Refutation of correctness from Pratt-Floyd-Hoare logic

Abstract: We evaluate the Pratt-Floyd-Hoare logic aimed at correctness of computer programs. For Pratt, four equations are tested, and for Hoare one is tested. None are tautologous. This refutes the Pratt-Floyd-Hoare logic for correctness.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v: p, q, a, b, p', q', R
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

From: Pratt, V.R. (1976). Semantical considerations of Floyd-Hoare logic. 17th IEEE Foundations of Computer Science Conference. boole.stanford.edu/pub/semcon.pdf pratt@cs.stanford.edu

Pratt weakest antecedent and strongest consequent:

P{a}Q ≡ ⊭(P⊃{a}Q) (W.1)

((p&r)&q)=~(p>(r&q)) ; `T`**`F`**`TF T`**`F`**`T`**`F`** `T`**`F`**`T`**`F`** `T`**`F`**`T`**`F`** (W.2)

Remark W: The proposition for in Eq. W.2 is *no*t tautologous.

Pratt axiom 2, in handwritten note on margin:

P{a∪b}Q ⊢P{a}Q (A2n.1)

((p&(r+s))&q)>((p&r)&q) ; `TTTT TTTT TTT`**`F`** `TTTT` (A2n.2)

Remark A2n: The axiom as a handwritten note is *not* tautologous and not equivalent to the axiom.

Pratt axiom 4:

Q{P}P/\Q (A4.1)

(q&p)&(p&q) ; **`FFF`**`T` **`FFF`**`T` **`FFF`**`T` **`FFF`**`T` (A4.2)

Pratt axiom 4, alternate:

$P \supset Q\{P\}Q$ (A4.alt.1)

p>((q&p)&q); T**F**TT T**F**TT T**F**TT T**F**TT (A4.alt.2)

Remark A4: The original and alternate axioms are *no*t tautologous and not equivalent.

Hoare "Rules of consequence":

$P \supset Q,\ Q\{a\}R \vdash P\{a\}R$ (HR.1.1)

((p>q)&((q&r)&v))>((p&r)&v) ; TTTT TTTT TTTT TTTT(4), TTTT TT**F**T TTTT TT**F**T(4) (HR.1.2)

Remark HR1: The rule of consequence is *not* tautologous.

Eqs. W2, A2.n.2, A4.2, A4.alt.2, HR.1.2 are *not* tautologous and contradict the respective alternates. This refutes the Pratt-Floyd-Hoare logic for correctness.