Una integral para Pi y algunos fractales

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Resumen

Esta nota muestra una integral para la constante π y algunos fractales.

La constante π se define por:

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.141592... \tag{1}$$

Una representación integral para π es:

$$\pi = 288\sqrt{6} \int_0^{1/\sqrt{3}} \frac{x^4\sqrt{1-2x^2-3x^4}}{\sqrt{1-x^2+\sqrt{1-2x^2-3x^4}}+\sqrt{1-x^2-\sqrt{1-2x^2-3x^4}}}\,dx \tag{2}$$

Sea $f(z), z \in \mathbb{C}$ definida por:

$$f(z) = \frac{z^4\sqrt{1-2z^2-3z^4}}{\sqrt{1-z^2+\sqrt{1-2z^2-3z^4}}+\sqrt{1-z^2-\sqrt{1-2z^2-3z^4}}} \tag{3}$$

El fractal (Newton) para $f(z)$ $, z \in (-2-2i, 2+2i)$ es:

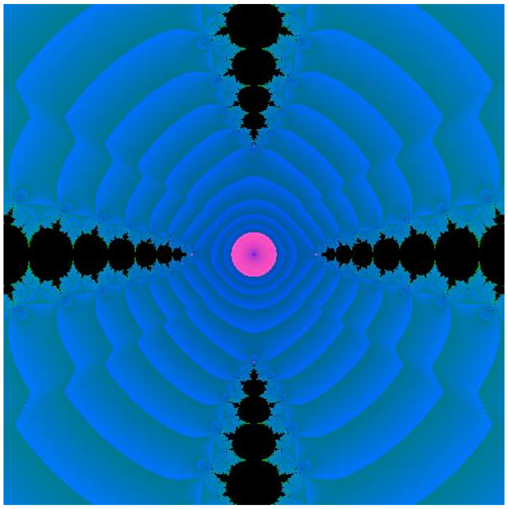

Fig. 1

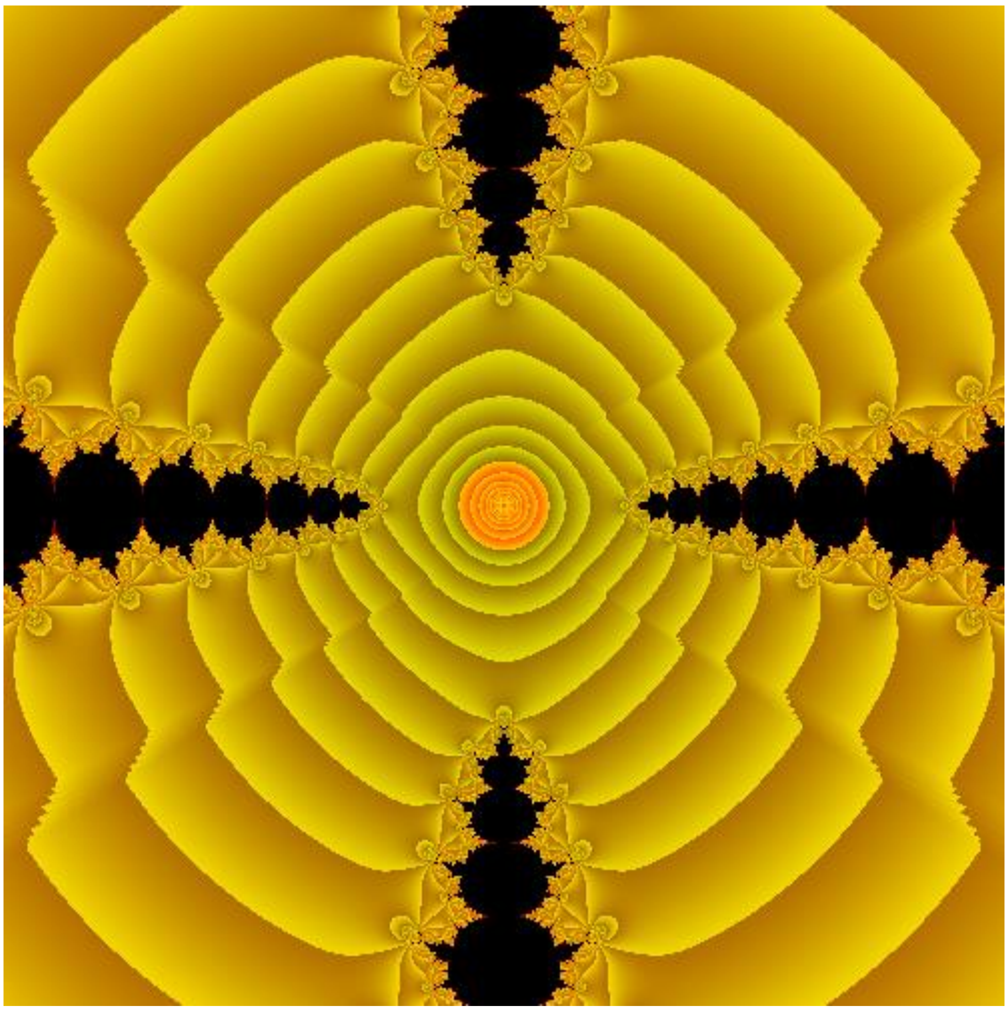

Fig. 2

Con un cambio de variable conveniente la integral (2) se transforma en:

$$\frac{\pi}{16\sqrt{2}\,n}=\int_0^1 \frac{x^{(5n-2)/2}\sqrt{3-2x^n-x^{2n}}}{\sqrt{3-x^n+\sqrt{9-6x^n-3x^{2n}}}+\sqrt{3-x^n-\sqrt{9-6x^n-3x^{2n}}}}dx \quad ,n>0 \qquad (4)$$

Sea $R(n,z)\,,z\in\mathbb{C},n>0$ definida por:

$$R(n,z)=\frac{z^{(5n-2)/2}\sqrt{3-2z^n-z^{2n}}}{\sqrt{3-z^n+\sqrt{9-6z^n-3z^{2n}}}+\sqrt{3-z^n-\sqrt{9-6z^n-3z^{2n}}}} \qquad (5)$$

El fractal (Newton) para $R(4/5,z)\,,z\in(-8-8i,8+8i)$ es:

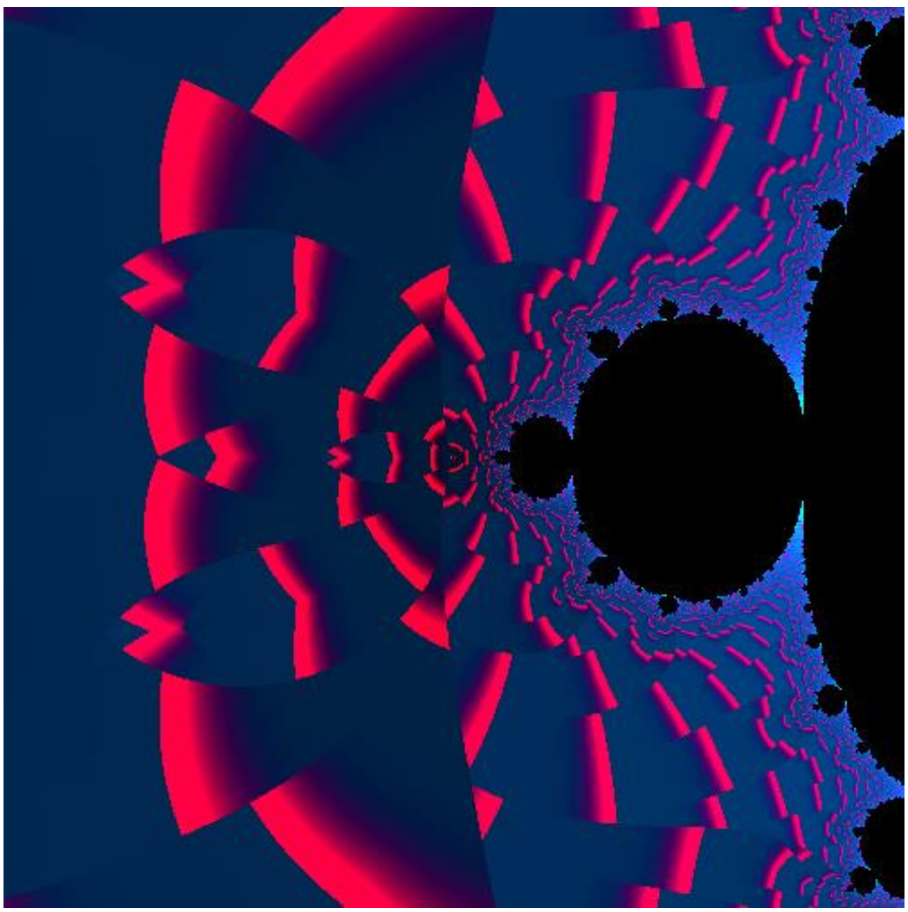

Fig. 3

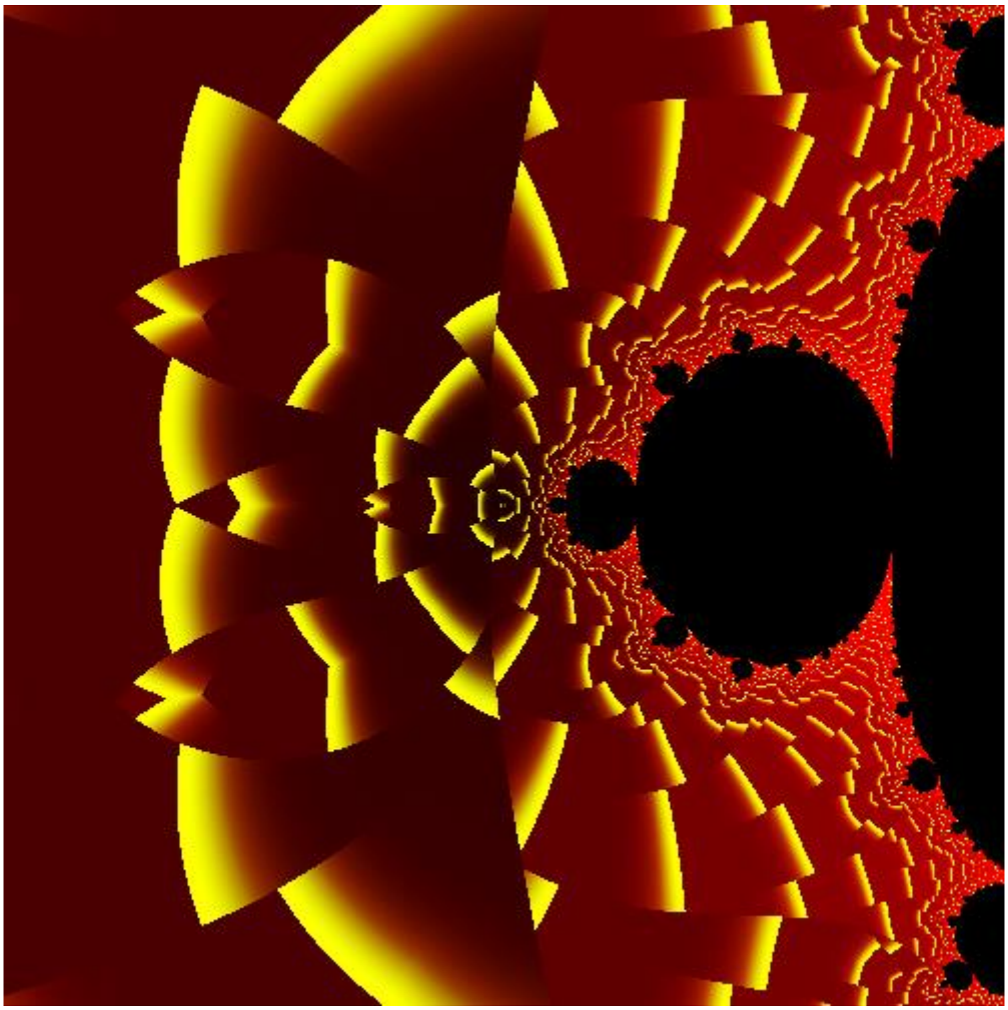

Fig. 4

El fractal (Newton) para $R(6/5,z)$, $z \in (-8-8i, 8+8i)$ es:

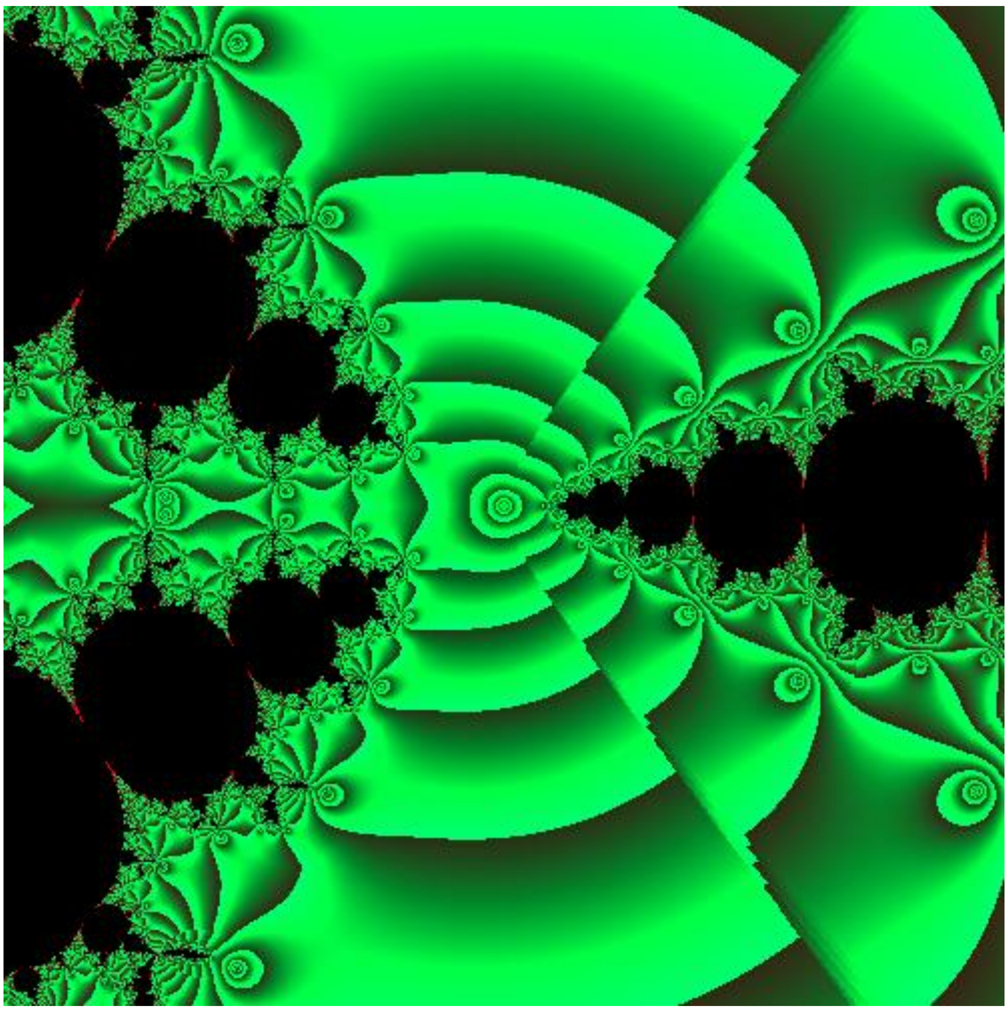

Fig. 5

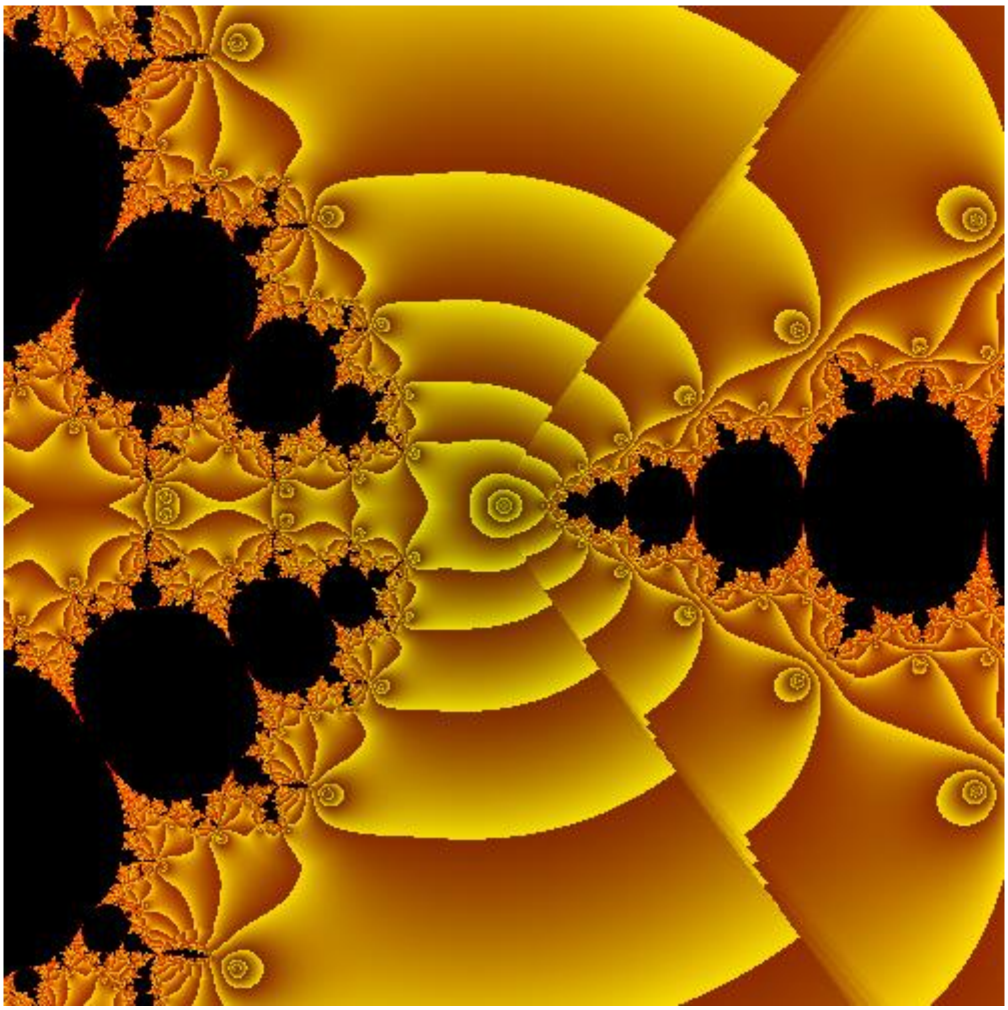

Fig. 6

El fractal (Newton) para $R(8/5, z), z \in (-8-8i, 8+8i)$ es:

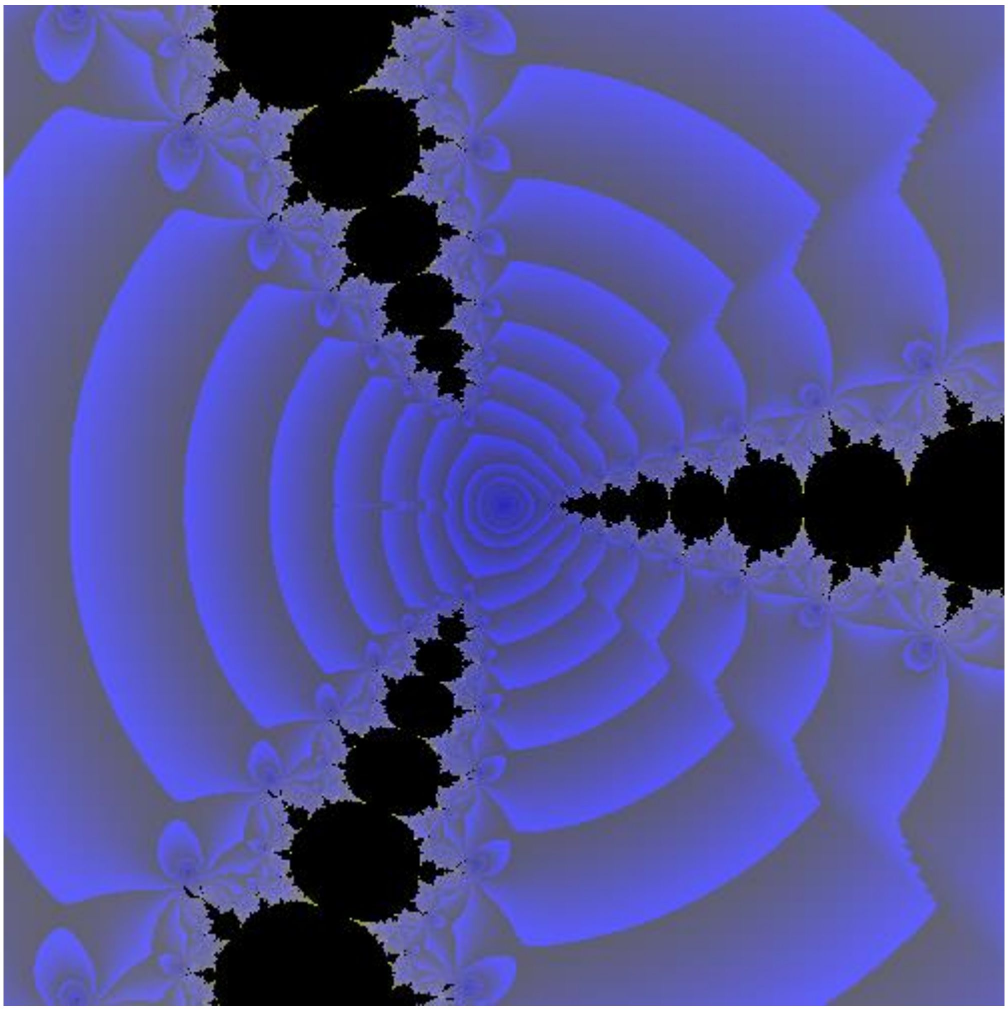

Fig. 7

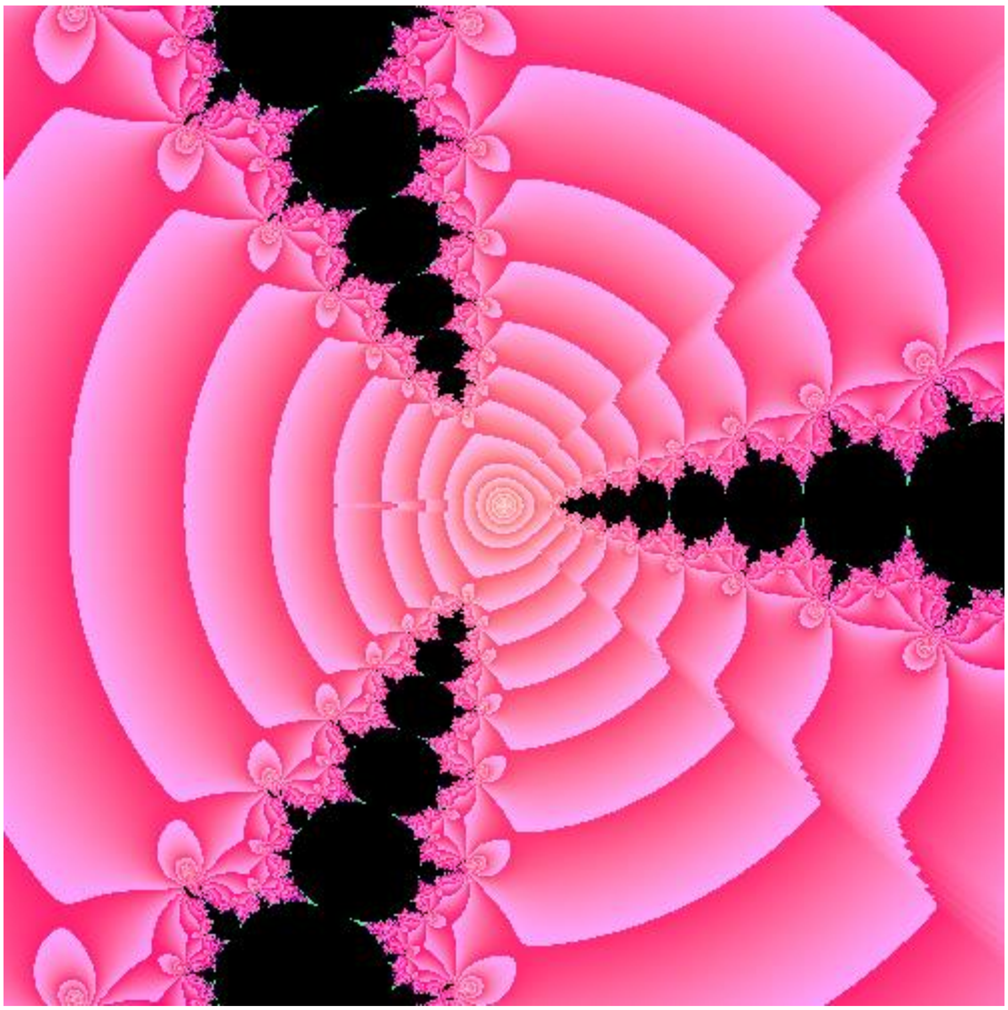

Fig. 8

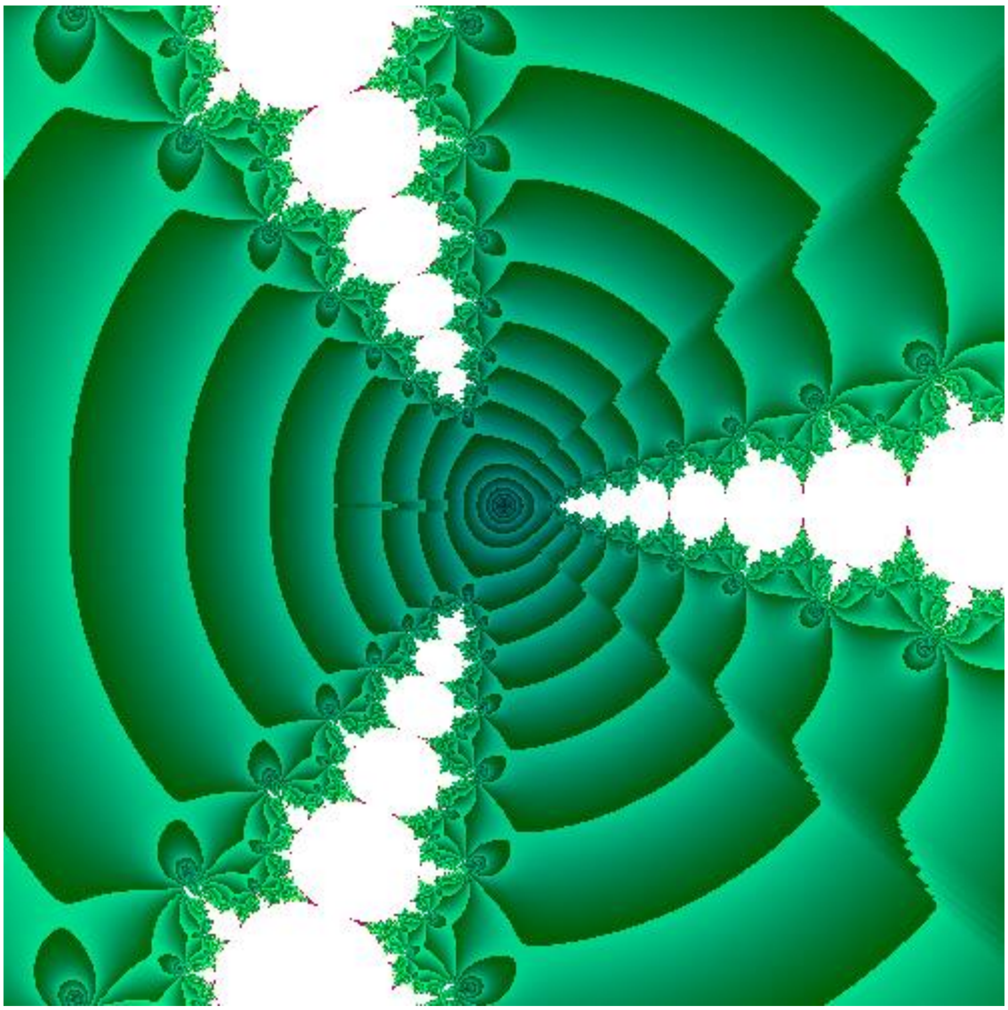

Fig. 9

El fractal (Newton) para $R(1,z)$, $z \in (-8-8i, 8+8i)$ es:

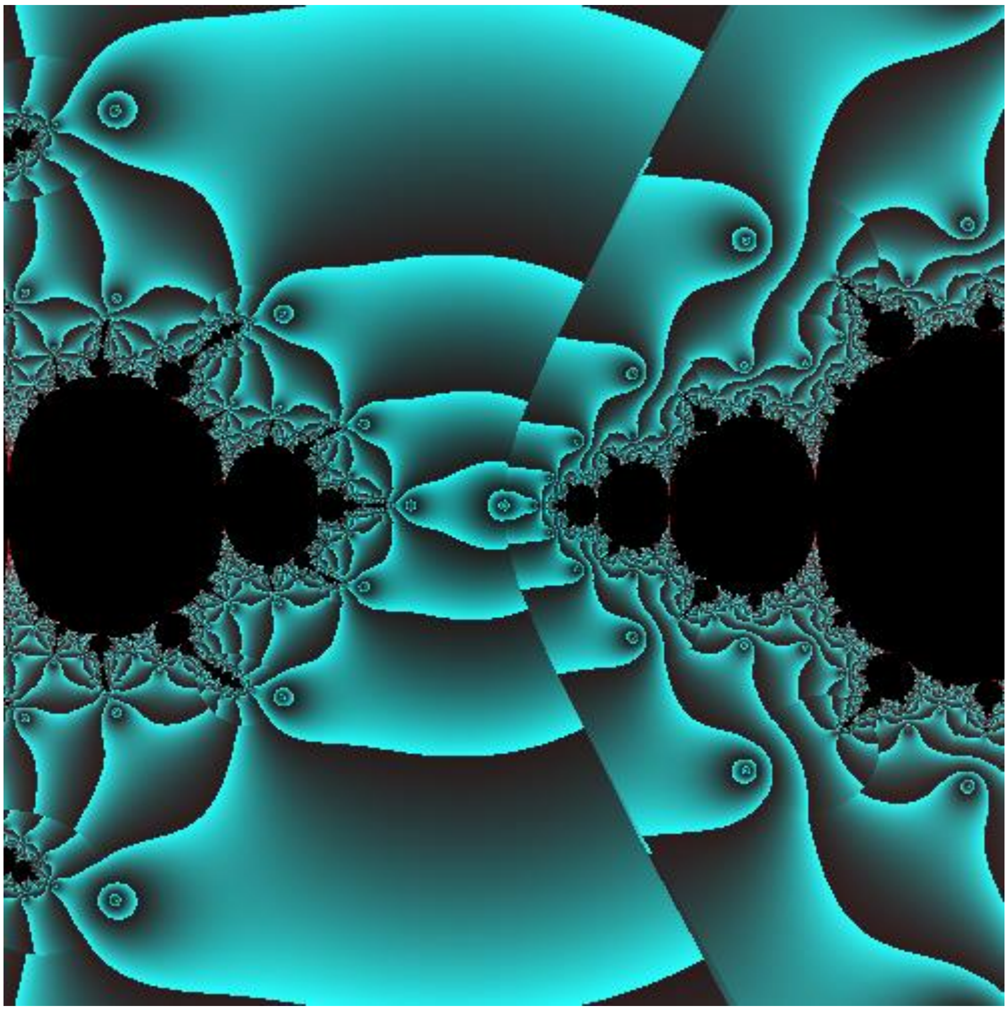

Fig. 10

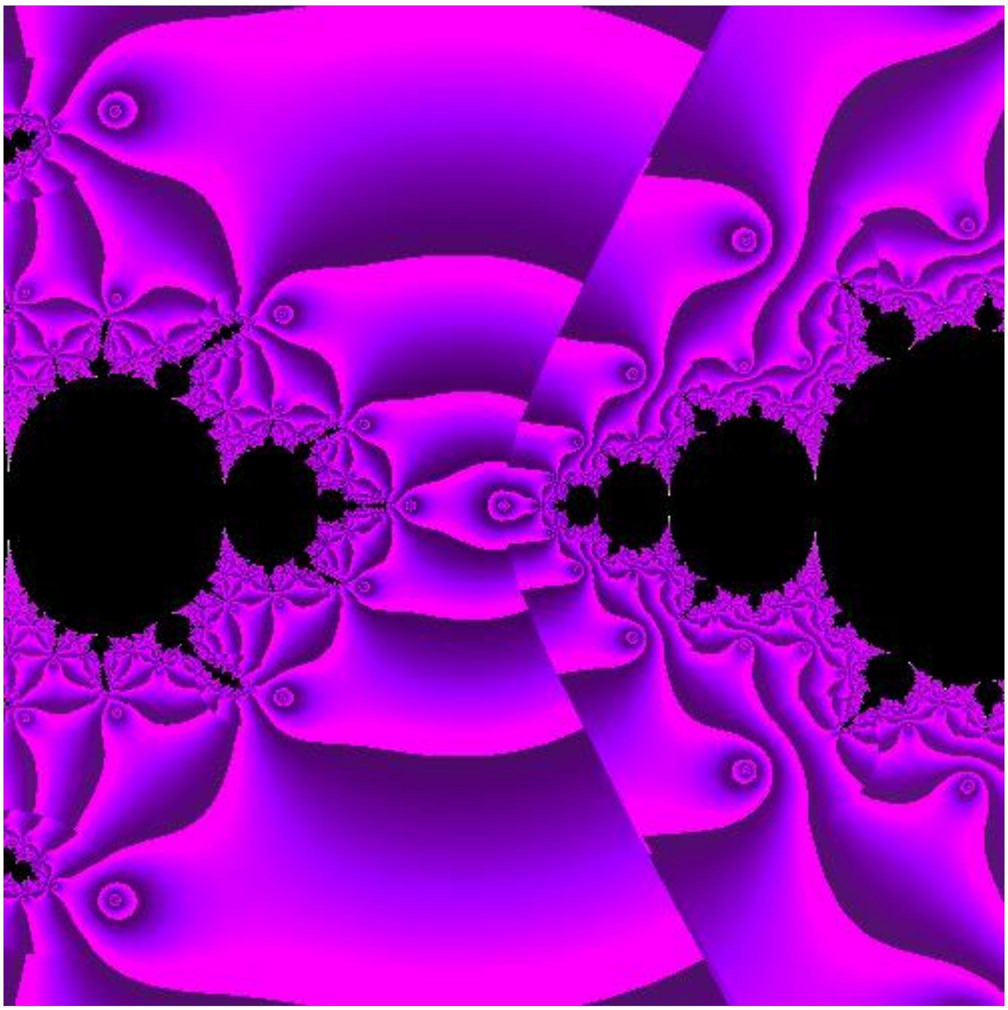

Fig. 11

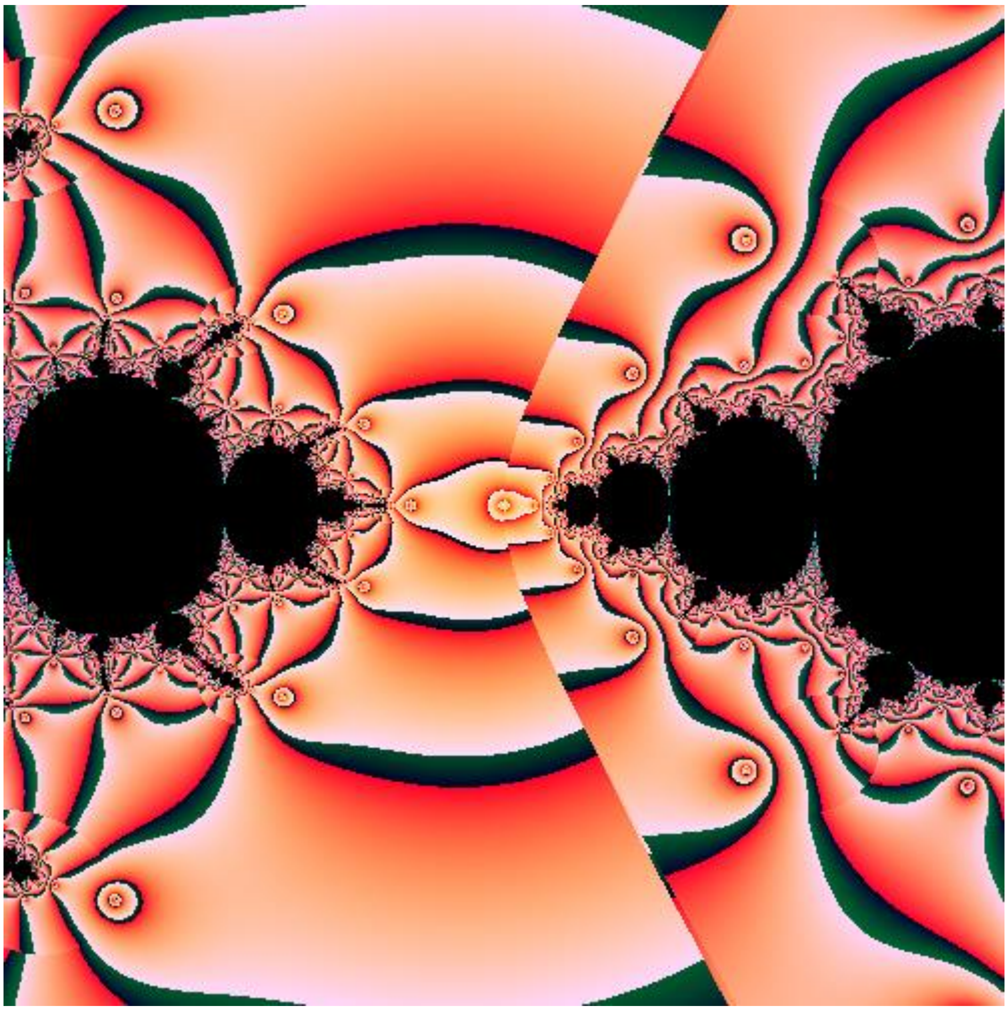

Fig. 12

Referencias

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