Refutation of dynamic modal logic

Abstract: We evaluate two definitions of dynamic modal logic on which axioms are built. The definitions are *not* tautologous and moreover logically equivalent. This refutes dynamic modal logic.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: p, q, a, b;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ↞;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

From: en.wikipedia.org/wiki/Dynamic_logic_(modal_logic)

Modal logic is characterized by the modal operators []p (box p) asserting that p is necessarily the case, and <> p (diamond p) asserting that p is possibly the case. Dynamic logic extends this by associating to every action a the modal operators [a] and <a>, thereby making it a multimodal logic. The meaning of [a]p is that after performing action a it is necessarily the case that p holds, that is, a must bring about p. The meaning of <a>p is that after performing a it is possible that p holds, that is, a might bring about p. These operators are related by

[a]p=~<a>~p and (0.1.1)

We write Eq. 0.1.1 as:
"Necessarily r applies to p implies the necessity of p" is equivalent to
"Not possibly r applies to p implies not the possibility of p".

((#r&p)>#p)=((~%r&p)>~%p) ; `TCTC TTTT TCTC TTTT` (0.1.2)

<a>p= ~[a]~p, (0.2.1)

We write Eq. 0.2.1 as:
"Possibly r applies to p implies the possibility of p" is equivalent to
"Not necessarily r applies to p implies not the necessity of p".

((%r&p)>%p)=((~#r&p)>~#p) ; `TCTC TTTT TCTC TTTT` (0.2.2)

analogously to the relationship between the universal (∀) and existential (∃) quantifiers.

Remark 0: Eqs. 0.1.2 and 0.2.2 as rendered as definitions are *no*t tautologous. In fact, they are logical equivalences. This refutes the definitions of dynamic modal logic. Those definitions as analogous to the quantifiers is thus denied.

A3. $[a \cup b]p \equiv [a]p \wedge [b]p$ (3.1)

(A3 says that if doing one of a or b must bring about p, then a must bring about p and likewise for b, and conversely.)

Remark 3.1: Eq. 3.1 is an obvious tautology and hence trivial as an axiom.

The stated definitions in Eqs. 0.1.2 and 0.2.2 are *not* tautologous and also logically identical. This on its face refutes dynamic modal logic.