SPACETIME TRANSCRIPTION OF PHYSICAL FIELDS

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Abstract: In this work we discuss possible relationships between physical fields and substructures of the spatiotemporal manifold. We show that physical fields are possibly formed according to designed patterns from the substructures of spacetime in a process may be termed as spacetime transcription. We will formulate and illustrate how spacetime transcriptions can be formulated for the case of the electromagnetic and Dirac field. Even though the formulation in this work is rather suggestive, and a rigorous representation would require a comprehensive development in terms of geometrical and topological dynamics in differential geometry and topology, the work initiates a new approach to establishing intimate relationships between physical fields and spacetime structures.

In nature, elementary particles can be created, destroyed, or decay by themselves. However, besides many observable properties associated with these physical processes there is a prominent feature that has been assumed, also from observation, without further explanation, that all elementary particles of a particular type have identical physical structure in regard to the way they react with other physical objects. Even though in the present development of physical theories elementary particles are assumed to possess no substructure, the fact that they are created identically immediately raises the question of whether there are designed patterns for their creation, similar to the synthesis of biological objects in the well-known biological processes. In physics, when we observe physical phenomena that occur in space normally we assume that they manifest by their own physical structures which have no direct relationships with possible structures of the spatiotemporal continuum, which is simply regarded as a stage for physical objects to display their appearance. Despite this view point of physical existence is satisfactory for the sole purpose of formulating the dynamics of macroscopic objects in classical physics, physical occurrences at the quantum level have shown that quantum dynamics requires a more intimate relationship between physical fields and spacetime structures. In fact, as shown in our previous works, in order to account for quantum dynamics we have suggested that quantum particles themselves can be described as differentiable manifolds and the total physical existence can be considered as a fiber bundle in which physical fields are the dynamics of the fibers of different types [1-7]. This view of the spatiotemporal manifold makes physical existence as a whole look more like a biological system in which physical fields are formed on the base space through a process which may be termed as spacetime transcription in the sense that they are created according to designed patterns in the spatiotemporal continuum. In this work we will discuss and illustrate how such spacetime transcription can be formulated for the case of the electromagnetic and Dirac field.

Even though the formulation in this work is rather suggestive, and a rigorous representation would require a comprehensive development in terms of geometrical and topological dynamics in differential geometry and topology, the work initiates a new approach to establishing intimate relationships between physical fields and spacetime structures.

Consider $n-1$ vectors $\mathbf{F}_i = f_{i1}\mathbf{i}_1 + f_{i2}\mathbf{i}_2 + \cdots + f_{in}\mathbf{i}_n$, $i = 1, \dots, n-1$, in the Euclidean space R^n. We denote $[\mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_{n-1}]$ as the vector product of the $n-1$ vectors that is defined so that whose norm is given as [8-9]

$$|[\mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_{n-1}]| = |\mathbf{F}_1||\mathbf{F}_2| \dots |\mathbf{F}_{n-1}|K, \quad K = \begin{vmatrix} 1 & \cos\alpha_{1,2} & \cdots & \cos\alpha_{1,n-1} \\ \cos\alpha_{2,1} & 1 & \cdots & \cos\alpha_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \cos\alpha_{n-1,1} & \cos\alpha_{n-1,2} & \cdots & 1 \end{vmatrix}^{\frac{1}{2}} \tag{1}$$

where $\cos\alpha_{i,j} = (\mathbf{F}_i . \mathbf{F}_j)/(|\mathbf{F}_i||\mathbf{F}_j|)$. As in the case of the vector product defined in three-dimensional space, the vector $[\mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_{n-1}]$ is perpendicular to all vectors $\mathbf{F}_i$. However, in this work we will restrict to the four-dimensional Euclidean space R^4 and in this case the vector product of three vectors $\mathbf{F}_i = f_{i1}\mathbf{i}_1 + f_{i2}\mathbf{i}_2 + f_{i3}\mathbf{i}_3 + f_{i4}\mathbf{i}_4$, $i = 1, 2, 3$, can be written in the form

$$[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3] = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \end{vmatrix} \tag{2}$$

whose norm is

$$|[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3]| = |\mathbf{F}_1||\mathbf{F}_2||\mathbf{F}_3|K, \quad K = \begin{vmatrix} 1 & \cos\alpha_{1,2} & \cos\alpha_{1,3} \\ \cos\alpha_{2,1} & 1 & \cos\alpha_{2,3} \\ \cos\alpha_{3,1} & \cos\alpha_{3,2} & 1 \end{vmatrix}^{\frac{1}{2}} \tag{3}$$

For the purpose of later discussions, first we expand Equation (2) to obtain

$$\begin{aligned}[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3] = {} & \big(f_{12}(f_{23}f_{34} - f_{24}f_{33}) - f_{13}(f_{22}f_{34} - f_{24}f_{32}) + f_{14}(f_{22}f_{33} - f_{23}f_{32})\big)\mathbf{i}_1 \\ & - \big(f_{11}(f_{23}f_{34} - f_{24}f_{33}) - f_{13}(f_{21}f_{34} - f_{24}f_{31}) + f_{14}(f_{21}f_{33} - f_{23}f_{31})\big)\mathbf{i}_2 \\ & + \big(f_{11}(f_{22}f_{34} - f_{24}f_{32}) - f_{12}(f_{21}f_{34} - f_{24}f_{31}) + f_{14}(f_{21}f_{32} - f_{22}f_{31})\big)\mathbf{i}_3 \\ & - \big(f_{11}(f_{22}f_{33} - f_{23}f_{32}) - f_{12}(f_{21}f_{33} - f_{23}f_{31}) \\ & + f_{13}(f_{21}f_{32} - f_{22}f_{31})\big)\mathbf{i}_4 \end{aligned} \tag{4}$$

In general, the vector $[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3]$ is a four-dimensional vector in the Euclidean space R^4. However, the vector $[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3]$ can be reduced to a three-dimensional vector if the coefficient of $\mathbf{i}_4$ in Equation (4) vanishes, i.e.

$$f_{11}(f_{22}f_{33} - f_{23}f_{32}) - f_{12}(f_{21}f_{33} - f_{23}f_{31}) + f_{13}(f_{21}f_{32} - f_{22}f_{31}) \equiv 0 \tag{5}$$

There are various possible relationships between the functions f_{ij} for the condition given in Equation (5) to be satisfied. In this work we will consider the following particular condition

$$f_{21} = kf_{31}, \quad f_{22} = kf_{32} \quad \text{and} \quad f_{23} = kf_{33} \tag{6}$$

The condition given in Equation (6) states that the projection on the three-dimensional space $(\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$ of the vector $\mathbf{F}_2$ is proportional to the projection on the three-dimensional space $(\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$ of the vector $\mathbf{F}_3$. However, to be shown in the following, in order to discuss the spacetime transcription of physical fields, we need to express the four-dimensional curl of a three-dimensional vector for the case of the electromagnetic field and the four-dimensional curl of a two-dimensional vector for the case of the Dirac field, both of which as shown in our previous works exhibit fluid state in their steady state which in turns can be used to represent a quantum particle. Using the definition of the vector product in the four-dimensional Euclidean space R^4 given in Equation (2), the curl of two vectors $(\mathbf{F}_1, \mathbf{F}_2)$ can be defined as follows

$$\nabla \times (\mathbf{F}_1, \mathbf{F}_2) = [\nabla \mathbf{F}_1 \mathbf{F}_2] = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \end{vmatrix} \tag{7}$$

The vector product that is defined as the curl of two vectors given in Equation (7) is written in an expanded form as

$$\begin{aligned}
&\nabla \times (\mathbf{F}_1, \mathbf{F}_2) \\
&= \left(\frac{\partial}{\partial x_2}(f_{13}f_{24} - f_{14}f_{23}) - \frac{\partial}{\partial x_3}(f_{12}f_{24} - f_{14}f_{22}) + \frac{\partial}{\partial x_4}(f_{12}f_{23} - f_{13}f_{22})\right)\mathbf{i}_1 \\
&- \left(\frac{\partial}{\partial x_1}(f_{13}f_{24} - f_{14}f_{23}) - \frac{\partial}{\partial x_3}(f_{11}f_{24} - f_{14}f_{21}) + \frac{\partial}{\partial x_4}(f_{11}f_{23} - f_{13}f_{21})\right)\mathbf{i}_2 \\
&+ \left(\frac{\partial}{\partial x_1}(f_{12}f_{24} - f_{14}f_{22}) - \frac{\partial}{\partial x_2}(f_{11}f_{24} - f_{14}f_{21}) + \frac{\partial}{\partial x_4}(f_{11}f_{22} - f_{12}f_{21})\right)\mathbf{i}_3 \\
&- \left(\frac{\partial}{\partial x_1}(f_{12}f_{23} - f_{13}f_{22}) - \frac{\partial}{\partial x_2}(f_{11}f_{23} - f_{13}f_{21}) \right. \\
&+ \frac{\partial}{\partial x_3}(f_{11}f_{22} \\
&\left. - f_{12}f_{21})\right)\mathbf{i}_4
\end{aligned} \tag{8}$$

In the following we will use the form of vector product given in Equation (8) to formulate the observable three-dimensional electromagnetic field and two-dimensional Dirac field in the four-dimensional Euclidean space R^4. In fact, the normal vector product in the three-dimensional space R^3 can also be expressed in the four-dimensional form given in Equation

(8). For example, for any three-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3$ and if we use the unit vector in the fourth dimension $\mathbf{S} = \mathbf{i}_4$ then the normal three-dimensional curl of the vector $\mathbf{T}$ can be expressed as

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \frac{\partial}{x_1} & \frac{\partial}{x_2} & \frac{\partial}{x_3} & \frac{\partial}{x_4} \\ T_1 & T_2 & T_3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= \left(\frac{\partial T_3}{\partial x_2} - \frac{\partial T_2}{\partial x_3}\right)\mathbf{i}_1 - \left(\frac{\partial T_3}{\partial x_1} - \frac{\partial T_1}{\partial x_3}\right)\mathbf{i}_2 + \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3 \qquad (9)$$

It is seen that in order for the curl of the three-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3$ to be expressed in a four-dimensional form we would need to choose a particular form for the four-dimensional vector $\mathbf{S}$. We may then ask the question about the physical nature of the vector $\mathbf{S}$ and what role it will play in the physical formulation of natural phenomena that appear in the three-dimensional space that we are living in if the whole spatiotemporal continuum has more than three spatial dimensions. As shown above the vector product of three four-dimensional vectors can appear as a three-dimensional vector if they satisfy the condition given in Equation (6). In the following we will used this condition into physical fields such as the electromagnetic field and Dirac quantum fields to argue that the three-dimensional electromagnetic field and the two-dimensional Dirac fields may in fact be the product of a spacetime transcription from designed patterns in a spatiotemporal manifold with four spatial dimensions.

First, we will identify the vector $\mathbf{T}$ either as the electric field $\mathbf{E}$ or as the magnetic field $\mathbf{B}$. It is seen that to be able to associate the electric and magnetic field with possible spacetime structures in four dimensions instead of the unit vector in the fourth dimension $\mathbf{S} = \mathbf{i}_4$ we would need to extend the vector $\mathbf{S}$ to include the three-dimensional components in the form $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + S_3\mathbf{i}_3 + \mathbf{i}_4$. Then the four-dimensional curl for the three-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3$ can be written as

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & T_3 & 0 \\ S_1 & S_2 & S_3 & 1 \end{vmatrix}$$

$$= \left(\frac{\partial T_3}{\partial x_2} - \frac{\partial T_2}{\partial x_3} + \frac{\partial}{\partial x_4}(S_3 T_2 - S_2 T_3)\right)\mathbf{i}_1$$

$$- \left(\frac{\partial T_3}{\partial x_1} - \frac{\partial T_1}{\partial x_3} + \frac{\partial}{\partial x_4}(S_3 T_1 - S_1 T_3)\right)\mathbf{i}_2$$

$$+ \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_4}(S_2 T_1 - S_1 T_2)\right)\mathbf{i}_3$$

$$- \left(\frac{\partial}{\partial x_1}(S_3 T_2 - S_2 T_3) - \frac{\partial}{\partial x_2}(S_3 T_1 - S_1 T_3) + \frac{\partial}{\partial x_3}(S_2 T_1 - S_1 T_2)\right)\mathbf{i}_4 \qquad (10)$$

It is observed from Equation (10) that the four-dimensional curl $\nabla \times \mathbf{T}$ can be reduced to a three-dimensional vector if the following condition is satisfied

$$\left(\frac{\partial}{\partial x_1}(S_3 T_2 - S_2 T_3) - \frac{\partial}{\partial x_2}(S_3 T_1 - S_1 T_3) + \frac{\partial}{\partial x_3}(S_2 T_1 - S_1 T_2)\right) = 0 \qquad (11)$$

The equation given in Equation (11) can be satisfied by either imposing various conditions on the four-dimensional vector **S** or establishing different relationships between the vector **S** and the vector **T**. In this work we are interested in the following relationship between the vector **S** and the vector **T**

$$T_1 = kS_1, \quad T_2 = kS_2 \quad \text{and} \quad T_3 = kS_3 \qquad (12)$$

The condition given in Equation (12) will be termed as a spacetime transcription which is a spacetime process from which the vector **T** is formed using the four-dimensional spatiotemporal vector **S** as a designed pattern. From the spacetime transcription, the curl of a three-dimensional vector expressed in a four-dimensional Euclidean space is reduced to the normal three-dimensional curl as follows

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & T_3 & 0 \\ S_1 & S_2 & S_3 & 1 \end{vmatrix}$$

$$= \left(\frac{\partial T_3}{\partial x_2} - \frac{\partial T_2}{\partial x_3}\right)\mathbf{i}_1 - \left(\frac{\partial T_3}{\partial x_1} - \frac{\partial T_1}{\partial x_3}\right)\mathbf{i}_2 + \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3 \qquad (13)$$

Now, if we identify the vector **T** with the electric field **E** and the magnetic field **B** then we may say that the three-dimensional electromagnetic field is a product obtained from a process that can be identified with a spacetime transcription. As shown in our work on the fluid state of a steady electromagnetic field only $\nabla \times \mathbf{E} = 0$ and $\nabla \times \mathbf{B} = 0$ are required to construct possible structure of the quantum particle of the electromagnetic field, namely the photon. However, we have shown only the process of a spatial transcription, and this process alone does not establish a complete three-dimensional electromagnetic field that would require other relationships between $\nabla \times \mathbf{E}$ and $\nabla \times \mathbf{B}$ and the temporal rates, as well as different physical entities, such as charge, as described by Maxwell field equations of electromagnetism $\nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = 0$, $\nabla \times \mathbf{B} - \epsilon\mu\,\partial \mathbf{E}/\partial t = \mu \mathbf{j}_e$. Such an anticipated formulation needs to undertake further investigation into the temporal transcription of physical fields in the temporal submanifold of the spatiotemporal continuum. In the mean time we will extend our discussion to the Dirac quantum field of elementary particles.

In order to formulate a similar spacetime transcription for Dirac quantum fields, we consider a two-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2$ and a four-dimensional spacetime vector $\mathbf{S}$ of the form $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + \mathbf{i}_3 + \mathbf{i}_4$. Then the four-dimensional curl $\nabla \times \mathbf{T}$ of the vector $\mathbf{T}$ takes the form

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix}$$
$$= \left(-\frac{\partial T_2}{\partial x_3} + \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2$$
$$+ \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_4}(S_2T_1 - S_1T_2)\right)\mathbf{i}_3$$
$$- \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_3}(S_2T_1 - S_1T_2)\right)\mathbf{i}_4 \qquad (14)$$

If we impose the spacetime transcription condition

$$T_1 = kS_1, \qquad T_2 = kS_2 \qquad (15)$$

then we have

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix}$$
$$= \left(-\frac{\partial T_2}{\partial x_3} + \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2 + \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3$$
$$- \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_4 \qquad (16)$$

It is seen from Equation (16) that the required condition for the curl $\nabla \times \mathbf{T}$ to be a two-dimensional vector is

$$\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} = 0 \tag{17}$$

To form a complete system in which the quantities T_1 and T_2 can be identified as the velocity potential and the stream function of a two-dimensional fluid flow for Dirac quantum fields as we have presented in our previous works, we need to consider a second two-dimensional vector field of the form $\mathbf{T} = T_1\mathbf{i}_1 - T_2\mathbf{i}_2$. The four-dimensional curl of the vector $\mathbf{T}$ takes the form

$$\begin{aligned}\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) &= \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & -T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix} \\ &= \left(\frac{\partial T_2}{\partial x_3} - \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2 \\ &+ \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_4}(S_2T_1 + S_1T_2)\right)\mathbf{i}_3 \\ &- \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_3}(S_2T_1 + S_1T_2)\right)\mathbf{i}_4 \end{aligned} \tag{18}$$

If we also impose the spacetime transcription condition

$$T_1 = kS_1, \quad T_2 = -kS_2 \tag{19}$$

then we have

$$\begin{aligned}\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) &= \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & -T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix} \\ &= \left(\frac{\partial T_2}{\partial x_3} - \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2 + \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3 \\ &- \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_4 \end{aligned} \tag{20}$$

It is seen that the condition for the four-dimensional curl $\nabla \times \mathbf{T}$ to be reduced to a two-dimensional vector is

$$\frac{\partial T_2}{\partial x_1} + \frac{\partial T_1}{\partial x_2} = 0 \tag{21}$$

The conditions given in Equations (17) and (21) together form the Cauchy-Riemann equations in the (x_1, x_2)-plane

$$\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} = 0, \qquad \frac{\partial T_2}{\partial x_1} + \frac{\partial T_1}{\partial x_2} = 0 \tag{22}$$

From the system of Cauchy-Riemann equations given in Equation (22) the quantities T_1 and T_2 can be identified as the velocity potential and the stream function of a two-dimensional fluid flow. For the case of Dirac fields, if we want to form a standing wave that can be used to represent a quantum particle then we would need another two vector fields of the forms $\mathbf{T} = T_3\mathbf{i}_1 + T_4\mathbf{i}_2$ and $\mathbf{T} = T_3\mathbf{i}_1 - T_4\mathbf{i}_2$. By using the four-dimensional spacetime vector $\mathbf{S}$ also of the form $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + \mathbf{i}_3 + \mathbf{i}_4$, then we have similarly the conditions for the four-dimensional curl of the vectors $\mathbf{T} = T_3\mathbf{i}_1 + T_4\mathbf{i}_2$ and $\mathbf{T} = T_3\mathbf{i}_1 - T_4\mathbf{i}_2$ to be two-dimensional vectors as follows

$$\frac{\partial T_3}{\partial x_1} + \frac{\partial T_4}{\partial x_2} = 0, \qquad \frac{\partial T_3}{\partial x_1} - \frac{\partial T_4}{\partial x_2} = 0 \tag{23}$$

The conditions given in Equation (23) shows that T_3 and T_4 can also be identified as a vector potential and stream function of a two-dimensional fluid flow, which are complementary to the quantities T_1 and T_2 with similar identification.

We have shown that both the electromagnetic field and Dirac field of quantum particles can be regarded as being formed from the processes of spacetime transcription that convert the four-dimensional patterns into three and two-dimensional physical objects, we may ask whether it is possible to form four-dimensional physical objects in similar processes of spacetime transcription. For example, if we consider a general four-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3 + T_4\mathbf{i}_4$ and $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + S_3\mathbf{i}_3 + S_4\mathbf{i}_4$ then the curl of the vector $\mathbf{T}$ with regard to the vector $\mathbf{S}$ takes the more general form

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & T_3 & T_4 \\ S_1 & S_2 & S_3 & S_4 \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x_2}(S_4T_3 - S_3T_4) - \frac{\partial}{\partial x_3}(S_4T_2 - S_2T_4) + \frac{\partial}{\partial x_4}(S_3T_2 - S_2T_3)\right)\mathbf{i}_1$$
$$- \left(\frac{\partial}{\partial x_1}(S_4T_3 - S_3T_4) - \frac{\partial}{\partial x_3}(S_4T_1 - S_1T_4) + \frac{\partial}{\partial x_4}(S_3T_1 - S_1T_3)\right)\mathbf{i}_2$$
$$+ \left(\frac{\partial}{\partial x_1}(S_4T_2 - S_2T_4) - \frac{\partial}{\partial x_2}(S_4T_1 - S_1T_4) + \frac{\partial}{\partial x_4}(S_2T_1 - S_1T_2)\right)\mathbf{i}_3$$
$$- \left(\frac{\partial}{\partial x_1}(S_3T_2 - S_2T_3) - \frac{\partial}{\partial x_2}(S_3T_1 - S_1T_3) + \frac{\partial}{\partial x_3}(S_2T_1 - S_1T_2)\right)\mathbf{i}_4 \tag{24}$$

Again, if we impose the conditions $T_1 = kS_1$, $T_2 = kS_2$, $T_3 = kS_3$ and $T_4 = kS_4$ then the curl $\nabla \times \mathbf{T} \equiv 0$, which indicates that there are no unified four-dimensional electromagnetic fields that exist in the four-dimensional Euclidean space R^4, similar to the existence of the three-dimensional electromagnetic field in the three-dimensional Euclidean space R^3. However, it can be seen that a steady four-dimensional photon can be constructed in the four-dimensional Euclidean space.

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