Denial of logic system PŁ4

Abstract: We evaluate the logic system PŁ4 as refuting and replacing VŁ4. Seven modal theses and four axioms are *not* tautologous and contrary to those of PŁ4. This denies that PŁ4 refutes VŁ4, refutes PŁ4, and justifies VŁ4 as containing the non bivalent fragment named PŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

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A strong and rich 4-valued modal logic without Łukasiewicz-type paradoxes.
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[Note that Springer sells this paper only to the public.]
readcube.com/articles/10.1007%2Fs11787-015-0130-z?
author_access_token=_L6Jv6iOFK12jKHv1HgYEve4RwlQNchNByi7wbcMAY59r2e7nCIsN
QegrHbnROJqffuaveeCk8TaBQuGOj5kVweSWjiGNTeHG-
kqV2rOibbvfbV1Lhsa3sYYaxKQxsG48f6c3kJbekbSQGkO5DxPSQ==

Proposition 7.11. Modal theses provable in PŁ4:

LA→A (T3.1)

%p>p ; NTNT NTNT NTNT NTNT (T3.2)

A→MA (T4.1)

p>#p ; TNTN TNTN TNTN TNTN (T4.2)

(MA→MB)→M(A→B) (T13.1)

(#p>#q)>#(p>q) ; NNNN NNNN NNNN NNNN (T13.2)

Remark T: Eqs. T3.2, T.4.2, and T13.2 are *not* tautologous

Proposition 7.13. Modal wffs not provable in PŁ4:

A→%A (F1.1)

p>%p ; TTTT TTTT TTTT TTTT (F1.2)

MA→A (F2.1)

#p>p ; TTTT TTTT TTTT TTTT (F2.2)

LMA→A (F3.1)

%#p>p ; TTTT TTTT TTTT TTTT (F3.2)

A→MLA (F4.1)

p>#%p ; TTTT TTTT TTTT TTTT (F4.2)

It is easy to check that each one of these wffs is invalidated in the matrix MPŁ4. Consequently, they are not provable in PŁ4 by the soundness theorems (cf. Corollary 5.7). Provability of F1-F4 would result in collapse, that is, in the provability

A ↔ LA (A ↔ MA). (F11.1)

p=(%p&(p=#p)) ; NNNN NNNN NNNN NNNN (F11.2)

Remark 11: PŁ4 is not supposed to prove Eqs. F1-F4. However VŁ4 proves F1.2-F4.2. This implies PŁ4 is non bivalent fragment of VŁ4. Furthermore VŁ4 finds Eq. F11 as *not* tautologous and *not* contradictory, but rather as in a state of non-contingency with the N value.

Then, we can add the following axioms to A1-A8 in Definition 3.1:

(A∧B) → A/(A∧B) → B (A9.1)

((p&q)>(p\(p&q)))>q ; **FF**TT **FF**TT **FF**TT **FF**TT (A9.2)

A → (A ∨B)/B → (A ∨B) (A11.1)

(p>((p+q)\q))>(p+q) ; **F**TTT **F**TTT **F**TTT **F**TTT (A11.2)

LA → A (A13.1)

%p>p ; NTNT NTNT NTNT NTNT (A13.2)

(LA ∧¬A) → B (A14.1)

(%p&~p)> q ; NTTT NTTT NTTT NTTT (A14.2)

After testing seven modal theses and four axioms, the results are contrary to those of PŁ4. This denies that PŁ4 refutes VŁ4, and further refutes logic system PŁ4.