Refutation of relativization by structural induction in weighted first order logic

Abstract: We evaluate a formula of relativization as defined by structural induction which is *not* tautologous. Its use in weighted first order logic is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ➝ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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arxiv.org/pdf/1902.08149.pdf
droste@informatik.uni-leipzig.de paul.gastin@lsv.fr paul.gastin@ens-paris-saclay.fr

We define below the relativizations $\phi^{<x}$, $\phi^{(x,y)}$ and $\phi^{>y}$... [for $\phi^{>y}$, read $\phi^{>x}$?]
The relativization is defined by structural induction on the formulas as follows:

$$... (\forall z\psi)^{<x} = \forall z(z < x \Rightarrow \psi^{<x}) \quad (4.1.1)$$

The relativizations $\phi^{(x,y)}$ and $\phi^{>x}$ are defined similarly. (4.2.1),(4.3.1)

Remark 4.1: We write the exponent in $\phi^{(x,y)}$ of Eq. 4.2.1 to mean variables x and y, such as each with a value of 1.

((#s&#p)&(q&r))=((#s<(q&r))>(p&(q&r))) ;
FFFF **FFFF** NNNN NN**F**N (4.2.2)

Eq. 4.2.2 as rendered is *not* tautologous. This means relativization in that context is refuted.