Refutation of subset models for justification logic

Abstract: We evaluate two axioms for justification logic which are not tautologous. This means subset models for justification logic are refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

Note: To preserve clarity, we usually distribute quantifiers to each variable so designated.

From: Lehmann, E.; Studer, T. (2019). Subset models for justification logic. arxiv.org/pdf/1902.02707.pdf tstuder@inf.unibe.ch

2.1 Syntax
We investigate a family of justification logics that differ in their axioms and how the axioms are justified.

We have two sets of axioms, the first axioms are:

[We take ":" to mean "such that" and mapped as the Imply connective.]

j+ s : A∨t : A→(s+t) : A; (2.1.2.1)

LET p, q, r, s: j, A, t, s.

((s>(q+r))>(q>(s+r)))>q ;
j+c: (((p+s)>(q+r))>(q>(s+r)))>q ; **FF**TT **FF**TT **FF**TT **FF**TT (2.1.2.2)

jc⋆ c⋆ : A∧c⋆ : (A→B)→c⋆ : B. (2.1.3.1)

LET p, q, r, s: j, c*, A, B.

(q>(r&q))>((r>s)>q))>s ; TT**FF** TTTT **FFFF** TTTT (2.1.3.2)

Eqs. 2.1.2.2 and 2.1.3.2 as rendered are *not* tautologous as axioms. This means subset models for justification logic are refuted.