The *Zitterbewegung* and the *de Broglie* wavelength

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12 February 2019

Summary: This paper explores some more philosophical aspects of the *Zitterbewegung* model of an electron, including a geometric interpretation of the *de Broglie* wavelength.

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Zitterbewegung basics

The *Zitterbewegung* model of an electron[1] is intuitive because it combines John Wheeler's 'mass without mass' idea with the idea of a pointlike electric charge. Indeed, the most basic *Zitterbewegung* model is the one Paul A.M. Dirac described: "a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us." He added: "As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."[2]

The reference to light scattering – *photon* scattering – is important because we do get the *Compton* radius of an electron when combining the E = mc^2, E = m$a^2\omega^2$ and E = ħω equations:

$$\mathrm{E} = \mathrm{m}a^2\omega^2 = \mathrm{m}a^2\frac{\mathrm{E}^2}{\hbar^2} \Leftrightarrow \hbar^2 = \mathrm{m}a^2\mathrm{E} = \mathrm{m}a^2\mathrm{m}c^2 = \mathrm{m}^2a^2c^2$$

$$\Leftrightarrow a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The idea is visualized in the illustration below (for which credit goes to the modern *zbw* theorists Celani, Vassallo and Di Tommaso). The illustration shows a rather particular implication of relativity: the *radius*

[1] We are hesitant to generalize to matter-particles (fermions) in general. Current theory suggests all *elementary* matter-particles (apart from electrons, we have quarks, and more) have some electric charge. However, all of our thinking has been focused on mainstream theory in the QED sector of the Standard Model only. Hence, that is the theory that deals with electrons and photons, and their interactions, basically. We always thought some basic *upgrade* of classical mechanics and electromagnetism, based on the idea of *the integrity of a cycle*, would do to deal with the 'quantum' in quantum electrodynamics, and we feel the *Zitterbewegung* hypothesis does exactly that.

[2] See: Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933

of the circulatory motion must diminish as the electron gains speed.[3] Note that there is no particular reason to assume the plane of oscillation should be perpendicular to the direction of propagation: in light of the results of the Stern-Gerlach experiment, it makes sense to think the plane of oscillation might actually be *parallel* to the direction of propagation, in which case the circular motion becomes an elliptical motion when the electron gains speed. In both cases, we can see that, as v approaches c, the circumference of the oscillation becomes a sort of wavelength. We will come back to that in a moment.

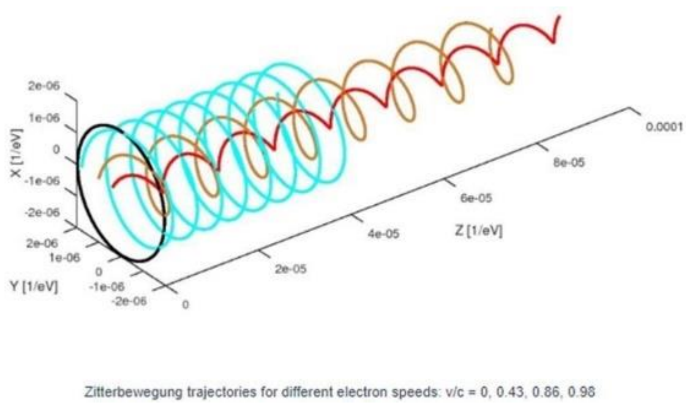

Figure 1: The Compton radius must decrease as the velocity of the electron increases

Note that pointlike does not necessarily mean *dimensionless*. There is also the classical electron radius, aka as the Lorentz radius, which is nothing but the *Thomson scattering radius*. When Thomson scattering happens, the electron's kinetic energy does *not* change, and the photon frequency also does not change – so there is no energy transfer whatsoever between the photon and the electron, unlike what happens when we have Compton scattering. We will not dwell on this as this is rather standard QED theory, but we will quickly recap the other calculations.

The amount of *physical* action – which we will denote by S as per the usual convention – that is associated with one loop along the *zbw* circumference over its cycle time is equal to Planck's constant. We calculate it as the product of the force, the distance (the *zbw* circumference) and the cycle time[4]:

$$S = F \cdot \lambda_C \cdot \mathrm{T} = \frac{\mathrm{E}}{\lambda_C} \cdot \lambda_C \cdot \frac{1}{f_C} = \mathrm{E} \cdot \frac{h}{\mathrm{E}} = h$$

This shows the *Zitterbewegung* hypothesis is consistent with the idea of the integrity of a cycle that we can associate with Planck's constant as the quantum of physical action. Of course, electrons are spin-1/2 particles, but we will show in a moment that's effectively the case if we're considering angular momentum. We will first calculate the force and other magnitudes, however.

Planck's constant h is equal to $6.62607015 \times 10^{-34}$ J·s. Hence, it is a small unit - but small and large are relative. In fact, because of the tiny time and distance scale, we have a rather enormous force here. We can calculate the force because the energy in the oscillator must be equal to the magnitude of the force

[3] The 1/GeV unit that is mentioned along the coordinate axes is a natural unit in cosmology. It is equal to 1.9733×10^{-14} cm.

[4] The formula uses the $\mathrm{E} = F \cdot s = F \cdot \lambda_C$ formula: energy is a force over a distance.

times the length of the loop, we can calculate the magnitude of the force, which is – effectively – rather enormous in light of the sub-atomic scale:

$$\mathrm{E} = \mathrm{F}\lambda_C \Leftrightarrow F = \frac{\mathrm{E}}{\lambda_C} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\ \mathrm{N}$$

The associated current is equally humongous:

$$\mathrm{I} = \mathrm{q_e}f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

A household-level current at the sub-atomic scale? The result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. As the oscillator model implies the effective mass of the electron will be spread over the circular disk, we should use the 1/2 form factor for the moment of inertia (*I*). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

As we mentioned already, the vector notation for **μ** and **L** (boldface) in the equation above should make us think about the plane of oscillation. This question is related to the question of how we should analyze all of this is a moving reference frame. This is the complicated question about which we want to develop a few thoughts in this paper. The Stern-Gerlach experiment suggests we may want to think of an oscillation plane that might be perpendicular to the direction of motion, as illustrated below.

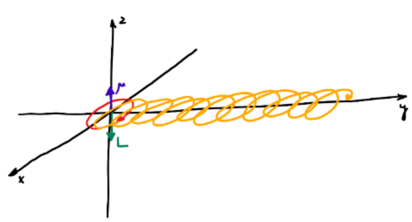

Figure 2: The *zbw* electron traveling through a Stern-Gerlach apparatus?

Of course, the Stern-Gerlach experiment assumes the application of a (non-homogenous) magnetic field. In the absence of such field, we may want to think of the plane of oscillation as something that is rotating in space itself. The idea, then, is that it sort of snaps into place when an external magnetic field is applied. Before we come back to this question, let us first recap some wavefunction math.

Applying wavefunction math

The illustration below shows how wavefunction math captures the *Zitterbewegung* idea. Think of the green dot as representing the circular oscillatory motion of the pointlike charge. As mentioned above, the electron is the *combined* idea of the pointlike charge and its motion.[5] Hence, the (rest) mass of the electron is the equivalent mass of the energy in the oscillation of the pointlike charge.

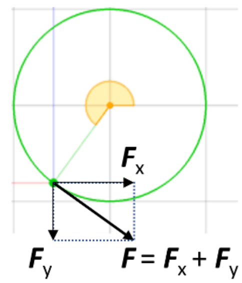

Figure 3: The description of the *Zitterbewegung* of an electron using the wavefunction

The pointlike charge must be driven by a force ***F***, whose nature must be electromagnetic because the force has only a charge to grab onto. We think of this charge as a pointlike object that has no rest mass. Hence, the charge spins around at the speed of light. We have a dual view of the reality of the wavefunction here:

1. On the one hand, it will describe the physical position (i.e. the *x*- and *y*-coordinates) of the pointlike charge – the green dot in the illustration, whose motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

As such, the (elementary) wavefunction may be viewed as an implicit function: it is equivalent to the $x^2 + y^2 = a^2$ equation, which describes the same circle.

2. On the other hand, the *zbw* model implies the circular motion of the pointlike charge is driven by a tangential force, which we write as:

$$\boldsymbol{F} = F_x \cdot \cos(\omega t + \pi/2) + i \cdot F_x \cdot \sin(\omega t + \pi/2) = F \cdot e^{i(\theta + \pi/2)}$$

What is the nature of this force? Where does it come from? We know it must be electromagnetic in its nature – because it grabs onto a force – but what keeps the pointlike charge in orbit?

In classical mechanics, a force may be defined as the (negative of the) derivative of a potential. Such potential may be gravitational or electrostatic. We write:

$$-\mathrm{d}U/\mathrm{d}x = F(x) = F_x$$

If we're considering the y-direction, then we write $-\mathrm{d}U/\mathrm{d}y = F(y) = F_y$.

What if we would – somehow – think of the $a \cdot e^{i\theta}$ function as some *complex* potential. Let us forget

[5] We have extended the model to electron orbitals in previous papers. We have also developed a new photon model re-using some of the concepts – most notably the idea of the integrity of a cycle. See: http://vixra.org/abs/1901.0105.

about the coefficient *a* for a while (we can plug it back in at a later stage), so we write:

$$\mathbf{U} = e^{i\theta}$$

Let us take the derivative in regard to the variable here, which is… What? It is the angle θ. It is a *real* number, so we will *not* be calculating the usual derivative of a complex exponential, which is $d(e^z)/dz = e^z$, with *z* a complex number. Instead, we calculate:

$$-d\mathbf{U}/d\theta = -d(e^{i\theta})/d\theta = -d(\cos\theta + i\cdot\sin\theta)/d\theta = -d(\cos\theta)/d\theta - i\cdot d(\sin\theta)/d\theta$$

$$= \sin\theta - i\cdot\cos\theta = \cos(\theta-\pi/2) + i\cdot\sin(\theta-\pi/2)$$

We get the sine and cosine factors of our formula, except the sign is right: the phase factor should be $+\pi/2$ instead of $-\pi/2$. That problem is solved if we drop the minus sign in front of the $-d\mathbf{U}/d\theta$ derivative:

$$d\mathbf{U}/d\theta = d(e^{i\theta})/d\theta = d(\cos\theta + i\cdot\sin\theta)/d\theta = d(\cos\theta)/d\theta + i\cdot d(\sin\theta)/d\theta$$

$$= -\sin\theta + i\cdot\cos\theta = \cos(\theta+\pi/2) + i\cdot\sin(\theta+\pi/2)$$

Why would we drop the minus sign? One may think it could be related to the other mathematical possibility: the rotation may be clockwise rather than counterclockwise. The mathematical formalism works out equally well, but it does not explain why we should drop the minus sign in front of the derivative. However, *if* we acknowledge there would be a minus sign if we would have adopted the convention of measuring angles clockwise rather than counterclockwise, *then* we see it's just a matter of convention, effectively.

The reader will wonder: what is the use of this weird exercise? The idea is the following: we think of the electron defining its own physical space. The idea is related to the idea of the quantization of space, as we will explain below.

The quantization of space

The issue here is nicely summarized in one of Dr. Burinskii's very first communications to me. He effectively wrote the following when I first contacted him on the viability of the *zbw* model:

> "I know many people who considered the electron as a toroidal photon[6] and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about *Zitterbewegung* [because of ideological reasons[7]], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?"[8]

[6] This is Dr. Burinskii's terminology: it does refer to the *Zitterbewegung* electron: a pointlike charge with no mass in an oscillatory motion – orbiting at the speed of light around some center.

[7] This refers to perceived censorship from the part of Dr. Burinskii. In fact, some of what he wrote me strongly suggests some of his writings have, effectively, been suppressed because – when everything is said and done – they do fundamentally question – directly or indirectly – some key assumptions of the mainstream interpretation of quantum mechanics.

[8] Email from Dr. Burinskii to the author dated 22 December 2018.

He noted that this fundamental flaw was (and still is) the main reason why had abandoned the simple *Zitterbewegung* model in favor of the much more sophisticated Kerr-Newman approaches to the (possible) geometry of an electron.

I am reluctant to make the move he made – mainly because I prefer simple math to the rather daunting math involved in Kerr-Newman geometries – and so that is why I am continuing to explore alternative explanations – such as this one. I feel there is scope here to complement the model with a third view of the wavefunction.

Why would we need a third view? What is this about? Let me explain. The *zbw* model offers a basic *interpretation* of the wavefunction by noting the various *aspects* of the (possible) reality that might correspond to the wavefunction. We referred to these aspects as the *dual view* of a wavefunction. That dual view consisted of (1) a description of the *position* of our pointlike particle and (2) a description of the *force* that makes it move. Both descriptions are descriptions in terms of a complex-valued function: the wavefunction itself. Hence, it would seem to be logical that we develop a *third* view now: the wavefunction as a description of the physical space that comes with the particle. How can we do that?

Perhaps Einstein provides some inspiration again. Indeed, we got that two-dimensional oscillator model (the flywheel model of an electron, as I used to call it) as a result of a *deep* exploration of the (possible) meaning of Einstein's mass-energy equivalence relation ($E = mc^2$), which comes out of *special* relativity theory. So… Well… Perhaps we should now explore some other intuition—an intuition-based Einstein's general relativity theory. What am I thinking of?

It's the following: if we can describe the particle itself by Euler's wavefunction – exploring different *aspects* of its reality, such as the position of the pointlike charge, and the nature of the force that makes it move along its circular orbit – then we should, perhaps, also explore how we can use it to describe the nature of the *space* that comes with the particle.

That's why we suggest to think of the $a \cdot e^{i\theta}$ function as describing some *complex* potential. Our terminology may not well be chosen, but the reader should sort of intuitively understand what is that we are trying to do here. If we take Euler's function to actually represent an elementary charged particle (practically speaking, we are talking an electron here), then the *Zitterbewegung* model suggests it comes with its own space. The *nature* of this space is – quite simply – this new concept: a complex-valued potential. The first reaction of the reader is predictable: this must be nonsense. I invite the reader to think about *why* he would say that – because my own initial reaction to my thoughts was the same: this is ridiculous. However, I then realized that my instinctive objection to my own thoughts was that it is somewhat hard to distinguish ontological or mathematical concepts here from what might (or might not) be reality – or *physical* concepts, I should say. In fact, the ambiguity is in the concept of a potential itself. It is *less* tangible than a force. It is like thinking of a force without thinking simultaneously about what it's going to grab onto – which is not an easy exercise. To put it differently, I have a strong feeling that my train of thought here involves implicit tautologies. Having said that, tautologies – when made explicit – may bring new insights.

We also need to relate this to another discussion, which would be worth some further exploration – but we can only touch upon here as we lack the more advanced math skills that are required to tackle the question. It concerns the magnitude of the force – and the associated field – and its likely impact on the geometry of space.

A black-hole model for an electron?

We should think some more about the nature of the force. The assumption is that the force grabs onto a pointlike charge. Hence, the force must be electromagnetic and we can write it as the product of the unit charge and the field (***E***). We write:

$$\boldsymbol{F} = q_e\boldsymbol{E}.$$

Because the force is humongous (a force of 0.0375 N is equivalent to a force that gives a mass of 37.5 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second), and the charge is tiny), we get an equally huge field strength:

$$E = \frac{F}{\mathrm{q_e}} \approx \frac{3.3743 \times 10^{-2}\ \mathrm{N}}{1.6022\ \times 10^{-19}\ \mathrm{C}} \approx 0.21 \times 10^{18}\ \mathrm{N/C}$$

Just as a yardstick to compare, we may note that the most powerful man-made accelerators may only reach field strengths of the order of 10^9 N/C (1 GV/m). Does this make sense? Can we calculate an energy density? Using the classical formula, we get:

$$u = \epsilon_0 E^2 \approx 8.854 \times 10^{-12} \cdot (0.21 \times 10^{18})^2 \frac{\mathrm{J}}{\mathrm{m}^3} = 0.36 \times 10^{24}\ \frac{\mathrm{J}}{\mathrm{m}^3} = 0.63 \times 10^{24}\ \frac{\mathrm{J}}{\mathrm{m}^3}$$

This amounts to about 7 kg per mm^3 (cubic *milli*meter). Is this a sensible value? Maybe. Maybe not. The rest mass of the electron is tiny, but then the *zbw* radius of an electron is also exceedingly small. It is very interesting to think about what might happen to the curvature of spacetime with such mass densities: perhaps our pointlike charge just goes round and round on a geodesic in its own (curved) space. We are not well-versed in general relativity and we can, therefore, only offer some general remarks here:

1. If we would pack all of the mass of an electron into a black hole, then the Schwarzschild formula gives us a radius that is equal to:

$$r_s = \frac{2\mathrm{Gm}}{c^2} \approx 1.35 \times 10^{-57}\mathrm{m}\ (meter)$$

This exceedingly small number has no relation whatsoever with the Compton radius. In fact, its scale has no relation with whatever distance one encounters in physics: it is *much* beyond the Planck scale, which is of the order of 10^{-35} meter and which, for reasons deep down in relativistic quantum mechanics, physicists consider to be the smallest possibly sensible distance scale.

2. We are intrigued, however, by suggestions that the Schwarzschild formula should not be used as it because an electron has angular momentum, a magnetic moment and other properties, perhaps, that do not apply when calculating, say, the Schwarzschild radius of the mass of a baseball. To be precise, we are particularly intrigued by models that suggest that, when incorporating the above-mentioned properties of an electron, the Compton radius might actually be the radius of an electron-sized black hole (Burinskii, 2008, 2016).

How to interpret the *de Broglie* wavelength?

Let us look at a moving electron now. Let us consider the idea of a particle traveling in the positive *x*-direction at constant speed *v*. This idea implies a pointlike concept of position and time: we think the particle will be somewhere at some point in time. The *somewhere* in this expression does not necessarily mean that we think the particle itself will be dimensionless or pointlike. It just implies that we can associate some *center* with it. In fact, that's what we have in our *zbw* model here: we have an oscillation around some center, but the oscillation has a *physical* radius, which we referred to as the Compton radius of the electron. Of course, two extreme situations may be envisaged: $v = 0$ or $v = c$.

However, let us consider the more general case. In our reference frame, we will have a position – a mathematical *point* in space, that is – which is a function of time: $x(t) = v \cdot t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t'. Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t', and the position and time in the two reference frames will be related as follows:

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1-\frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows:

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(\frac{t}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{E_0}{\hbar}t'$$

We have just shown that the argument of the wavefunction is relativistically invariant (E_0 is, obviously, the rest energy and, because $p' = 0$ in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself). That's what made us think that the argument of the wavefunction and – therefore – the wavefunction itself – might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution.

Let us explore this further. We have been interpreting the wavefunction as an implicit function again: for each *x*, we have a *t*, and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by $x(t) = v \cdot t$. We will get some linear motion. If we look at the $\psi = a \cdot \cos(p \cdot x/\hbar - E \cdot t/\hbar) + i \cdot a \cdot \sin(p \cdot x/\hbar - E \cdot t/\hbar)$ once more, we can write $p \cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what *x* this phase factor $\Delta = p \cdot x/\hbar$ will be equal to 2π. Hence, we write:

$$\Delta = p \cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi \cdot \hbar/p = h/p = \lambda$$

We now get a meaningful interpretation of the *de Broglie* wavelength. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.[9]

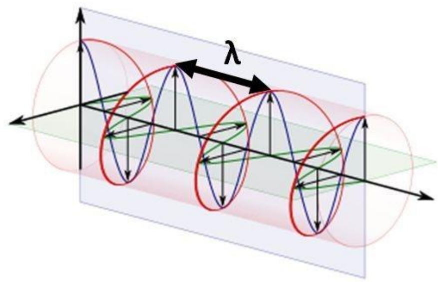

Figure 4: An interpretation of the *de Broglie* wavelength

Of course, we should probably think of the plane of oscillation as being *perpendicular* to the plane of motion – or as oscillating in space itself – but that doesn't matter. Let us explore some more. We can, obviously, re-write the argument of the wavefunction as a function of *time* only:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}vt\right) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)t = \sqrt{1-\frac{v^2}{c^2}}\cdot\frac{\mathrm{E}_0}{\hbar}t$$

We recognize the *inverse* Lorentz factor here, which goes from 1 to 0 as *v* goes from 0 to *c*, as shown below.

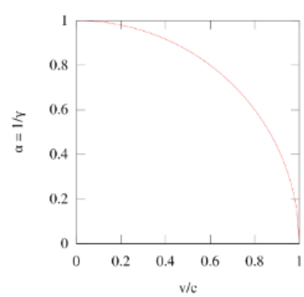

Figure 5: The inverse Lorentz factor as a function of (relative) velocity (*v*/*c*)

[9] We are actually *not* satisfied with this description. I write that it's the distance between crests of the wavefunction, but it cannot be. Note that it converges to the Compton wavelength as *v* goes to *c*: λ = h/p = h/m*c* = *a* for *v* = *c*. The interpretation of the meaning of the *de Broglie* wavelength remains a tricky matter. The standard interpretation of quantum physics (mainstream or Copenhagen) always brings some complicated argument involving uncertainty – but we do not have any uncertainty in the *Zitterbewegung* model (we can introduce uncertainty later but – at this stage – we're really looking at an electron model without uncertainty). So… Well… It requires some further thinking. At a minimum, I guess we should measure time and distance in equivalent units to say something meaningful about the λ = h/p relation. Of course, if *v* = *c*, and we measure x and t in equivalent units, then we get the λ = h/p relation from the universal λ = *c*/*f* relation for a wave and the Planck-Einstein relation (E = mc^2 = h*f*). We can then write: λ = *c*/f = *c*h/mc^2 = h/m*c* = h/p. Perhaps it's that simple. Any thoughts? Anyone?

Note the shape of the function: it is a simple circular arc. This result should not surprise us, of course, as we also get it from the Lorentz formula:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t - \frac{v^2}{c^2}t}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{1-\frac{v^2}{c^2}} \cdot t$$

In fact, we had already introduced this formula when we were talking about the difference between coordinate time and proper time.

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} = \frac{\mathrm{dt}}{\mathrm{d}\tau}$$

We just used different symbols for it: the time in *our* reference frame (t) is the *coordinate time*, and the time in the reference frame of the object itself (τ) is referred to as the *proper time*. Why are we talking about this? What does it all mean? Let us first go through a simple numerical example to think through that formula above. Let us assume that, for example, that we are able to speed up an electron to, say, about one tenth of the speed of light. Hence, the Lorentz factor will then be equal to $\gamma = 1.005$. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0: $E_v = \gamma E_0 \approx 1.005 \cdot 0.511$ MeV ≈ 0.5135 MeV. The relativistic momentum will then be equal to $m_v v = (0.5135\ \mathrm{eV}/c^2)\cdot(0.1\cdot c) = 5.135\ \mathrm{eV}/c$. We get:

$$\theta = \frac{E_0}{\hbar}t' = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = 0.955\frac{E_0}{\hbar}t$$

This is interesting: we get an explanation for time dilation. A more interesting question is what happens to the *radius* of the oscillation. Does it change? It must, but how should we interpret this? In the moving reference frame, we measure higher mass and, therefore, higher energy – as it includes the kinetic energy. The $c^2 = a^2\cdot\omega^2$ identity must now be written as $c^2 = a'^2\cdot\omega'^2$. Instead of the rest mass m_0 and rest energy E_0, we must now use $m_v = \gamma m_0$ and $E_v = \gamma E_0$ in the formulas for the Compton radius and the Einstein-Planck frequency, which we just write as m and E in the formula below:

$$\mathrm{m}a'^2\omega'^2 = \mathrm{m}\frac{\hbar^2}{\mathrm{m}^2c^2}\frac{\mathrm{m}^2c^4}{\hbar^2} = \mathrm{m}c^2$$

This is easy to understand intuitively: we have the mass factor in the denominator of the formula for the Compton radius, so it must increase as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, increase with velocity. It is interesting to note that we have a simple (inverse) proportionality relation here.

Can the velocity go to *c*? Of course! That's what we mentioned already: in the limit, the circumference of the oscillation becomes a wavelength! This links the *zbw* electron model with the photon model which we developed in previous publications.[10] which we will explain later. We first need to talk about orbital electron motion. Before we do so, we will resume the model that we have here.

[10] See our reference above: http://vixra.org/abs/1901.0105.

We should note that the center of the *Zitterbewegung* was plain nothingness and we must, therefore, assume some two-dimensional oscillation makes the charge go round and round. The angular frequency of the *Zitterbewegung* rotation is given by the Planck-Einstein relation ($\omega = E/\hbar$) and we get the *Zitterbewegung* radius (which is just the Compton radius $a = r_C = \hbar/mc$) by equating the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equation. The energy and, therefore, the (equivalent) mass is in the oscillation and we, therefore, should associate the momentum $p = E/c$ with the electron as a whole or, if we would really like to associate it with a single mathematical *point* in space, with the center of the oscillation – as opposed to the rotating massless *charge*.

END