Confirmation of the failure of modus ponens when the consequent is itself a conditional sentence

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Abstract: We confirm the failure of modus ponens when the consequent is itself a conditional sentence. The reason a repeated consequent does not produce tautology is because it dilutes the original sentence to assume incorrectly other plausible consequents.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r: Shakespeare, Hobbes, Hamlet;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↪, ⊃, ⊖, ⊣, ⊳ < Not Imply, less than, ∈, ⊊, ⊆, ⊐, ⊑;
≡ Equivalent, ≡, ↔, ⇌, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ♦, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) F as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y); (A=B) (A~B).

From: en.wikipedia.org/wiki/Modus_ponens

[The following is attributed to Vann McGee, but without a proper footnote in the article.]

Either Shakespeare or Hobbes wrote Hamlet. (1.1.1)

\((p+q)>r\); \begin{array}{cccc}
\text{T} & \text{FFF} & \text{TTTT} & \text{TFFFF} & \text{TTTTT}
\end{array} (1.1.2)

If Shakespeare didn't do it, Hobbes did. (1.2.1)

\~(p>r)>(q>r) ; \begin{array}{cccc}
\text{TTTT} & \text{TTTT} & \text{TTTF} & \text{TTTT}
\end{array} (1.2.2)

If either Shakespeare or Hobbes wrote Hamlet, then if Shakespeare didn't do it, Hobbes did. (2.1.0)

We write this as (Eq. 1.1.1 implies 1.2.1). (2.1.1)

\(((p+q)>r)>(\~(p>r)>(q>r))\); \begin{array}{cccc}
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTTTT}
\end{array} (2.1.2)

Therefore, if Shakespeare didn't write Hamlet, Hobbes did it. (3.1.0)

We write this as (Eq. 1.1.1 implies 1.2.1) implies 1.2.1. (3.1.1)

\(((p+q)>r)>(\~(p>r)>(q>r)))>(\~(p>r)>(q>r))\); \begin{array}{cccc}
\text{TTTT} & \text{TTTF} & \text{TTTT} & \text{TTTF}
\end{array} (3.2.1)
Eq. 2.1.2 for (Eq. 1.1.2 implies 1.2.2) is tautologous. Eq. 3.1.1 supplements 2.1.2 with an additional consequent 1.2.2 as a conditional sentence. We call this a repeated consequent. However 3.1.1 ((Eq. 1.1.1 implies 1.2.1) implies 1.2.1) is not tautologous. Therefore, the repeated consequent dilutes the tautology of the original sentence.

**Remark 3:**

The wiki consortium writes:

"But the conclusion [3.1.0] is dubious, because if Shakespeare is ruled out as *Hamlet's* author, there are many more plausible alternatives than Hobbes."

That is mistaken because it makes an assumption, and should read:

"But the conclusion [3.1.0] is dubious, because if Shakespeare is ruled out as *Hamlet's* author, for Shakespeare to be ruled again does not imply the dubious assumption of many more plausible alternatives than Hobbes."