Denial of logic system PŁ4

Abstract: We evaluate the logic system PŁ4 as refuting and replacing VŁ4. Eight modal theses and two axioms are *not* tautologous and contrary to those of PŁ4. This denies that PŁ4 refutes VŁ4, refutes PŁ4, and justifies VŁ4 as containing the non bivalent fragment named PŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

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A strong and rich 4-valued modal logic without Łukasiewicz-type paradoxes.
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[Note that Springer sells this paper only to the public.]
readcube.com/articles/10.1007%2Fs11787-015-0130-z?author_access_token=_L6Jv6iOFK12jKHv1HgYEve4RwlQNchNByi7wbcMAY59r2e7nCIsNQegrHbnROJqffuaveeCk8TaBQuGOj5kVweSWjiGNTeHG-kqV2rOibbvfbV1Lhsa3sYYaxKQxsG48f6c3kJbekbSQGkO5DxPSQ==

Proposition 7.11. Modal theses provable in PŁ4:

A→(¬A∨LA) (T18.1)

p>(~p+#p) ; `TNTN TNTN TNTN TNTN` (T18.2)

(¬LA∧A)→¬A (T19.1)

(~#p&p)>~p ; `TNTN TNTN TNTN TNTN` (T19.2)

Remark T: Eqs. T18.2 and T19.2 are *not* tautologous

Proposition 7.13. Modal wffs not provable in PŁ4:

(A→B)→(MA→MB) (F5.1)

(p>q)>(%p>%q) ; `TTTT TTTT TTTT TTTT` (F5.2)

(A→B)→(LA→LB) (F6.1)

(p>q)>(#p>#q) ; TTTT TTTT TTTT TTTT (F6.2)

$(MA \wedge MB) \rightarrow M(A \wedge B)$ (F7.1)

(%p&%q)>%(p&q) ; TTTT TTTT TTTT TTTT (F7.2)

$L(A \vee B) \rightarrow (LA \vee LB)$ (F8.1)

#(p+q)>(#p+#q) ; TTTT TTTT TTTT TTTT (F8.2)

$LA \rightarrow (B \rightarrow LB)$ (F9.1)

#p>(q>#q) ; TTTT TTTT TTTT TTTT (F9.2)

$LA \rightarrow (MB \rightarrow B)$ (F10.1)

#p>(%q>q) ; TTTT TTTT TTTT TTTT (F10.2)

It is easy to check that each one of these wffs is invalidated in the matrix MPŁ4. Consequently, they are not provable in PŁ4 by the soundness theorems (cf. Corollary 5.7). Provability of F1-F4 would result in collapse, that is, in the provability

Remark 11: PŁ4 is not supposed to prove Eqs. F5-F10. However VŁ4 proves F5.2-F10.2. This implies PŁ4 is a non bivalent fragment of VŁ4. Furthermore VŁ4 finds Eq. F11 as *not* tautologous.

Then, we can add the following axioms to A1-A8 in Definition 3.1:

$(A \wedge B) \rightarrow A/(A \wedge B) \rightarrow B$ (A9.1)

((p&q)>(p\(p&q)))>q ; **FF**TT **FF**TT **FF**TT **FF**TT (A9.2)

$A \rightarrow (A \vee B)/B \rightarrow (A \vee B)$ (A11.1)

(p>((p+q)\q))>(p+q) ; **F**TTT **F**TTT **F**TTT **F**TTT (A11.2)

After testing eight modal theses and two axioms, the results are contrary to those of PŁ4. This denies that PŁ4 refutes VŁ4, and further refutes logic system PŁ4.