 19841 | 20001 | 28961 | 29121 | 38521 | 38681 | | | | | | |

Fig. 5. Tabular summary of numbers \( u_r \) generated from numbers \( z_r \) - postulate No 2.

str. 4
- with red in the tabular summary of numbers $u_r$ (Figure 5.), check the repeated numbers being the results of a two-element combination with the repetitions from the numbers $z_r$ in the range $(0, x >)$ and affecting the generation of numbers $u_r$ in the range $(0, x >)$;

- for this purpose, let us use conditional formatting (one of the interesting functions of the EXCEL program) to capture duplicate values of the number $u_r$;

- please note that in the first row of the table there are only numbers $z_r$ raised to the square. Because in the tabular summary we do not capture the results of the two-element combinations being their mirror reflection in the record (reminder: the $ab = ba$ combination);

- the idea of tabular summary of numbers $u_r$ (Figure 6a, 6b) presents schemes:

![Tabular Summary Table](image)

**Fig. 6a. The idea tabular summary of number $u_r$ - postulate No. 2. (tabular sets of numbers $u_r$ represent only the above-mentioned black boxes)**

- the idea of tabular summary of numbers $u_r$ (Figure 6a, 6b) presents schemes:

![Tabular Summary Table](image)

**Fig. 6b. The idea tabular summary of number $u_r$ - postulate No. 2. (after changing cell values - field conversion)**

**STEP 3.**

Marking (underlining) only those values of numbers $u_r$, which are within the tested range $(0, x >)$. 

str. 5
for this purpose, let's also use conditional formatting to mark values of numbers $u_r$ less or equal to $x$. 

Fig. 7: Tabular summary of numbers $u_r$, smaller than the number $x$. 

str. 6
- the numbers 0 seen at the bottom of the table are caused by the offsets of the rows of subsequent columns in accordance with the illustration of the idea of tabular summary of number \( u_r \) (Fig. 6);
- comparing two figures 5 and 7, we come to the conclusion that among the generated numbers \( u_r \) in the range \((0, x >)\) there are still repeating values.

**STEP 4.**

Determination of the multiplicity of repetitions of the numbers \( u_r \) in the interval \((0, x >)\):
- using very useful formulas of the EXCEL program we are able to count the duplication times (repetitions) of individual values of the number \( u_r \) (see Fig. 9.);
- result:

<table>
<thead>
<tr>
<th>ILOŚĆ</th>
<th>WKR</th>
<th>Zr</th>
</tr>
</thead>
<tbody>
<tr>
<td>206</td>
<td>1</td>
<td>206</td>
</tr>
<tr>
<td>48</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>54</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

*Fig. 8. The multiplicity of the repetitions of generated \( u_r \) numbers (results).*

- according to the table in Fig. 8. and the postulate No 2 from the numbers \( z_r \) falling within the range \((0, x >)\) we are able to generate in the numerical range \((0, x >)\) up to 312 numbers \( u_r \), with the proviso that:
  - 48 numbers \( u_r \) are repeated twice;
  - 54 numbers \( u_r \) are repeated three times;
  - 4 numbers \( u_r \) are repeated four times;
- if the number of repeated numbers of each group divide by their multiplicity, we will get an interesting set of numbers \( u_r \);
- finally we receive information that the number of generated and different numbers \( u_r \) (i.e. without repetitions) in the range \((0, x >)\) is exactly 249;

**STEP 5.**

Calculation of the power of a set of prime numbers, which at this moment becomes the proverbial "bread roll", because it is enough to determine which number according to the formula (9) is the number \( x \) and subtract from it the number 249 received above (see formula 12);
Fig. 9. Multiplicity of duplicate values of numbers \( u_r \) less than \( x \).
\[ z_{r,k} = O_r + 6 \sum_{1}^{k} (-1)^{k+1} k \] (9)

\[ z_{r,k} = 1411 \]

\[ k = 470 \]

- for those who still accept the number \( t_r \) for the proper prime numbers - add the number two to the result obtained:

\[ Z_r - U_r + T_r = P_r \] (12)

\[ 470 - 249 + T_r = 211 + T_r \]

\[ \pi(1411) = 211 + T_r \]

**MATHEMATICAL CALCULATIONS:**

**REFERENCE TO STEP 2.**

In order to resolve item 3 from example No. 1, the number of numbers \( z_r \) included in the set should be determined by using the formula \((0, \frac{x}{5} >)\):

\[ n(Z_{r_i}) = \left[ \frac{x - 5}{30} \right] + \left[ \frac{x + 5}{30} \right] \] (16)

— where:

- \( n(Z_{r_i}) \) — power of the set \( Z_{r_i} \);
- \( i \) — (auxiliary record)
  
  index informing about the number of \( z_r \) in the set \((0, \frac{x}{5} >)\)
  
  the value of the index is synonymous with the table height;
- \( [] \) — means the result trait (integer part) used for next activities;

The above result of the amount \( z_{r_i} \) is at the same time the height (indicates the number of rows) of the tabular summary of the number \( u_r \) (Fig.5.).

The width (number of columns) of the tabular summary of number \( u_r \) (Fig.5.) is able to be determined using a very similar formula:
\[ n(Z_{rj}) = \left[ \frac{\sqrt{x} - 1}{6} \right] + \left[ \frac{\sqrt{x} + 1}{6} \right] \]  
(17)

where:
- \( n(Z_{rj}) \) — power of the set \( Z_{rj} \);
- \( j \) — (auxiliary record) index informing about the number of \( z_r \) in the set \( (0, \sqrt{x} >) \), the value of the index is synonymous with the table width;
- \( [ ] \) — means the result trait (integer part) used for next activities;

**EXAMPLE 2:**

**Table’s height:**

1) Let \( x \) be 1411.

2) If we set 1411 under the formula 16 we get 93 – number of numbers \( z_{r_1} = \text{number of rows} \).

3) If the numbers from 1 to 93 are substituted by the formula (9), then we obtain the values of all numbers \( z_{r_1} \) of the set \( (0,282,2 >) \), where the largest is \( z_{r_{93}} = 281 \).

**Table’s width:**

1) If we set 1411 under the formula 17 we get 12 – number of numbers \( z_{r_j} = \text{number of columns} \).

2) If the numbers from 1 to 12 are substituted by the formula (9), then we obtain the values of all numbers \( z_{r_j} \) of the set \( (0; 37,56 >) \), where the largest is \( z_{r_{12}} = 37 \).

**REFERENCE TO STEP 3.**

The formula for calculating the total number of numbers \( u_r \) being a numerical combination (Fig. 7.) smaller than the numer \( x \) is a combination of three formulas: 9, 16 i 17:

\[ n(U_{rk}) = \sum_{j=1}^{n(Z_{rj})} \left[ \frac{x - |z_{rj}|}{6|z_{rj}|} \right] + \left[ \frac{x + |z_{rj}|}{6|z_{rj}|} \right] - (j - 1) \]  
(18)

gdzie:
- \( n(U_{rk}) \) — power of the set \( U_{rk} \);
- \( n(Z_{rj}) \) — power of the set \( Z_{rj} \) according to formula 17.
- \( [ ] \) — means the result trait (integer part) used for next activities;
- $z_{rj}$ — individual numbers $z_r$ with the index above interval $j$;
- $j$ — ordinal number of the number $z_{rj}$ ($j \in C$);

REFERENCE TO STEP 4.

Dispose of the multiplicity of repetitions of the numbers $u_r$ constituting the numerical combination (Fig. 7.). For this purpose, we can use the following general formula of the set $U_r$ consisting of the value of the function $f(z_{rj}z_{ri})$:

$$U_r \leftrightarrow f(z_{rj}z_{ri})$$ (19)

$$\bigwedge_{z_{rj}} j \in <1; n(z_{rj}) > \bigwedge_{z_{ri}} i \in < j; \left\lceil \frac{x - |z_{rj}|}{6|z_{rj}|} \right\rceil \left\lceil \frac{x + |z_{rj}|}{6|z_{rj}|} \right\rceil + (j - 1) \rightarrow f(z_{rj}z_{ri})$$ (20)

where $f(z_{rj}z_{ri}) = z_{rj}z_{ri}$ (21)

— gdzie:
- $U_r$ — a set consisting of unique values of the number $u_r$;
- $i$ — ordinal number of the number $z_{ri}$;
  where $i \in < j; \left\lceil \frac{x - |z_{rj}|}{6|z_{rj}|} \right\rceil \left\lceil \frac{x + |z_{rj}|}{6|z_{rj}|} \right\rceil + (j - 1) \land i \in C$;
- $j$ — ordinal number of the number $z_{rj}$;
  where $j \in <1; n(z_{rj}) > \land j \in C$;

In this way, we get all the data (values) necessary to calculate the power of prime numbers using the formula 12 - which has its confirmation in the work of the mathematical calculator (EXCEL).

2. VERIFICATION OF THE "PRIME" NUMBER $z_r$

The proverbial "icing on the cake" would be the solution to the problem of verifying the number $z_r$ in terms of its primacy, i.e. whether it is the prime number or the imaginary prime number.
I believe that it is logically possible if we follow the algorithm below - if we compare the cell values of the highest rows of all the columns of the table summary of numbers $u_r$ with the number $x$ being the right boundary of the whole set of integers numbers:

$$\bigwedge_{j \in < 1; n(Z_{rj})} \bigvee_{z_{ri}} i = \left\lfloor \frac{x - |z_{rj}|}{6|z_{rj}|} \right\rfloor + \left\lceil \frac{x + |z_{rj}|}{6|z_{rj}|} \right\rceil + (j - 1) \rightarrow f(z_{rj}z_{ri})$$  \hspace{1cm} (22)

$$f(z_{rj}z_{ri}) = z_{rj}z_{ri}$$  \hspace{1cm} (23)

where:

- $i$ — ordinal number of the number $z_{ri}$;
- $j$ — ordinal number of the number $z_{rj}$ ($j \in C$);

if:

- $f(z_{rj}, z_{ri}) < x$ — number $z_r$ is the proper prime number $p_r$;
- $f(z_{rj}, z_{ri}) = x$ — number $z_r$ is the improper prime number $u_r$;

The number of comparisons made in accordance with the above algorithm equals only the harvesting power $n(Z_{rj})$, and its display shows blue ellipses on the table summary of numbers $u_r$ (Fig. 5.)

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