Refutation of a complete axiomatization of reversible Kleene lattices

Abstract: We evaluate the main result theorem and two inequations as valid but not derivable from Kleene lattices. The theorem has an identical antecedent and consequent.. The two inequations are *not* tautologous and hence *not* valid, regardless of derivability without the union operator.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

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[W]e say that the equation e ≃ f is *valid* if the corresponding equality holds universally.

Remark 20.0: We write this as: □(e = f) > (e ≃ f). (20.0)

We may now prove the main result of this paper:
Theorem 20 (Main result). ∀e, f ∈ E_X, e ≡ f ⇔ e ≃ f. (20.1)

LET p, q, r, s: e, f, E, x.

(q<(r&s))&((#p=q)=(#p=q)) ; **FF**TT **FF**TT **FF**TT **FFFF** (20.2)

(#p=q)=(#p=q) ; TTTT TTTT TTTT TTTT (20.3)

Remark 20.3: If the connective symbols =, ≡, ⇔, and ≃ are equivalents, then Eq. 20.3 is a trivial equality.

Example 22 (Levi's lemma). ... the following inequation holds:

$(e_1 \cdot e_2) \cap (f_1 \cdot f_2) \lesssim (e_1 \cdot \top \cdot f_2) + (f_1 \cdot \top \cdot e_2)$. (22.1)

LET p, q, r, s: e_1, e_2, f_1, f_2.

~((((p&(p=p))&s)+((r&(p=p))&q))<((p&q)&(r&s))) = (p=p) ;
TTTT TT**FF** T**F**T**F** T**FF**T (22.2)

Example 23 (Factorisation). Another troubling example is the following:

$(a \cdot b) \cap (a \cdot c)\ a \lesssim ((\top \cdot b) \cap (\top \cdot c))$.

LET p, q, r: a, b, c. (23.1)

(p&(((p=p)&q)&((p=p)&r))) < ~((p&q)&(p&r)) ;
FFTT **FFFF** **FF**TT **FF**FT (23.2)

As before, this inequation is valid, but it is not derivable, and it does not involve unions.

Eq. 20.2, 22.2, and 23.2 are *not* tautologous. Eq. 20.3 is a trivial equality. This means Eqs. 22.2 and 23.2 are *not* valid as claimed, regardless of their derivability status without unions.