Refutation of the axiom of extensionality

Abstract: We evaluate the axiom of extensionality. The equation we tested is *not* tautologous. This refutes the axiom of extensionality.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r: x, A, B;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

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Set theory is based on the classical first order logic. Basic entities are sets[,] and they are completely determined by their elements: the Axiom of Extensionality states that for any sets A and B, $[\forall x(x\in A \Longleftrightarrow x\in B)]\Longleftrightarrow A=B$. (3.1)

((#p<q)=(#p<r))=(q=r) ; TT**F**N **F**NTT TT**F**N **F**NTT (3.2)

Eq. 3.2 as rendered is not tautologous. This refutes the axiom of extensionality.