

Confirmation of the failure of modus ponens when the consequent is itself a conditional sentence

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Abstract: We confirm the failure of modus ponens when the consequent is itself a conditional sentence. The reason a repeated consequent does not produce tautology is because it dilutes the original sentence to assume incorrectly other plausible consequents.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r : Shakespeare, Hobbes, Hamlet;
 \sim Not, \neg ; + Or, \vee, \cup ; - Not Or; & And, \wedge, \cap ; \ Not And;
 $>$ Imply, greater than, $\rightarrow, \mapsto, \succ, \supset, \vdash, \models, \Rightarrow$; $<$ Not Imply, less than, $\in, \prec, \subset, \neq, \neq, \Leftarrow$;
 $=$ Equivalent, $\equiv, :=, \iff, \leftrightarrow, \triangleq$ @ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists, \diamond, M ; # necessity, for every or all, \forall, \square, L ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, $\emptyset, \text{Null}, \perp$, zero;
 $(\%z<\#z)$ **C** as contingency, Δ , ordinal 1; $(\%z>\#z)$ **N** as non-contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$); $(A=B)$ ($A \sim B$).

From: en.wikipedia.org/wiki/Modus_ponens

[The following is attributed to Vann McGee, but without a proper footnote in the article.]

Either Shakespeare or Hobbes wrote Hamlet. (1.1.1)

$(p+q)>r$; TFFF TTTT TFFF TTTT (1.1.2)

If Shakespeare didn't do it, Hobbes did. (1.2.1)

$\sim(p>r)>(q>r)$; TTTF TTTT TTTF TTTT (1.2.2)

If either Shakespeare or Hobbes wrote Hamlet, then if Shakespeare didn't do it, Hobbes did. (2.1.0)

We write this as (Eq. 1.1.1 implies 1.2.1). (2.1.1)

$((p+q)>r)>(\sim(p>r)>(q>r))$; TTTT TTTT TTTT TTTT (2.1.2)

Therefore, if Shakespeare didn't write Hamlet, Hobbes did it. (3.1.0)

We write this as (Eq. 1.1.1 implies 1.2.1) implies 1.2.1. (3.1.1)

$((((p+q)>r)>(\sim(p>r)>(q>r)))>(\sim(p>r)>(q>r)))$; TTTT TTTF TTTT TTTF (3.2.1)

Eq. 2.1.2 for (Eq. 1.1.2 implies 1.2.2) is tautologous. Eq. 3.1.1 supplements 2.1.2 with an additional consequent 1.2.2 as a conditional sentence. We call this a repeated consequent. However 3.1.1 ((Eq. 1.1.1 implies 1.2.1) implies 1.2.1) is *not* tautologous. Therefore, the repeated consequent dilutes the tautology of the original sentence.

Remark 3:

The wiki consortium writes:

"But the conclusion [3.1.0] is dubious, because if Shakespeare is ruled out as *Hamlet's* author, there are many more plausible alternatives than Hobbes."

That is mistaken because it makes an assumption, and should read:

"But the conclusion [3.1.0] is dubious, because if Shakespeare is ruled out as *Hamlet's* author, for Shakespeare to be ruled again does not imply the dubious assumption of many more plausible alternatives than Hobbes."