The title

Proof to the twin prime conjecture

Authors

ZhangAik, Leet_Noob

Abstract

The elementary proof to the twin prime conjecture.

The content of the article

Let p_s denote the s'th prime and P_s the product of the first s primes.

Define A_s to be the set of all positive integers less than P_s which are relatively prime to P_s.

1. Each A_s, for $s \geq 3$, contains two elements which differ by 2.

2. Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. More than half of the elements are prime.

3. Combining 1) and 2), there is always a pair of twin primes which are relatively prime to P_s, and therefore infinitely many twin primes.

For every pair of values a, b in A_s differing by d, there exist at least $p_{s+1} - 2$ pairs of values in A_{s+1} differing by d. (And exactly that many when d is not divisible by p_{s+1}).

Given this, the claim follows using induction with $d = 2$, noting for the base case that $11, 13$ are both in A_3.

The proof is as follows: Suppose a and b are in A_s, with $b - a = d$. Consider the set of values $a + mP_s$, where $0 \leq m < p_{s+1}$. These are all less than P_{s+1}, and since P_s is relatively prime to p_{s+1}, there is a unique value $m1$ with $a + m1P_s$ divisible by p_{s+1}. Similarly, there is a unique value $m2$ with $b + m2P_s$ divisible by p_{s+1}. Furthermore, if $m1 = m2$, then $(b + m2P_s) - (a + m1P_s) = d$ would be divisible by p_{s+1}. So when d is not divisable by p_{s+1}, for the $p_{s+1} - 2$ values of $0 \leq m < p_{s+1}$ which are not equal to $m1$ or $m2$, the pair $(a + mP_s, b + mP_s)$ are a pair in A_{s+1} differing by d.

Proof of 2

Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. More than half of the elements are prime.

The largest number generate by $a + mP_s = {P_s}^2 - 1$ is when $a = P_s - 1$ and $m = P_s - 1$

Therefore all non-prime greater than 1 generated by arithmetic progression $a + mP_s$ must have an odd factor ≥ 3 and $\leq P_s - 1$

Consider the finite arithmetic progression $a + mP_s$, where $n \leq m < n + f$. If there exist a number divisable by f then there is a unique value $m1$ with $a + m1P_s$ divisible by f.

Therefore in arithmetic progression $P_s - 1 + mP_s$ and $0 \leq m < P_s$ only has two numbers $P_s - 1 + 0$ and $P_s - 1 + (P_s - 1) \times P_s$ which are divisable by $P_s - 1$.

Able to approximate all the non-prime numbers generated by the arithmetic progression P_s-1+mP_s where $0 \leq m < P_s$ with arithmetic progression $0+n(P_s-1)$ where $1 \leq n \leq P_s+1$.

Note that the common difference of the actual arithmetic progression is larger than the common difference of the approximate arithmetic progression. $P_s > P-s-1$

Therefore approximate arithmetic progression generates more elements than the actual progression between and including the values of P_s-1 and $(P_s-1)(P_s+1)$.

The actual non-prime numbers will either be greater than or equal to or less than or equal to the terms in the approximate arithmetic progression.

For each arithmetic progression term P_s-1+mP_s there exist unique $n1$ such that $0+n1(P_s-1) \leq P_s-1+mP_s \leq 0+(n1+1)(P_s-1)$.

The first and last terms are mapped to 1 and P_s+1: $P_s-1+0 \times P_s = 0+1(P_s-1)$ and $P_s-1+(P_s-1)P_s = 0+(P_s+1)(P_s-1)$

If $P_s-1+mP_s < frac(P_s)^2 2$ then map P_s-1+mP_s to $0+(n1+1)(P_s-1)$

If $P_s-1+mP_s > frac(P_s)^2 2$ then map P_s-1+mP_s to $0+(n1)(P_s-1)$

Assume the first and last terms are non-prime: $P_s-1+0 \times P_s = 0+1(P_s-1)$ and $P_s-1+(P_s-1)P_s = 0+(P_s+1)(P_s-1)$.

Applying the constriants of non-prime numbers found in actual arithmetic progression to the approximate arithmetic progression all non-prime numbers must not be divisable by an even number.

1) Therefore the estimate of times non-prime numbers generate in P_s-1+mP_s is equal to or less then the number of intervals.

One interval is $\geq 0+1(P_s-1)$ and $< 0+2(P_s-1)$. Another interval is $> 0+P_s(P_s-1)$ and $\leq 0+(P_s+1)(P_s-1)$ The intervals $> 0+(n1+1)(P_s-1)$ and $< 0+(n1+3)(P_s-1)$ when $1 \leq n1 < P_s+1$ and $n1$ is odd.

Total number of times when n is odd where $1 \leq n \leq P_s+1$ is $\frac{P_s+1+1}{2}$

2) The interval of $\geq 0+1 \times (P_s-1)$ and $< 0+2 \times (P_s-1)$ and the interval of $> 0+2 \times (P_s-1)$ and $< 0+4 \times (P_s-1)$ can be combined into one interval reducing the total estimate of non-prime numbers generated in P_s-1+mP_s by 1 because in P_s-1+mP_s all numbers are not divisable by 3 when $s \geq 2$

3) The interval of $> 0+P_s \times (P_s-1)$ and $\leq 0+(P_s+1) \times (P_s-1)$ and the interval of $> 0+(P_s-2) \times (P_s-1)$ and $\leq 0+(P_s-1) \times (P_s-1)$ can be combined into one interval reducing the total estimate of non-prime numbers generated in P_s-1+mP_s by 1 because in P_s-1+mP_s the only numbers divisable by (P_s-1) are P_s-1+0 and $P_s-1+(P_s-1)P_s$.

Therefore the maximum number of possible non-prime which could be non-prime numbers in actual arithmetic progression is $\frac{P_s+1+1}{2} - 2$

$$\frac{P_s + 1 + 1}{2} - 2 < \frac{P_s}{2}$$

Now consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$ and $a \neq P_s - 1$.

The approximate arithmetic progress $0 + n(P_s - 1)$ can be adjusted to become $-(P_s - 1) + a + n(P_s - 1)$ where $1 \leq n \leq P_s + 1$.

For example $1 + mP_s$ has the approximate arithmetic progression $-(P_s - 1) + 1 + n(P_s - 1)$.

The value of a is relatively prime to $P_s - 1$. Therefore for all odd numbers in approximate arithmetic progression $-(P_s - 1) + a + n(P_s - 1)$ where $3 \leq n \leq P_s - 1$ to be odd non-prime numbers then for each $-(P_s - 1) + a + n(P_s - 1)$ must be divisable by n. But $-(P_s - 1) + a + 3(P_s - 1)$ is not divisable by 3. But $-(P_s - 1) + a + (P_s - 1)(P_s - 1)$ is not divisable by $(P_s - 1)$. Therefore there are at least 2 odd numbers in $-(P_s - 1) + a + n(P_s - 1)$ which are prime.

Therefore the maximum number of possible non-prime generated which could be non-prime numbers in actual arithmetic progression is $\frac{P_s+1+1}{2} - 2$

$$\frac{P_s + 1 + 1}{2} - 2 < \frac{P_s}{2}$$

The adjusted approximate arithmetic progression does not generate more non-prime numbers because in each of the actual arithmetic progression where $a \neq P_s - 1$ and $0 \leq m < P_s$ there is unique $m1$ where $a + m1P_s$ is divisable by $P_s - 1$. Therefore the count of non-prime numbers in arithmetic progression when $a = P_s - 1$ can be applied to arithmetic progressions when $a \neq P_s - 1$.

https://www.reddit.com/r/badmathematics/comments/aljw4b /elementary_proof_to_the_twin_prime_conjecture_to/

User Leet_Noob rewrote proof structure and proof to 1.