A Set of Sets and Quantification of Logic
—Towards a Truly Thinking Machine—
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Abstract
I describe relationship between a set of sets and quantification of logic. And I show a thesis which is essentially identical to an axiom of first order logic. Logic which is given in this paper is extension of propositional logic.

1. Preparation
A and B are existences. I call nothing an existence like I call zero a number.
Implication: A→B
Negation: ¬A = A→nothing
Relationship between all existences and nothing: whole = ¬nothing
Disjunction: A∨B
Conjunction: A∧B

1.1 A Truth Function
Use of a truth function is very technical. A calculation of a truth function is a calculation of a ramification. For example, a truth function of A→B is given as follows.

\[(A→B) = (A→B)∧(A∧¬B ∨ ¬A∧B ∨ ¬A∧¬B)\]
\[= (¬A∨B)∧(A∧¬B ∨ ¬A∧B ∨ ¬A∧¬B)\]
\[= A∧B ∨ nothing ∨ ¬A∧B ∨ ¬A∧¬B\]

2. A Set of Sets and Quantification of Logic
I define that a, b, c, and d are elements whose size are 1. And I define that whole=(a b c d) and A=(a b). The sentence that ‘one of A exists’ is represented by a set of sets as follows.

\[((a) (b) (a b) (a c) (a d) (b c) (b d) (a b c) (a b d) (a c d) (b c d) (a b c d))\]

An existence which is ¬A is represented by a set of sets as follows.

\[((nothing) (c) (d) (c d))\]

This set is a power set of ¬A. These two sets of sets are connected by relation of negation. I describe that ‘one of A exists’ as one(A) and a power set of A as A’. If so, the following thesis would be right generally.

¬one(A) = (∼A’)

I think that this thesis is essentially identical to an axiom of first order logic. When |A| is the size of A, this thesis is essentially identical to the following thesis.

¬(|A|≥1) = (|A|=0)

‘two of A exists’, ‘three of A exists’, etc. can be described in the same way.
2.1 Another Case in which a Set of Sets is Necessary
The sentence that ‘I eat bread or rice’ can be represented by a simple set, but I think that
the sentence that ‘I always eat bread, or I always eat rice’ can be represented only by a set
of sets. For example, I think that it might be represented as $A' \lor B'$.

2.2 Quantification and Probability
Logic is essentially probabilistic. For example, the sentence that ‘A can be B’ is essentially
identical to the sentence that ‘one of $A \land B$ exists’. Therefore, first order logic might include
modal logic.

3 Conclusion
I described relationship between a set of sets and quantification of logic. And I showed a
thesis which is essentially identical to an axiom of first order logic. Logic which was given
in this paper is extension of propositional logic. I will think about more general logic.