Denial of the refutation of coherence in modal logic

Abstract: We evaluate the refutation of coherence in modal logic as based on weakly transitive logics using a ternary term to admit finite chains. The term is *not* tautologous, thereby denying the refutation. What follows is that K, KT, K4, and S4 (and S5) are tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

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Abstract : A variety is said to be coherent if the finitely generated subalgebras of its finitely presented members are also finitely presented. ... In this paper, a more general criterion is obtained and used to prove the failure of coherence and uniform deductive interpolation for a broad family of modal logics, including K, KT, K4, and S4.

4.2 Weakly Transitive Logics ... We therefore make use here of the ternary term

t(x,y,z)=□(y∨□(z∨x))∨x (4.2.1)

LET p, q, r, s: x, y, z, t;

(s&((p&q)&r))=(#(q+#(r+p))+p) ; `T`**F**`C`**F** `C`**F**`C`**F** `T`**F**`C`**F** `C`**F**`CT` (4.2.2)

Lemma 4.2: Let L be a modal logic admitting finite chains, and let t(x,y,z) be as defined above.

Eq. 4.2.2 as rendered is not tautologous. This means the ternary term is not a theorem on which finite terms are admitted, thereby denying the refutation of coherence in modal logic. What follows is that K, KT, K4, and S4 (and S5) are tautologous fragments of the universal logic VŁ4.