Refutation of the Banach-Tarski paradox

Abstract: We evaluate the crucial claim of the proof in Step 3, as a fleshed out detail. It is not tautologous, nor is it contradictory. This means the claim is a non tautologous fragment of the universal logic $VŁ4$ and constitutes the briefest known refutation of the Banach-Tarski paradox.

We assume the method and apparatus of Meth8/$VŁ4$ with Tautology as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Some details, fleshed out ... [for Step 3 of 4]

Proof. Let $\lambda$ be some line through the origin that does not intersect any point in $D$. This is possible since $D$ is countable. Let $J$ be the set of angles, $\alpha$, such that for some natural number $n$, and some $P$ in $D$, $r(n\alpha)P$ is also in $D$, where $r(n\alpha)$ is a rotation about $\lambda$ of $n\alpha$. Then $J$ is countable. So there exists an angle $\theta$ not in $J$. Let $\rho$ be the rotation about $\lambda$ by $\theta$. Then $\rho$ acts on $S^2$ with no fixed points in $D$, i.e., $\rho^n(D)$ is disjoint from $D$, and for natural $m<n$, $\rho^n(D)$ is disjoint from $\rho^m(D)$. Let $E$ be the disjoint of $\rho^n(D)$ over $n = 0, 1, 2, \ldots$. Then

\[
S^2 = E \cup (S^2 - E) \sim p(E) \cup (S^2 - E) = (E - D) \cup (S^2 - E) = S^2 - D,
\]

(3.1)

where $\sim$ denotes "is equidecomposable to".

Remark 3.2: We write "$\sim$" as "equivalent to". Eq. 3.2 as rendered is not tautologous. Because it is the crucial claim of the proof, the result is that the Banach-Tarski paradox is also not contradictory, and hence a non tautologous fragment of the universal logic $VL4$. 

From: en.wikipedia.org/wiki/Banach–Tarski_paradox