Quantization of Poisson brackets

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Abstract

We show a quantization of the Poisson brackets. A non-commutativity is introduced and generalizes the classical brackets.

1 The classical Poisson brackets

We take a classical symplectic manifold $(M,w)$, the Poisson brackets are defined:

$$\{f, g\} = w(df, dg)$$

We have:

$$\{f, g\} = \sum_i (e_i f)(e'_i g) - (e'_i f)(e_i g)$$

2 The quantization

We consider a fiber bundle with flat connection $(E, \nabla)$. Then, the quantization is given by a symplectic form $W$:

$$W \in \Lambda^2(TM) \otimes \text{End}(E)^3$$

We have the non-commutativ Poisson brackets:

$$\{f, g\} = W(\nabla f, \nabla g)$$

with $f, g$ two endomorphisms.

$$[f, g] = \sum_i (\nabla e_i f) \circ (\nabla e'_i g) - (\nabla e'_i f) \circ (\nabla e_i g)$$

$$\{f, g\} = [f, g] - [g, f]$$

Due to the fact that $W$ is closed and that the connection is flat, we have a non-commutativ Jacobi identity.

3 Bibliography