The title
Proof to the twin prime conjecture

Abstract
The elementary proof to the twin prime conjecture.

The content of the article Let \( p_s \) denote the \( s \)'th prime and \( P_s \) the product of the first \( s \) primes.

Define \( A_s \) to be the set of all positive integers less than \( P_s \) which are relatively prime to \( P_s \).

1. Each \( A_s \), for \( s \geq 3 \), contains two elements which differ by 2.
2. Consider the finite arithmetic progression \( \{a + mP_s\} \), where \( a \) is in \( A_s \) and \( 0 \leq m < P_s \). More than half of the elements are prime.
3. Combining 1) and 2), there is always a pair of twin primes which are relatively prime to \( P_s \), and therefore infinitely many twin primes.

For every pair of values \( a, b \) in \( A_s \) differing by \( d \), there exist at least \( p_s^2 - 2 \) pairs of values in \( A_{s+1} \) differing by \( d \). (And exactly that many when \( d \) is not divisible by \( p_s+1 \)).

Given this, the claim follows using induction with \( d = 2 \), noting for the base case that 11, 13 are both in \( A_3 \).

The proof to 1 is as follows: Suppose \( a \) and \( b \) are in \( A_s \), with \( b - a = d \).

Consider the set of values \( a + mP_s \),

where \( 0 \leq m < p_{s+1} \). These are all less than \( P_{s+1} \), and since \( P_s \) is relatively prime to \( p_{s+1} \),

there is a unique value \( m_1 \) with \( a + m_1P_s \) divisible by \( p_{s+1} \).

Similarly, there is a unique value \( m_2 \) with \( b + m_2P_s \) divisible by \( p_{s+1} \).

Furthermore, if \( m_1 = m_2 \), then \( (b + m_2P_s) - (a + m_1P_s) = d \) would be divisible by \( p_{s+1} \).

So when \( d \) is not divisible by \( p_{s+1} \), for the \( p_{s+1} - 2 \) values of \( 0 \leq m < p_{s+1} \) which are not equal to \( m_1 \) or \( m_2 \), the pair \( (a + mP_s, b + mP_s) \) are a pair in \( A_{s+1} \) differing by \( d \).

The proof to 2 is as follows:

Consider the finite arithmetic progression \( \{a + mP_s\} \), where \( a \) is in \( A_s \) and \( 0 \leq m < P_s \). More than half of the elements are prime.

Suppose \( a = P_s - 1 \)

Able to approximate all the non-prime numbers generated by the arithmetic progression \( P_s - 1 + mP_s \) where \( 0 \leq m < P_s \) with arithmetic progression \( 0 + n(P_s - 1) \) where \( 1 \leq n \leq P_s + 1 \).

Note that the first and last terms of the two arithmetic progressions are equal:

\[
P_s - 1 + 0 \times P_s = 0 + 1 \times (P_s - 1) \quad \text{and} \quad P_s - 1 + (P_s - 1)P_s = 0 + (P_s + 1)(P_s - 1).
\]

All terms of \( 0 + n(P_s - 1) \) when \( n > 1 \) are non-prime numbers.

Assume \( 0 + 1 \times (P_s - 1) \) is non-prime.
Apply the restriction that all non-prime numbers must be odd in arithmetic progression $P_s - 1 + mP_s$ to arithmetic progression $0 + n(P_s - 1)$

There are $\frac{P_s+2}{2}$ odd numbers out of $P_s + 1$ terms in $0 + n(P_s - 1)$

Therefore there are already naively at least $(P_s) - \frac{P_s+2}{2}$ prime numbers in arithmetic progression $P_s - 1 + mP_s$ after converting terms of $0 + n(P_s - 1)$ into terms of $P_s - 1 + mP_s$.

To convert form arithmetic progression $0 + n(P_s - 1)$ to arithmetic progression $P_s - 1 + mP_s$ where $1 \leq n \leq P_s + 1$ and $0 \leq m < P_s$.

The first and last terms of the two arithmetic progressions are equal: $P_s - 1 + 0 \times P_s = 0 + 1 \times (P_s - 1)$ and $P_s - 1 + (P_s - 1)P_s = 0 + (P_s + 1)(P_s - 1)$.

Reduce the number of terms from $P_s + 1$ to $P_s$ by removing the middle term $0 + \frac{P_s+1+1}{2} \times (P_s - 1)$.

All other terms are converted as such.

Not all odd values of $o(P_s - 1)$ where $1 \leq o < \frac{P_s+1+1}{2}$ can be non-prime in $P_s - 1 + mP_s$.

The approximate arithmetic progression to $a + mP_s$ is $-(P_s - 1) + a + n(P_s - 1)$ where $1 \leq n \leq P_s + 1$ and $0 \leq m < P_s$.

Suppose $a \neq P_s - 1$

Again apply the restriction that all non-prime numbers must be odd in $a + mP_s$.

Not all odd values of $-(P_s - 1) + a + o(P_s - 1)$ where $1 \leq o < \frac{P_s+1+1}{2}$ can be non-prime in $a + mP_s$.

To convert form arithmetic progression $0 + n(P_s - 1)$ to arithmetic progression $P_s - 1 + mP_s$ where $1 \leq n \leq P_s + 1$ and $0 \leq m < P_s$.

The first and last terms of the two arithmetic progressions are equal: $P_s - 1 + 0 \times P_s = 0 + 1 \times (P_s - 1)$ and $P_s - 1 + (P_s - 1)P_s = 0 + (P_s + 1)(P_s - 1)$.

Reduce the number of terms from $P_s + 1$ to $P_s$ by removing the middle term $0 + \frac{P_s+1+1}{2} \times (P_s - 1)$.

All other terms are converted as such.

Not all odd values of $-(P_s - 1) + a + o(P_s - 1)$ where $1 \leq o < \frac{P_s+1+1}{2}$ can be non-prime in $a + mP_s$.

Therefore the number of prime numbers in arithmetic progression $a + mP_s$ to be at least $P_s - (\frac{P_s+2}{2} - 2)$