

# Extended GRA Strategy for Multi-Attribute Decision Making with Trapezoidal Neutrosophic Numbers

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**Abstract.** Multi-attribute decision making(MADM) strategy has been proposed to handle uncertain decision making problem .The most extensively used models of Grey system theory is grey relational analysis (GRA). This strategy was flourished by Chinese Professor J.Deng. This strategy also known as Deng’s Grey Incidence Analysis model. GRA uses a generic concept of intelligence. It describes any circumstance as, no information as black, and perfect information as white. Nevertheless, these idealized situations ever appear in real world problem. In this paper, we extend GRA strategy for multi attribute decision making in trapezoidal neutrosophic number (TrNN) environment. Here, we describe score and accuracy function for TrNNs. Then Hamming distance for twoTrNNs are also described. Lastly, a numerical problem is solved to explain thepertinence of the proposed strategy.

**Keywords:** Neutrosophic set, Trapezoidal neutrosophic fuzzy number, Multi-attribute decision making, VIKOR strategy.

## 1 Introduction

In 1998, Smarandache [1] consecrated the notion of neutrosophic set by incorporating philosophy of neutrosophy [1] in mathematical arena. Thereafter, Wang et al. [2] defined single valued neutrosophic set. Neutrosophic set and its various extensions and hybrid sets have been widely employed in decision making problems [3-58], conflict resolution [59], image processing [60], medical diagnosis [61], social science [62], etc.

Ye [63] introduced trapezoidal neutrosophic number (TrNN) in 2017. The TrNN and the single valued neutrosophic set (SVNS) [2] are very effective mathematical tools to deal with indeterminacy, incomplete, and inconsistent information. Single valued trapezoidal neutrosophic number (SVTrNN) [63, 64] is an extension of SVNS. Each element of SVTrNN is distinguished by trapezoidal number with truth membership degree, indeterminacy membership degree and falsity membership degree. Biswas et al. [65] documented value and ambiguity based ranking strategy for SVTrNN and employed the strategy to handle multi-attribute decision making (MADM) problem. Biswas et al. [66] developed a technique for order of preference by similarity to ideal solution (TOPSIS) strategy for MADM with TrNNs. Biswas et al. [67] presented distance measure based MADM strategy with interval trapezoidal neutrosophic numbers (ITrNNs).

In 1982, Deng [68, 69] introduced a grey relation analysis (GRA) to deal with uncertainty. Rao and Singh [70] introduced modified GRA strategy for decision making in manufacturing situation. In 2011, Pramanik and Mukhopadhyaya [71] studied a GRA based multi criteria group decision making (MCGDM) strategy for teacher selection in intuitionistic fuzzy set environment. In 2011, Wei [72] introduced a GRA strategy for intuitionistic fuzzy MCDM. Biswas et al. [73] discussed an entropy related GRA for MADM strategy in SVNS environment. Dey et al. [74] developed a GRA based MCGDM strategy for weaver selection in Khadi institutions in intuitionistic fuzzy environment in 2015. In 2015, Pramanik and Mondal [75] proposed a GRA for MADM strategy in an interval neutrosophic set environment. Dey et al. [76] studied an extended GRA for neutrosophic MADM strategy in interval uncertain linguistic setting. Banerjee et al. [77] constructed an MADM model via GRA for neutrosophic cubic set envinment.

GRA based MADM strategy is not proposed in the literature. To fill the research gap, we propose an extended GRA based MADM strategy to deal decision making problems in TrNN environment.

This paper is organized as follows: In section 2, we introduce some definitions relating to neutrosophic set and trapezoidal neutrosophic number. In section 3, we develop an extended GRA strategy for MADM. In section 5, an illustrative example is discussed to demonstrate the applicability of the proposed strategy. Lastly, section 6 represents the concluding remarks.

## 2 Preliminaries

In this section, we recall some basic definitions related to neutrosophic sets, trapezoidal neutrosophic set and GRA strategy.

**Definition 2. 1.** Let  $Z$  be a universal set. A single-valued neutrosophic set [2]  $Y$  in  $Z$  is given by

$$Y = \{z, \langle T_Y(z), I_Y(z), F_Y(z) \rangle \mid z \in Z\} \quad (2)$$

where  $T_Y(z): Z \rightarrow [0,1]$ ,  $I_Y(z): Z \rightarrow [0,1]$  and  $F_Y(z): Z \rightarrow [0,1]$  with the condition  $0 \leq T_Y(z) + I_Y(z) + F_Y(z) \leq 3$  for all  $z \in Z$ . The functions  $T_Y(z)$ ,  $I_Y(z)$  and  $F_Y(z)$  are respectively, the truth membership function, the indeterminacy membership function and the falsity membership function of the element  $z$  to the set  $Y$ .

**Definition 2. 2.** Let  $X$  be a single valued trapezoidal neutrosophic number [63, 64]. Then, its truth membership function is

$$T_X(z) = \begin{cases} \frac{(z-e)t_x}{(f-e)}, e \leq z < f \\ t_x, f \leq z \leq g \\ \frac{(h-z)t_x}{(h-g)}, g < z \leq h \\ 0, otherwise \end{cases} \quad (3)$$

Its indeterminacy membership function is

$$I_X(z) = \begin{cases} \frac{(f-z) + (z-e)i_x}{(f-e)}, e \leq z < f \\ i_x, f \leq z \leq g \\ \frac{z-g + (h-z)i_x}{h-g}, g < z \leq h \\ 0, otherwise \end{cases} \quad (4)$$

And its falsity membership function is

$$F_X(z) = \begin{cases} \frac{f-z + (z-e)f_x}{f-e}, e \leq z < f \\ f_x, f \leq z \leq g \\ \frac{z-g + (h-z)f_x}{h-g}, g < z \leq h \\ 0, otherwise \end{cases} \quad (5)$$

Where  $0 \leq T_X(z) \leq 1$ ,  $0 \leq I_X(z) \leq 1$  and  $0 \leq F_X(z) \leq 1$  and  $0 \leq T_X(z) + I_X(z) + F_X(z) \leq 3$ ;  $e, f, g, h \in R$ . Then  $X = ([e, f, g, h]; t_x, i_x, f_x)$  is called a neutrosophic trapezoidal number.

If  $0 \leq e \leq f \leq g \leq h$ , then  $X$  is called a positive TrNN. If  $e \leq f \leq g \leq h \leq 0$ , then  $X$  is called a negative TrNN. If  $0 \leq e \leq f \leq g \leq h \leq 1$  and  $T_X, I_X, F_X \in [0,1]$ , then  $X$  is called a normalized TrNN, which is used in this paper.

**Definition 2. 3.** Let  $K = \langle (e, f, g, h); T_K, I_K, F_K \rangle$  be TrNN. Then the score function [63]  $S(K)$  of TrNN is defined by

$$S(K) = \frac{1}{12}(e+f+g+h)(2+T_K - I_K - F_K), S(K) \in [0,1] \quad (6)$$

**Definition 2. 4.** The accuracy function [78]  $Ac(K)$  of TrNN  $K = \langle (e, f, g, h); T_K, I_K, F_K \rangle$  is defined by

$$Ac(K) = \frac{1}{6}(g+h-e-f)(2+T_K - F_K), Ac(K) \in [0,1] \quad (7)$$

**Definition 2. 5. Comparison of two TrNNs:**

Let  $K_1 = \langle (e_1, f_1, g_1, h_1); T_{K_1}, I_{K_1}, F_{K_1} \rangle$  and  $K_2 = \langle (e_2, f_2, g_2, h_2); T_{K_2}, I_{K_2}, F_{K_2} \rangle$  be two TrNN.

The comparison between the two TrNNs [78] is stated as follows:

1. If  $Sc(K_1) > Sc(K_2)$ , then  $K_1 > K_2$ .
2. If  $Sc(K_1) = Sc(K_2)$  and  $Ac(K_1) > Ac(K_2)$ , then  $K_1 > K_2$ .

3.If  $Sc(K_1) = Sc(K_2)$  and  $Ac(K_1) = Ac(K_2)$ , then  $K_1 = K_2$ .

**Definition 2. 6.** Let  $K_1 = ([e_1, f_1, g_1, g_1]; t_{K_1}, i_{K_1}, f_{K_1})$  and  $K_2 = ([e_2, f_2, g_2, h_2]; t_{K_2}, i_{K_2}, f_{K_2})$  be two neutrosophic trapezoidal fuzzy numbers, its Hamming distance [66] between  $K_1$  and  $K_2$  is defined as follows:

$$d(K_1, K_2) = \frac{1}{12} \left( |e_1(2 + t_{K_1} - i_{K_1} - f_{K_1}) - e_2(2 + t_{K_2} - i_{K_2} - f_{K_2})| + |f_1(2 + t_{K_1} - i_{K_1} - f_{K_1}) - f_2(2 + t_{K_2} - i_{K_2} - f_{K_2})| \right. \\ \left. + |g_1(2 + t_{K_1} - i_{K_1} - f_{K_1}) - g_2(2 + t_{K_2} - i_{K_2} - f_{K_2})| + |h_1(2 + t_{K_1} - i_{K_1} - f_{K_1}) - h_2(2 + t_{K_2} - i_{K_2} - f_{K_2})| \right) \tag{9}$$

**3. Standardize the decision matrix**

Let  $D = (c_{ij})_{p \times q}$  be a neutrosophic decision matrix, where the SVTrNN  $\tilde{c}_{ij} = ([c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4]; t_{\tilde{c}_{ij}}, i_{\tilde{c}_{ij}}, f_{\tilde{c}_{ij}})$  is the rating value of alternative  $Y_i$  w.r.t.attribute  $Z_j$ . Now to eliminate the effect from different physical dimension into decision making process, we should standardize the decision matrix  $(c_{ij})_{p \times q}$  based on two common types of attribute such as benefit and cost type attribute. We consider the following technique to obtain the standardized decision matrix  $Z^* = (\tilde{z}_{ij})_{p \times q}$ , in which the component  $z_{ij}^k$  of the entry  $\tilde{z}_{ij} = ([z_{ij}^1, z_{ij}^2, z_{ij}^3, z_{ij}^4]; t_{\tilde{z}_{ij}}, i_{\tilde{z}_{ij}}, f_{\tilde{z}_{ij}})$  in the matrix  $Z$  are considered as:

i. For benefit types attribute:

$$\tilde{z}_{ij} = \left( \left[ \frac{c_{ij}^1}{q_j^+}, \frac{c_{ij}^2}{q_j^+}, \frac{c_{ij}^3}{q_j^+}, \frac{c_{ij}^4}{q_j^+} \right]; t_{\tilde{z}_{ij}}, i_{\tilde{z}_{ij}}, f_{\tilde{z}_{ij}} \right) \tag{10}$$

ii. For cost type attribute:

$$\tilde{z}_{ij} = \left( \left[ \frac{q_j^-}{c_{ij}^4}, \frac{q_j^-}{c_{ij}^3}, \frac{q_j^-}{c_{ij}^2}, \frac{q_j^-}{c_{ij}^1} \right]; t_{\tilde{z}_{ij}}, i_{\tilde{z}_{ij}}, f_{\tilde{z}_{ij}} \right) \tag{11}$$

Where  $q_j^+ = \max\{c_{ij}^4 | i = 1, 2, \dots, p\}$  and  $q_j^- = \min\{c_{ij}^1 | i = 1, 2, \dots, p\}$  for  $j=1, 2, \dots, q$ .

Then we obtain the following standardized decision matrix:

$$Z = (\tilde{z}_{ij})_{m \times n} = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1n} \\ z_{21} & z_{22} & \dots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1} & z_{m2} & \dots & z_{mn} \end{pmatrix} \tag{12}$$

**4. GRA strategy for solving MCDM problem under trapezoidal neutrosophic number environment:**

Assume that  $\tilde{B} = \{\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_p\}$  be the p alternatives and  $R' = \{R'_1, R'_2, \dots, R'_q\}$  be the set of q attributes. Also assume that the rating values each of the alternative corresponding to each of the attribute are expressed in the form of  $m_{ij} = ([b_1, b_2, b_3, b_4]; t_{m_{ij}}, i_{m_{ij}}, f_{m_{ij}})$ . Using the following steps ( see figure1), we describe GRA strategy for

TrNN by considering the weight vector  $\tilde{w} = \{\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_q\}^T$  of attributes where  $\tilde{w}_b \in [0, 1]$  and  $\sum_{b=1}^q \tilde{w}_b = 1$ .

Step-1: First we defined decision matrix as follows:

$$Q = \begin{pmatrix} & R'_1 & R'_2 & \dots & R'_q \\ \tilde{B}_1 & m_{11} & m_{12} & \dots & m_{1q} \\ \tilde{B}_2 & m_{21} & m_{22} & \dots & m_{2q} \\ \dots & \dots & \dots & \dots & \dots \\ \tilde{B}_p & m_{p1} & m_{p2} & \dots & m_{pq} \end{pmatrix} \tag{13}$$

Step-2: Generally decision making problem consists of cost and benefit attributes. So we need to standardize the decision matrix. To standardize in benefit criteria we use the equation (10) and for cost criteria we use (11). After standardizing, the decision matrix reduces to

$$Q' = \begin{pmatrix} R'_1 & R'_2 & \dots & R'_q \\ \tilde{B}_1 & m'_{11} & m'_{12} & \dots & m'_{1q} \\ \tilde{B}_2 & m'_{21} & m'_{22} & \dots & m'_{2q} \\ \dots & \dots & \dots & \dots & \dots \\ \tilde{B}_p & m'_{p1} & m'_{p2} & \dots & m'_{pq} \end{pmatrix} \quad (14)$$

Step-3: In this step we calculate score value and accuracy value using equation (8) and (9).

Step-4: Here, we describe the positive ideal solution (PIS) and negative ideal solution (NIS) for TrNN.

$$T^+ = ([b_1^+, b_2^+, b_3^+, b_4^+]; \max t_{m_{ij}}, \min i_{m_{ij}}, \min f_{m_{ij}}) \quad (15)$$

$$T^- = ([b_1^-, b_2^-, b_3^-, b_4^-]; \min t_{m_{ij}}, \max i_{m_{ij}}, \max f_{m_{ij}}) \quad (16)$$

Step-5: Determine the grey relation coefficient of each alternative from  $T^+$  and  $T^-$  by the following equations:

$$\chi_{bc}^+ = \frac{\min_{1 \leq b \leq p} \min_{1 \leq c \leq p} D(x_{bc}, T_c^+) + \rho \max_{1 \leq b \leq p} \max_{1 \leq c \leq p} D(x_{bc}, T_c^+)}{D(x_{bc}, T_c^+) + \rho \max_{1 \leq b \leq p} \max_{1 \leq c \leq p} D(x_{bc}, T_c^+)} \quad (17)$$

$$\chi_{bc}^- = \frac{\min_{1 \leq b \leq p} \min_{1 \leq c \leq p} D(x_{bc}, T_c^-) + \rho \max_{1 \leq b \leq p} \max_{1 \leq c \leq p} D(x_{bc}, T_c^-)}{D(x_{bc}, T_c^-) + \rho \max_{1 \leq b \leq p} \max_{1 \leq c \leq p} D(x_{bc}, T_c^-)} \quad (18)$$

where the identification coefficient is considered as  $\rho = 0.5$ .

Step-6: In this step, we employ the pre-determined weight vector of attributes as  $w = \{w_1, w_2, \dots, w_q\}$  and

$$\sum_{c=1}^q w_c = 1$$

Step-7: In this step, we determine the degree of grey relation coefficient of each alternative  $\tilde{B}_b$  ( $b=1,2,\dots, p$ ) from  $\chi_{bc}^+$  and  $\chi_{bc}^-$  by the following equations:

$$\mathcal{G}_b^+ = \sum_{c=1}^q w_c \chi_{bc}^+ \quad (19)$$

$$\mathcal{G}_b^- = \sum_{c=1}^q w_c \chi_{bc}^- \quad (20)$$

Step-8: Evaluating the relative closeness co-efficient  $\mathcal{G}_b$  for each alternative  $\tilde{B}_b$  ( $b=1,2,\dots, p$ ) w.r.t. the positive ideal solution  $T^+$  as

$$\mathcal{G}_b = \frac{\mathcal{G}_b^+}{\mathcal{G}_b^+ + \mathcal{G}_b^-} \quad (21)$$

for  $b=1,2, \dots, p$ .

Step-9: Ranking the alternative according to the relative closeness coefficient  $\mathcal{G}_b$  ( $b=1,2,\dots, p$ )

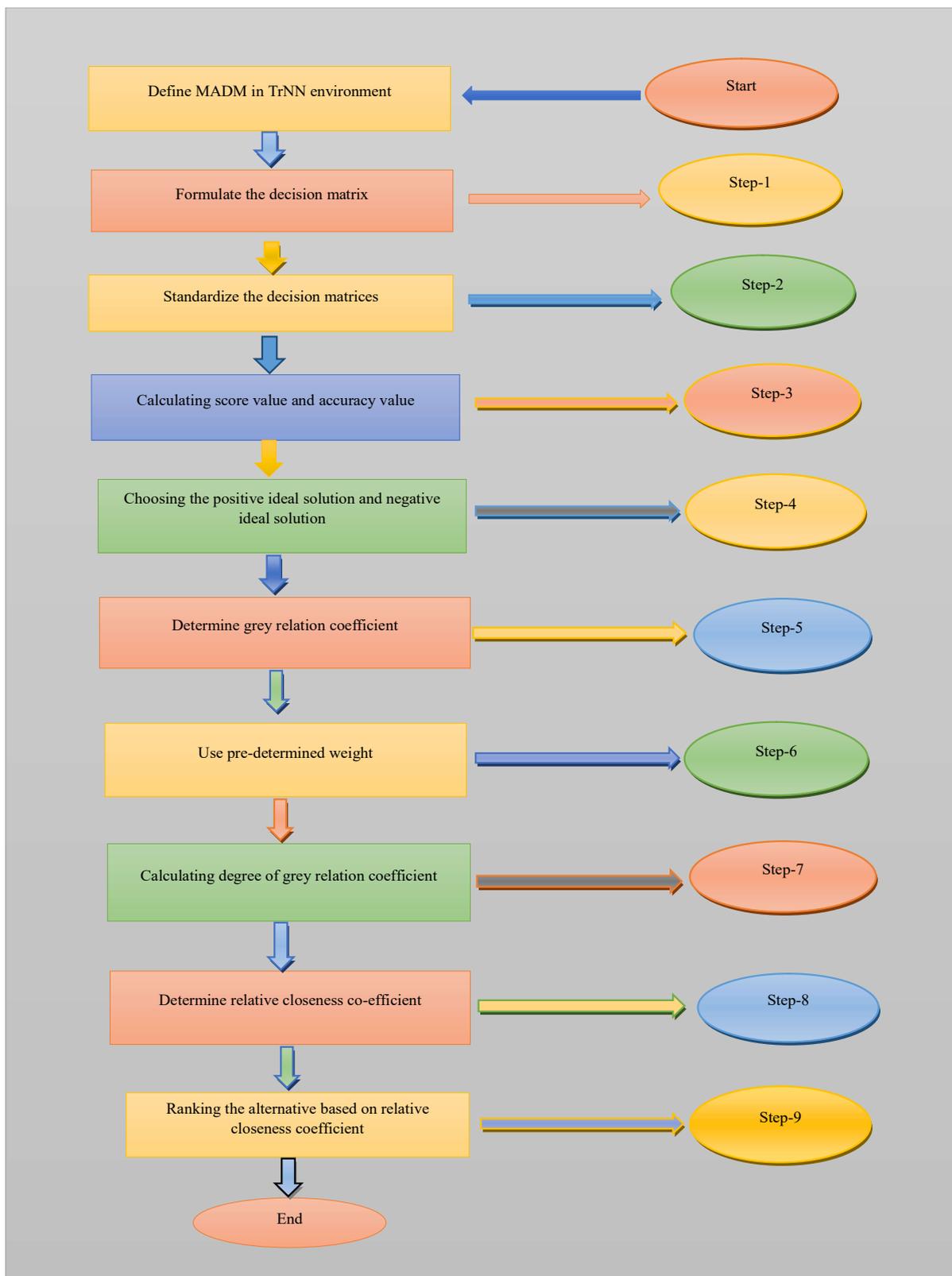


Fig 1: GRA strategy based on MADM in trapezoidal neutrosophic number.

### 5. Numerical problem

Here, we describe trapezoidal neutrosophic number MADM to illustrate the applicability and effectiveness of the proposed strategy. We solve a decision making problem adapted from [65] which is stated as follows. A customer intends to buy a tablet from the set of elementarily chosen four tablets  $\tilde{B}_i$  ( $i=1, 2, 3, 4$ ). The customer considers four attributes which include: features ( $R'_1$ ), hardware specification ( $R'_2$ ), affordable price ( $R'_3$ ) and customer care( $R'_4$ ).

Step-1: Here we defined decision matrix

$$Q = \begin{pmatrix} & R'_1 & R'_2 & R'_3 & R'_4 \\ \tilde{B}_1 & ([0.5,0.6,0.7,0.8];0.65,0.25,0.20) & ([0.7,0.8,0.8,0.9];0.60,0.35,0.30) & ([0.4,0.5,0.5,0.6];0.48,0.26,0.20) & ([0.4,0.5,0.6,0.7];0.55,0.42,0.20) \\ \tilde{B}_2 & ([0.7,0.8,0.8,0.9];0.80,0.20,0.15) & ([0.6,0.7,0.8,0.9];0.50,0.40,0.35) & ([0.3,0.4,0.5,0.6];0.50,0.45,0.35) & ([0.6,0.6,0.7,0.8];0.70,0.40,0.15) \\ \tilde{B}_3 & ([0.4,0.5,0.6,0.7];0.50,0.40,0.30) & ([0.7,0.8,0.9,0.9];0.85,0.30,0.25) & ([0.6,0.6,0.7,0.7];0.65,0.22,0.18) & ([0.6,0.6,0.7,0.7];0.85,0.25,0.15) \\ \tilde{B}_4 & ([0.6,0.7,0.7,0.8];0.70,0.35,0.25) & ([0.5,0.6,0.6,0.7];0.65,0.35,0.30) & ([0.5,0.6,0.6,0.7];0.60,0.40,0.30) & ([0.5,0.6,0.7,0.7];0.70,0.30,0.20) \end{pmatrix}$$

Step-2: The selected four attribute are benefit type attribute. Thus we can standardized the decision matrix  $(Q_{ij})_{4 \times 4}$  to  $(Q'_{ij})_{4 \times 4}$  by using equation (11). The standardized decision matrix represented as follow.

$$Q' = \begin{pmatrix} & R'_1 & R'_2 & R'_3 & R'_4 \\ \tilde{B}_1 & ([0.62,0.75,0.88,1];0.65,0.25,0.20) & ([0.78,0.89,0.89,1];0.60,0.35,0.30) & ([0.67,0.83,0.83,1];0.48,0.26,0.20) & ([0.57,0.71,0.85,1];0.55,0.42,0.20) \\ \tilde{B}_2 & ([0.78,0.89,0.89,1];0.80,0.20,0.15) & ([0.67,0.78,0.89,1];0.50,0.40,0.35) & ([0.50,0.67,0.83,1];0.50,0.45,0.35) & ([0.75,0.75,0.86,1];0.70,0.40,0.15) \\ \tilde{B}_3 & ([0.57,0.71,0.86,1];0.50,0.40,0.30) & ([0.78,0.89,1,1];0.85,0.30,0.25) & ([0.86,0.86,1,1];0.65,0.22,0.18) & ([0.86,0.86,1,1];0.85,0.25,0.15) \\ \tilde{B}_4 & ([0.75,0.88,0.88,1];0.70,0.35,0.25) & ([0.71,0.86,0.86,1];0.65,0.35,0.30) & ([0.71,0.86,0.86,1];0.60,0.40,0.30) & ([0.71,0.85,1,1];0.70,0.30,0.20) \end{pmatrix}$$

Step-3: Calculating score value and accuracy value by using equation (7) and (8)

$$Sc(Q') = \begin{pmatrix} & R'_1 & R'_2 & R'_3 & R'_4 \\ \tilde{B}_1 & 0.60 & 0.58 & 0.56 & 0.50 \\ \tilde{B}_2 & 0.73 & 0.49 & 0.42 & 0.60 \\ \tilde{B}_3 & 0.47 & 0.70 & 0.69 & 0.75 \\ \tilde{B}_4 & 0.61 & 0.53 & 0.53 & 0.65 \end{pmatrix}$$

$$Ac(Q') = \begin{pmatrix} & R'_1 & R'_2 & R'_3 & R'_4 \\ \tilde{B}_1 & .21 & 0.08 & 0.12 & 0.22 \\ \tilde{B}_2 & .097 & 0.158 & 0.23 & 0.15 \\ \tilde{B}_3 & 0.12 & 0.14 & 0.11 & 0.13 \\ \tilde{B}_4 & 0.10 & 0.11 & 0.11 & 0.18 \end{pmatrix}$$

Step-4: Identifying positive ideal solution (PIS)  $T^+$  and negative ideal solution (NIS)  $T^-$  using equation (15) and (16)

$$T^+ = \begin{pmatrix} & R'_1 & R'_2 & R'_3 & R'_4 \\ ([0.78,0.89,0.89,1];0.80,0.20,0.15) & ([0.78,0.89,1,1];0.85,0.30,0.25) & ([0.86,0.86,1,1];0.65,0.22,0.18) & ([0.86,0.86,1,1];0.85,0.25,0.15) \end{pmatrix}$$

$$T^- = \begin{pmatrix} & R'_1 & R'_2 & R'_3 & R'_4 \\ ([0.57,0.71,0.86,1];0.50,0.40,0.30) & ([0.67,0.78,0.89,1];0.50,0.40,0.35) & ([0.50,0.67,0.83,1];0.50,0.45,0.35) & ([0.57,0.71,0.85,1];0.55,0.42,0.20) \end{pmatrix}$$

Step-5: Using (17) and (18) we calculate grey relation coefficient:

$$\chi^+ = \begin{pmatrix} 0.33 & 0.56 & 0.52 & 0.37 \\ 1 & 0.41 & 0.36 & 0.52 \\ 0.58 & 1 & 1 & 1 \\ 0.58 & 0.54 & 0.54 & 0.58 \end{pmatrix}$$

$$\chi^- = \begin{pmatrix} 0.49 & 0.6 & 0.49 & 1 \\ 0.35 & 1 & 1 & 0.55 \\ 1 & 0.38 & 0.33 & 0.34 \\ 0.49 & 0.55 & 0.52 & 0.47 \end{pmatrix}$$

Step-6: In this step, we assume the weights are

$$w_1 = 0.22, w_2 = 0.22, w_3 = 0.33, w_4 = 0.22$$

Step-7: Calculate the degree of grey relation coefficient by equation (19) and (20)

$$\mathcal{G}_1^+ = 0.4488, \mathcal{G}_2^+ = 0.5434, \mathcal{G}_3^+ = 0.8976, \mathcal{G}_4^+ = 0.536$$

$$\mathcal{G}_1^- = 0.6215, \mathcal{G}_2^- = 0.748, \mathcal{G}_3^- = 0.4873, \mathcal{G}_4^- = 0.5038$$

Step-8: Using equation (21) we evaluate the relative closeness co-efficient

$$\mathcal{G}_1 = 0.6215, \mathcal{G}_2 = 0.4208, \mathcal{G}_3 = 0.6481, \mathcal{G}_4 = 0.5155$$

Step-9: The ranking of the alternative based on relative closeness co-efficient is

$$\tilde{B}_3 > \tilde{B}_4 > \tilde{B}_2 > \tilde{B}_1$$

We see that  $\tilde{B}_3$  has highest value. Therefore,  $\tilde{B}_3$  is the best solution.

## 6. Conclusion

In this paper we have investigated MADM strategy in single valued trapezoidal neutrosophic number environment. We have developed an extended GRA based MADM strategy for MADM problem. A numerical example has been provided to show the applicability and effectiveness of the proposed strategy. In future, the developed strategy can be applied to real-world problems such as teacher selection [71], brick selection [79], pattern recognition [80], clustering analysis [81], etc.

Reference:

- [1] F. Smarandache. A unifying field of logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth 1998.
- [2] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multi-space and Multi-structure, 4 (2010), 410-413.
- [3] P. Biswas, S. Pramanik, and B. C. Giri. A new methodology for neutrosophic multi-attribute decision making with unknown weight information. Neutrosophic Sets and Systems, 3 (2014), 42-52.
- [4] K. Mondal, and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment. Neutrosophic Sets and Systems, 6(2014), 28-34.
- [5] K. Mondal, and S. Pramanik. A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. Neutrosophic Sets and Systems, 5(2014), 21-26. doi.org/10.5281/zenodo.571272.
- [6] P. Biswas, S. Pramanik, and B.C. Giri. TOPSIS method for multi-attribute group decision making under single-valued neutrosophic environment. Neural Computing and Applications, 27(3)(2015), 727-737. DOI: 10.1007/s00521-015-1891-2.
- [7] K. Mondal, and S. Pramanik. Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. Neutrosophic Sets and Systems, 9(2015), 64-71.
- [8] K. Mondal, and S. Pramanik. Neutrosophic tangent similarity measure and its application to multiple attribute decision making. Neutrosophic Sets and Systems, 9(2015), 80-87. doi.org/10.5281/zenodo.571578
- [9] S. Pramanik, S. Dalapati, and T. K. Roy. Logistics center location selection approach based on neutrosophic multi-criteria decision making. In F. Smarandache, & S. Pramanik (Eds.), New trends in neutrosophic theory and applications. Pons Editions, Brussels, 2016, 161-174.
- [10] F. Smarandache, and S. Pramanik (Eds). New trends in neutrosophic theory and applications, Brussels: Pons Editions, 2016.
- [11] S. Pramanik, P. Biswas, and B.C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, 28 (5)(2017), 1163-1176
- [12] K. Mondal, S. Pramanik, and F. Smarandache. Role of neutrosophic logic in data mining. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications . Pons Editions, Brussels, 2016, 15-23.
- [13] S. Pramanik, R. Mallick, and A. Dasgupta. Contributions of selected Indian researchers to multi-attribute decision making in neutrosophic environment. Neutrosophic Sets and Systems, 20(2018), 108-131.
- [14] K. Mondal, S. Pramanik, and B. C. Giri. Single valued neutrosophic hyperbolic sine similarity measure based MADM strategy. Neutrosophic Sets and Systems, 20 (2018), 3-11. <http://doi.org/10.5281/zenodo.1235383>.

- [15] K. Mondal, S. Pramanik, and B. C. Giri. Hybrid binary logarithm similarity measure for MAGDM problems under SVN assessments. *Neutrosophic Sets and Systems*, 20 (2018), 12-25.  
<http://doi.org/10.5281/zenodo.1235365>.
- [16] S. Pramanik, P. P Dey, and B. C. Giri. An extended grey relational analysis based interval neutrosophic multi-attribute decision making for weaver selection. *Journal of New Theory* 9(2015), 82-93.
- [17] P. P. Dey, S. Pramanik, and B. C. Giri. Extended projection based models for solving multiple attribute decision making problems with interval valued neutrosophic information. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications* . Pons Editions, Brussels, 2016, 127-140.
- [18] S. Dalapati, S. Pramanik, S. Alam, F. Smarandache, and T. K. Roy. IN-cross entropy based MAGDM strategy under interval neutrosophic set environment. *Neutrosophic Sets and Systems*, 18( 2017), 43-57.  
<http://doi.org/10.5281/zenodo.1175162>.
- [19] K. Mondal, S. Pramanik, and B. C. Giri. Interval neutrosophic tangent similarity measure based MADM strategy and its application to MADM problems. *Neutrosophic Sets and Systems*, 19 (2018), 47-56.  
<http://doi.org/10.5281/zenodo.1235201>
- [20] S. Pramanik, P. P Dey, and B. C. Giri. TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 10(2015), 88-95.
- [21] S. Pramanik, P. P Dey, and B. C. Giri. Neutrosophic soft multi-attribute decision making based on grey relational projection method. *Neutrosophic Sets and Systems*, 11(2016), 98-106.
- [22] S. Pramanik, P. P Dey, and B. C. Giri. Neutrosophic soft multi-attribute group decision making based on grey relational analysis method. *Journal of New Results in Science*, 10(2016), 25-37.
- [23] S. Pramanik, and S. Dalapati. GRA based multi criteria decision making in generalized neutrosophic soft set environment. *Global Journal of Engineering Science and Research Management*, 3(5)(2016), 153-169.
- [24] K. Mondal, and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, 7(2015), 8-17.
- [25] K. Mondal, and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. *Neutrosophic Sets and Systems* 8(2015), 14-21.
- [26] K. Mondal, and S. Pramanik. Tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. *Critical Review*, 11(2015), 26-40.
- [27] K. Mondal, S. Pramanik, and F. Smarandache. Rough neutrosophic TOPSIS for multi-attribute group decision making. *Neutrosophic Sets and Systems*, 13(2016), 105-117.
- [28] K. Mondal, S. Pramanik, and F. Smarandache. Several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications* . Pons Editions, Brussels, 2016, 93-103.
- [29] K. Mondal, S. Pramanik, and F. Smarandache. Multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. *Neutrosophic Sets and Systems*, 13(2016), 3-17.
- [30] K. Mondal, S. Pramanik, and F. Smarandache. Rough neutrosophic hyper-complex set and its application to multiattribute decision making. *Critical Review* 13(2016), 111-126.
- [31] S. Pramanik, R. Roy, T. K. Roy, and F. Smarandache. Multi criteria decision making using correlation coefficient under rough neutrosophic environment. *Neutrosophic Sets and Systems*, 17(2017), 29-36.
- [32] S. Pramanik, R. Roy, and T. K. Roy. Multi criteria decision making based on projection and bidirectional projection measures of rough neutrosophic sets. In F. Smarandache, & S. Pramanik (Eds., vol.2), *New trends in neutrosophic theory and applications* . Pons Editions, Brussels, 2018, 175-187.
- [33] K. Mondal, and S. Pramanik. Decision making based on some similarity measures under interval rough neutrosophic environment. *Neutrosophic Sets and Systems*, 10(2015), 46-57.  
[doi.org/10.5281/zenodo.571358](http://doi.org/10.5281/zenodo.571358).
- [34] S. Pramanik, R. Roy, T. K. Roy and F. Smarandache. Multi attribute decision making strategy on projection and bidirectional projection measures of interval rough neutrosophic sets. *Neutrosophic Sets and Systems*, 19(2018),101-109.
- [35] S. Pramanik, R. Roy, T. K. Roy, and F. Smarandache. Multi-attribute decision making based on several trigonometric hamming similarity measures under interval rough neutrosophic environment. *Neutrosophic Sets and Systems*, 19(2018), 110-118.
- [36] K. Mondal, and S. Pramanik. Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. *Journal of New Theory*, 8(2015), 41-50.

- [37] K. Mondal, and S. Pramanik. Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making. *Global Journal of Advanced Research*, 2(2)(2015), 486-494.
- [38] S. Pramanik, D. Banerjee, and B. C. Giri. Multi – criteria group decision making model in neutrosophic refined set and its application. *Global Journal of Engineering Science and Research Management*, 3(6)(2016), 12-18.
- [39] S. Pramanik, D. Banerjee, and B. C. Giri. TOPSIS approach for multi attribute group decision making in refined neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds.), *New trends in neutrosophic theory and applications* . Pons Editions, Brussels, 2016, 79-91.
- [40] S. Pramanik, P. P. Dey, and B. C. Giri. Hybrid vector similarity measure of single valued refined neutrosophic sets to multi-attribute decision making problems. . In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2018, 156-174.
- [41] K. Mondal, S. Pramanik, and B. C. Giri. Multi-criteria group decision making based on linguistic refined neutrosophic strategy. In F. Smarandache, & S. Pramanik (Eds., vol.2), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2018, 125-139.
- [42] P. Biswas, S. Pramanik, and B. C. Giri. Some distance measures of single valued neutrosophic hesitant fuzzy sets and their applications to multiple attribute decision making. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2016, 55-63.
- [43] P. Biswas, S. Pramanik, and B. C. Giri. GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2016, 55-63.
- [44] P. P. Dey, S. Pramanik, and B. C. Giri. TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2016, 65-77.
- [45] S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache. Bipolar neutrosophic projection based models for solving multi-attribute decision-making problems. *Neutrosophic Sets and Systems*, 15(2017), 70-79. doi.org/10.5281/zenodo.570936.
- [46] S. Pramanik, S. Dalapati, S. Alam, and T. K. Roy. VIKOR based MAGDM strategy under bipolar neutrosophic set environment .*Neutrosophic Sets and Systems*, 19(2018),57-69.
- [47] S. Pramanik, S. Dalapati, S. Alam and T. K. Roy. TODIM method for group decision making under bipolar neutrosophic set environment. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2018, 140-155.
- [48] S. Pramanik, P. P. Dey, F. Smarandache and J. Ye. Cross entropy measures of bipolar and interval bipolar neutrosophic sets and their application for multi-attribute decision-making. *Axioms*, 7( 21)(2018),1-25; doi:10.3390/axioms7020021.
- [49] S. Pramanik, P. P. Dey, and F. Smarandache. Correlation coefficient measures of interval bipolar neutrosophic sets for solving multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 19(2018),70-79.
- [50] S. Pramanik, S. Dalapati, S. Alam, T. K. Roy, and F. Smarandache. Neutrosophic cubic MCGDM method based on similarity measure. *Neutrosophic Sets and Systems*, 16 (2017), 44-56. doi.org/10.5281/zenodo.831934.
- [51] S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache. An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. *Neutrosophic Sets and Systems*, 17(2017), 20-28.
- [52] S. Pramanik, S. Dalapati, S. Alam, and T. K. Roy. NC-VIKOR based MAGDM strategy under neutrosophic cubic set environment. *Neutrosophic Sets and Systems*, 20(2018), 95-108.
- [53] S. Dalapati, and S. Pramanik. A revisit to NC-VIKOR based MAGDM strategy in neutrosophic cubic set environment. *Neutrosophic Sets and Systems*, 21( 2018), 131-141. https://doi.org/10.5281/zenodo.1408665
- [54] P. Biswas, S. Pramanik, and B. C. Giri. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. *Neutrosophic Sets and Systems*, 12(2016), 20-40.
- [55] P. Biswas, S. Pramanik, and B. C. Giri. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. *Neutrosophic Sets and Systems*, 8(2015), 46-56. doi.org/10.5281/zenodo.571274.
- [56] P. Biswas, S. Pramanik, and B. C. Giri. Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. In F. Smarandache, & S. Pramanik (Eds., vol.2), *New trends in neutrosophic theory and applications*. Pons Editions, Brussels, 2018, 103-124.
- [57] S. Pramanik, and R. Mallick. VIKOR based MAGDM strategy with trapezoidal neutrosophic number.

Neutrosophic Sets and Systems, 22(2018), 118-130.

- [58] F. Smarandache, and S. Pramanik (Eds). New trends in neutrosophic theory and applications, Vol.2. Brussels: Pons Editions, 2018.
- [59] S. Pramanik, and T. K. Roy. Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. *Neutrosophic Sets and Systems*, 2(2014), 82-101.
- [60] G. Xu, S. Wang, T. Yang, and W. Jiang. A neutrosophic approach based on TOPSIS method to image segmentation. *International Journal Of Computers Communications & Control*, 13(6)(2018), 1047-1061.
- [61] K. Mondal, and S. Pramanik. Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. *Global Journal of Advanced Research*, 2(1)(2015), 212-220.
- [62] S. Pramanik, and S. N. Chackrabarti. A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. *International Journal of Innovative Research in Science, Engineering and Technology*, 2(11)(2013), 6387-6394.
- [63] J. Ye. Some weighted aggregation operator of trapezoidal neutrosophic number and their multiple attribute decision making method. *Informatica*, 28 (2) (2017), 387-402.
- [64] I. Deli, and Y. Subas. A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *International Journal of Machine Learning and Cybernetics*, (2016). doi:10.1007/s13042016-0505-3.
- [65] P. Biswas, S. Pramanik, and B. C. Giri. Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. *Neutrosophic Sets and Systems* 12 (2016), 127-138.
- [66] P. Biswas, S. Pramanik, and B. C. Giri. TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. *Neutrosophic Sets and Systems*, 19(2018), 29-39.
- [67] P. Biswas, S. Pramanik, and B. C. Giri. Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. *Neutrosophic Sets and Systems*, 19( 2018), 40-46.  
<http://doi.org/10.5281/zenodo.1235165>
- [68] J. L. Deng. Introduction to grey system theory. *The Journal of Grey System*, 1(1) (1989), 1–24.
- [69] J. L. Deng. The primary methods of grey system theory. Huazhong University of Science and Technology Press, Wuhan, (2005).
- [70] R. V. Rao, and D. Singh. An improved grey relational analysis as a decision making method for manufacturing situations. *International Journal of Decision Science, Risk and Management*, 2(2010), 1–23
- [71] S. Pramanik, and D. Mukhopadhyaya. Grey relational analysis based intuitionistic fuzzy multi criteria group decision making approach for teacher selection in higher education. *International Journal of Computer Applications*, 34 (10) (2011), 21 – 29.
- [72] G. W. Wei. Grey relational analysis method for intuitionistic fuzzy multiple attribute decision making. *Expert Systems with Applications*, 38(2011), 11671-11677.
- [73] P. Biswas, S. Pramanik, and B.C. Giri. Entropy based grey relational analysis method for multi-attribute decision – making under single valued neutrosophic assessments. *Neutrosophic Sets and Systems*, 2 (2014), 102 – 110.
- [74] P.P. Dey, S. Pramanik, and B.C. Giri. Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi institution. *Journal of Applied and Quantitative Methods*, 10(4) (2015), 1-14.
- [75] S. Pramanik, and K. Mondal. Interval neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, 9(2015), 13-22.
- [76] P.P. Dey, S. Pramanik, and B.C. Giri. An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting. *Neutrosophic Sets and Systems*, 11 (2016), 21-30.
- [77] D. Banerjee, B. C. Giri, S. Pramanik, and F. Smarandache. GRA for multi attribute decision making in neutrosophic cubic set environment. *Neutrosophic Sets & Systems*, 15(2017), 60-69.
- [78] S. Pramanik, and R. Mallick. TODIM strategy for multi attribute group decision making in trapezoidal neutrosophic number environment. *Complex and Intelligent Systems*. Submitted.
- [79] K. Mondal, and S. Pramanik. Intuitionistic fuzzy multicriteria group decision making approach to quality-brick selection problem. *Journal of Applied Quantitative Methods*, 9(2)(2014), 35-50.
- [80] M. Alia, I. Delib, and F. Smarandache. The theory of neutrosophic cubic sets and their applications in pattern recognition. *Journal of Intelligent & Fuzzy Systems*, 16 (2016), 1064-1246.
- [81] Li. Qiaoyan, Ma. Yingcang, F. Smarandache, and Z. Shuangwu. Single-valued neutrosophic clustering algorithm based on Tsallis entropy maximization. *Axioms*, 7(57)(2018), 1-12.