Refutation of bisimulation

Abstract: We evaluate two definitions of bisimularity, both *not* tautologous. That refutes bisimulation, along with its proof tools of coinduction and Howe's congruence method.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Pitts, A. (2011). Howe's method for higher-order languages.
In D. Sangorgi, J. Rutten (eds.), Advanced topics in bisimulation and coinduction,
Cambridge tracts in theoretical computer science. no. 52, chapter 5, pages 197-232.
cl.cam.ac.uk/~amp12/papers/howmho/howmho.pdf andrew.pitts@cl.cam.ac.uk

5. Howe's method for higher-order languages
5.1 Introduction
[A]lthough it is usually easy to see that a bisimilarity ≃ satisfies

$$\forall Q.\ P(x) \simeq P'(x) \Rightarrow P(Q) \simeq P'(Q) \tag{5.1.1.1}$$

LET p, q, r, s, t: P, Q, P', x, Q'

((p&s)=(r&s))>((p&#q)=(r&#q)) ; TTTC TTCT TTTT TTTT (5.1.1.2)

for compatibility of ≃ we have to establish the stronger property

$$\forall Q,Q'.\ P(x) \simeq P'(x) \wedge Q \simeq Q' \Rightarrow P(Q) \simeq P'(Q') \tag{5.1.2.1}$$

(((p&s)=(r&s))&(#q=#t))>((p&#q)=(r&#t)) ;
TTTC TTCT TTTT TTTT,
TTTT TTTT TTTT TTTT (5.1.2.1)

This is often hard to prove directly from the definition of ≃.

Eqs. 5.1.1.2 and 5.1.2.2 as rendered are *not* tautologous. This refutes bisimularity, along with its proof tools of coinduction and Howe's congruence method.