Constant $e \cdot c / 2\pi \alpha$ determines
magnetic flux quantum in charged leptons

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Abstract

The constant $e \cdot c / 2\pi \alpha$ is a common characteristic of charged leptons $(e, \mu, \tau)$ resulting from their identical fraction $\hat{m} / \lambda_C$ of magnetons $\hat{m}$ to Compton-wavelengths $\lambda_C$, in spite of their largely differing $\hat{m}$ and $\lambda_C$. However the physical interpretation of this constant remained uncertain, but now clarified: It is proven that $e \cdot c / 2\pi \alpha$ is an alternative and equivalent definition of the magnetic flux quantum $\hbar / 2e$ which makes up the dipole-fields of charged leptons.

1 Introduction, problem and goal

Comprehensive consideration and incorporation of magnetic flux quantization in theories of charged leptons like $e, \mu, \tau$ has not yet been established. (1) On the one hand, the magnetic dipole-moments $\hat{m}$ (magnetons) assignable to $e, \mu$ and $\tau$ belong to the theoretically and experimentally most precisely determined constants. (CODATA) On the other hand, only rough estimates exist for the magnetic flux included in their dipole-fields. However the existence of a presumably constant magnetic flux in the dipole-fields of leptons is reasonable. Moreover, consideration of flux quantization would support the conjecture of magnetic flux conservation in the dipole-fields of charged leptons. Nonetheless current electron-models like the ”mathematical” or ”dressed” electron don’t take into account the magnetic flux included in its dipole-field.

If the principle of magnetic flux quantization was applied to charged leptons the ab-initio postulate that each of their dipole-fields comprises identical magnetic flux amounting at least one magnetic flux quantum $\Phi_0 = \hbar / 2e$ would have to be proven. This initial assumption is supported by the finding that the constant $e \cdot c / 2\pi \alpha$ - being assigned to charged leptons - is an implicit definition of the magnetic flux quantum presumably located in charged leptons. The approach will essentially consist of an analysis of the dipole-fields of charged leptons - focusing on their magnetic flux $\Phi_l$ - with the aim to clarify the physical interpretation of the constant $e \cdot c / 2\pi \alpha$. It can be anticipated that there exists a fundamental relationship among $e \cdot c / 2\pi \alpha$, the dipole-field of charged leptons,
2 Modeling of the dipole-field of charged leptons

2.1 Frame of reference

For modeling and analysis of the dipole-field of charged leptons the choice of an adequate frame and plane of reference is crucial. Thus a spherical \( r, \Theta, \varphi \) and a cartesian \( x - y - z \) - frame will be used. Their polar axis coincide with the dipole-axis.

2.2 Equatorial plane of reference \( p_e \)

The most convenient plane of reference for determination of the magnetic flux \( \Phi \) penetrating this plane is the equatorial (azimuthal) \( x - y \) plane \( p_e \) of the dipole. Let \( \vec{n} \) designate the normal unit-vector of \( p_e \). Among the specific advantages of \( p_e \) is that it is a symmetry-plane and that the angle of incidence of the induction \( \vec{B} \) field into \( p_e \) always is perpendicular to \( p_e \), i.e. \( \vec{B} \perp p_e \) or \( \vec{B} \parallel \vec{n} \).

2.3 Induction-field of a point-like dipole

The induction- or \( \vec{B} \)- field of a point-like dipole \( \hat{m} \) can most conveniently be represented by the \( \vec{B} \)- field generated by a microscopic circular current-loop of radius \( r_x \) located in \( p_e \), where its axis is centered with the \( z \)-axis. Generally, any dipole-field traversing the equatorial plane \( p_e \) comprises mutually opposed internal and an external \( \vec{B} \)- field sections designated \( \vec{B}_{int} \) and \( \vec{B}_{ext} \), corresponding to magnetic fluxes \( \Phi_{int} \) and \( \Phi_{ext} \). \( \Phi_{int} \) and \( \Phi_{ext} \) are delimited by their generating circular current-loop in \( p_e \). The equatorial plane \( p_e \) will serve as a reference plane for determination of \( \Phi_{int} \) and \( \Phi_{ext} \). According to Maxwell’s equation \( \nabla \cdot \vec{B} = 0 \rightarrow \Phi_{int} = -\Phi_{ext} \). Hence it will suffice to determine \( \Phi_{ext} \) so there is no need to dwell with \( \Phi_{int} \). \( \Phi_{ext} \) will be identified with total magnetic flux of a dipole-field.

3 External magnetic flux \( \Phi_{ext} \) of a dipole-field

The external induction field of a dipole is

\[
\vec{B}_{ext} \approx \frac{\mu_0 \hat{m}}{4\pi r^3} \left( 2 \cos \Theta + \sin \Theta \right)
\]

where \( \mu_0 = 1/\epsilon_0 c^2 \) is vacuum permeability, \( \hat{m} \) the dipole-moment or magneton, \( \Theta \) the polar angle and \( r > r_c \) the radial distance of a point from the origin.
Generally, the magnetic flux $\Phi$ of an induction field $\vec{B}$ penetrating a given surface $A$ of arbitrary orientation $\vec{n}$ is

$$\Phi = \int_A d\Phi = \int_A \vec{B} \cdot d\vec{a}$$

(2)

where $d\vec{a}$ is a surface element differential.

In $p_e$, the following applies: $\vec{n} \parallel \vec{B}$ or $\vec{n} \cdot \vec{B} = |\vec{B}|$.

For all points in $p_e$ with $r > r_c$:

$\Theta = \pi/2 \rightarrow \cos \Theta = 0$, $\sin \Theta = 1$, $\vec{B}(\vec{r}) \parallel z$ and $\vec{B}(\vec{r}) \perp p_e$.

Thus the first term in brackets in (1) vanishes and (1) is reduced to

$$B_{ext} = \frac{\mu_0 \bar{m}_e}{4 \pi r^3}$$

(3)

Total external magnetic flux $\Phi_{ext}$ through $p_e$ results from integration of (3) over $p_e$ from $r_c$ to $\infty$.

$$\Phi_{ext} = \int_{r_c}^\infty \vec{B}_r da = 2\pi \int_{r_c}^\infty \vec{B}_r r dr$$

(4)

where

$$da = 2\pi r dr$$

(5)

After substitution with (5) and integration of (4)

$$\Phi_{ext} = \frac{\mu_0 \bar{m}_e}{2r_c}$$

(6)

It should be pointed out that (6) reveals how $\Phi_{ext}$ is determined by a singular variable $r_c$ (classical electron- or Compton-radius). However $r_c$ should not be misinterpreted as the radius of a classical sphere thus more of a self-determined length-unit allocable to the electron without any geometrical meaning.

### 3.1 Delimiting "critical" radius $r_c$

The radius $r_c$ in (4) and (6) is a lower or "critical" integration limit in the dipole-field being identical with the radius of a.m. circular current-loop. As $r_c$ delimits the mutually opposed internal flux $\Phi_{int}$ from the external flux $\Phi_{ext}$, $r_c$ is the key variable to be determined. For the electron the author already proved in [4] that $r_c$ is identical with the "classical" or "charge" radius $r_e$.

$r_e$ can also be expressed as a function of the Compton-wavelength $\lambda_{Ce}$ thus being a self-defined length-unit of the electron:

$$r_c = r_e = \frac{\alpha \lambda_{Ce}}{2\pi} = \alpha \frac{\hbar}{m_e c}$$

(7)
where $\alpha$ = fine-structure constant, $m_e = \text{electron-mass}$ and $\lambda_{Ce} = \text{electron Compton-wavelength}$.

Substitution with (7) in (6):

$$\Phi_{\text{ext}} = \frac{\mu_0}{2\alpha} \cdot \frac{\hat{m}_e}{\lambda_{Ce}} = \frac{\hat{m}_e}{2\alpha \cdot \epsilon_0 c^2 \lambda_{Ce}} \quad (8)$$

To facilitate interpretation of (8) it can be split into more meaningful factors:

$$\lambda_{Ce} = \frac{h}{m_e c} \quad (8 \ a)$$

$$\alpha = \frac{e^2}{4 \pi \epsilon_0 \hbar c} \quad (8 \ b)$$

$$\hat{m}_e = \frac{e \hbar}{2m_e} = \frac{e \hbar}{4 \pi m_e} \quad (8 \ c)$$

By substitution of (8a), (8b) and (8c) in (8) the result reveals

$$\Phi_{\text{ext}} = \frac{h}{2e} = \Phi_0 \quad (9)$$

where $\Phi_0$ is the magnetic flux quantum!

Remarkably in (8), $\Phi_{\text{ext}}$ is exclusively determined by the constants $\epsilon_0$, $c$, $\alpha$, $\hat{m}_e$, $\lambda_{Ce}$.

Let

$$C_\Phi = \frac{\hat{m}_e}{\alpha \lambda_{Ce}} = \frac{e \cdot c}{2\pi \alpha} \quad (10)$$

Substitution with (10) in (8) yields

$$\Phi_{\text{ext}} = \frac{\mu_0}{2} \cdot C_\Phi = \mu_0 \cdot \frac{e \cdot c}{4\pi \alpha} = \Phi_0 \quad (11)$$

where $\Phi_0$ is the magnetic flux quantum.

Note that $\Phi_0 = \Phi_{\text{ext}} = -\Phi_{\text{int}}$.

In $p_e$, $\Phi_{\text{ext}}$ is delimited from $\Phi_{\text{int}}$ by a circle of Compton-radius $\alpha \lambda_C/2\pi$ (7).

### 3.2 Critical radius of Myons and Tauons

It is evident from (8) that above approach for the electron would also apply for myons and tauons if their individual masses $m_l$, magnetons $\hat{m}_l$ and Compton-wavelengths $\lambda_{Cl}$ were used.

Substitution in (8c) with a generalized lepton-mass $m_l$ (instead of $m_e$) and a general lepton-magneton $\hat{m}_l$ (instead of $\hat{m}_e$) yields

$$\hat{m}_l = \frac{e \hbar}{4\pi m_l} \quad (12)$$
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Charged leptons comprise magnetic flux quantum

as well as a generalized Compton-wavelength $\lambda_{Cl}$

$$\lambda_{Cl} = \frac{h}{m_l c}$$  \hfill (13)

The ratio (12) to (13) $\times \alpha^{-1}$ delivers a constant $C_\Phi$ for all charged leptons:

$$C_\Phi = \frac{\hat{m}_l}{\alpha \lambda_{Cl}} = \frac{e \cdot c}{2 \pi \alpha} \left[ \frac{A}{m} m^2 \right]$$  \hfill (14)

As (14) is identical with (10) constant magnetic flux $\Phi_l$ of all charged leptons can be inferred.

The dimensionality of $C_\Phi$ corresponds to a total flux of a magnetic field-strength vector-field $\vec{H}$ through $p_e$ being in proportion to total flux of the induction-field $\vec{B} = \mu_0 \vec{H}$ through $p_e$.

Substitution in (8) with (10) and (12) yields

$$\Phi_l = \frac{\mu_0}{2} C_\Phi = \frac{h}{2e} = \Phi_0$$  \hfill (15)

In conclusion (14) and (15) prove that all charged Leptons carry one magnetic flux quantum $\Phi_0$.

4 Summary and Comment

A new commonality among charged leptons $e, \mu, \tau$ is revealed: Each of their dipole-fields coincides with one magnetic flux quantum being determined by their common constant $e \cdot c / 2\pi \alpha$ being given by the fraction $\hat{m}_l / \lambda_{Cl}$ of their individual magnetons $\hat{m}_l$ and Compton-wavelengths $\lambda_{Cl}$.

This conclusion is based on analysis of their magnetic dipole-fields which unravels that the constant $e \cdot c / 2\pi \alpha$ is an equivalent (hidden) definition of the magnetic flux-quantum $h/2e$. It denotes identical flux of the leptons dipole fields through their equatorial planes - outside of their Compton-radii - being equivalent to one magnetic flux quantum $h/2e$.

The mathematical procedure includes breaking of the fine-structure constant $\alpha$ into its factors (8 b). The analysis further reveals a remarkable and unexpected role of the “classical” or “Compton” electron radius $r_e$ (and its equivalents for myons and tauons), being intimately related to the magnetic flux quantum, Josephson’s constant, spin-angular momentum $\hbar/2$ and the constant $e \cdot c / 2\pi$, even if $r_e$ is regarded as a fictitious length. [5]

The task could also be inverted by asking for a critical radius $r_c$ which delimits one fluxon $\Phi_0$ in $p_e$ following a transformation of (6): $r_c = \mu_0 \hat{m}_l / 2 \Phi_0$.

It deserves a final remark that (14), (15) would also apply for protons.
References


