Denial of logic system $PL_4$

Abstract: We evaluate the logic system $PL_4$ as refuting and replacing $VL_4$. Eight modal theses and two axioms are not tautologous and contrary to those of $PL_4$. This denies that $PL_4$ refutes $VL_4$, refutes $PL_4$, and justifies $VL_4$ as containing the non bivalent fragment named $PL_4$. We assume the method and apparatus of Meth8/V$L_4$ with Tautology as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Let $\sim$ Not, $\neg$; $+$ Or, $\lor$; $-$ Not Or; $\&$ And, $\land$; $\backslash$ Not And; $>$ Imply, greater than, $\rightarrow$, $\Rightarrow$, $\supset$, $\supseteq$, $\leftarrow$, $\Leftarrow$; $<$ Not Imply, less than, $\leftarrow$, $\Leftarrow$, $\subset$, $\subseteq$, $\#$ possibility, for one or some, $\exists$, $\checkmark$, $\ast$, $\%$; $\#$ necessity, for every or all, $\forall$, $\square$, $\lozenge$; $(z=z)$ $T$ as tautology, $\top$, ordinal 3; $(z\neq z)$ $F$ as contradiction, $\bot$, Null, $\perp$, zero; $(\%z<\#z)$ $C$ as contingency, $\Delta$, ordinal 1; $(\%z>\#z)$ $N$ as non-contingency, $\nabla$, ordinal 2; $\sim(y<x)$ ($x \leq y$), ($x \leq y$); $(A=B)$ $(A\sim B)$.

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[Note that Springer sells this paper only to the public.]
readcube.com/articles/10.1007%2Fs11787-015-0130-z?

Proposition 7.11. Modal theses provable in $PL_4$:

$$A \rightarrow (\neg A \lor LA)$$  \hspace{1cm} (T18.1)

$$p > (\sim p + \# p) ;$$  \hspace{1cm} TNTN TNTN TNTN TNTN  \hspace{1cm} (T18.2)

$$\neg (LA \land A) \rightarrow \neg A$$  \hspace{1cm} (T19.1)

$$\sim (\# p \land p) > \sim p ;$$  \hspace{1cm} TNTN TNTN TNTN TNTN  \hspace{1cm} (T19.2)

Remark T: Eqs. T18.2 and T19.2 are not tautologous.

Proposition 7.13. Modal wffs not provable in $PL_4$:

$$(A \rightarrow B) \rightarrow (MA \rightarrow MB)$$  \hspace{1cm} (F5.1)

$$(p > q) > (\% p > \% q) ;$$  \hspace{1cm} TTTT TTTT TTTT TTTT  \hspace{1cm} (F5.2)

$$(A \rightarrow B) \rightarrow (LA \rightarrow LB)$$  \hspace{1cm} (F6.1)
(p>q)>(#p>#q);  

(MA∧MB)→M(A∧B)  

(%p&%q)>%(p&q);  

L(A∨B)→(LA∨LB)  

#(p+q)>(#p+#q);  

LA→(B→LB)  

#p>(q>#q);  

LA→(MB→B)  

#p>(%q>q);  

It is easy to check that each one of these wffs is invalidated in the matrix MPŁ4. Consequently, they are not provable in PL4 by the soundness theorems (cf. Corollary 5.7). Provability of F1-F4 would result in collapse, that is, in the provability

**Remark 11:** PL4 is not supposed to prove Eqs. F5-F10. However VL4 proves F5.2-F10.2. This implies PL4 is a non bivalent fragment of VL4. Furthermore VL4 finds Eq. F11 as *not* tautologous.

Then, we can add the following axioms to A1-A8 in Definition 3.1:

\[(A∧B) → A/(A∧B) → B\]  

\[((p&q)>((p'(p&q))))>q\];  

\[A → (A ∨ B)/B → (A ∨ B)\]  

\[(p>((p+q)q))>(p+q)\];  

After testing eight modal theses and two axioms, the results are contrary to those of PL4. This denies that PL4 refutes VL4, and further refutes logic system PL4.