Refutation of the spin-statistics theorem

Abstract: We evaluate the spin-statistics theorem assuming two variables are not equivalent for the equations of commute and anti-commute fields. The equations are logically equivalent meaning the status of the two variables is irrelevant and unnecessary. Therefore the theorem is refuted, casting doubt on the logical foundations of QFT.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, ψ, x, y;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

From: en.wikipedia.org/wiki/Spin–statistics_theorem

Let us assume that

x ≠ y (0.1.1)

r@s ; **FFFF** TTTT TTTT **FFFF** (0.1.2)

and the two operators take place at the same time ...

Remark 0: We use Eq. 0.1.1 as the antecedent implying 1.1-2.1 for 1.2-2.2.

If the fields **commute**, meaning that the following holds:

$$\phi (x) \phi (y) = \phi (y) \phi (x)$$

then only the symmetric part of ψ contributes, so that

$$\psi (x , y) = \psi (y , x) , \tag{1.1}$$

(r@s)>((((p&r)&(p&s))=((p&s)&(p&r)))>((q&(r&s))=(q&(s&r)))) ;
TTTT TTTT TTTT TTTT (1.2)

and the field will create bosonic particles.

> **Remark 1:** Because Eq. 1.1 contains an antecedent and consequent that are respective equivalents, the tautologous equation for commute fields is expected and trivial.
> If the antecedent in Eq. 1 is r<s, r>s, or r=s, the table results are the same as for 1.2.

On the other hand, if the fields **anti-commute**, meaning that ϕ has the property that

$$\phi(x)\,\phi(y) = -\,\phi(y)\,\phi(x)\,,$$

then only the antisymmetric part of ψ contributes, so that

$$\psi(x, y) = -\,\psi(y, x)\,, \tag{2.1}$$

(r@s)>((((p&r)&(p&s))=~((p&s)&(p&r)))>((q&(r&s))=~(q&(s&r)))) ;
TTTT TTTT TTTT TTTT (2.2)

and the particles will be fermionic.

> **Remark 2:** Because Eq. 2.1 contains an antecedent and consequent that are respective equivalents, the tautologous equation for anti-commute fields is expected and trivial.
> If the antecedent in Eq. 2 is r<s, r>s, or r=s, the table results are the same as for 2.2.

> **Remark 3:** Because of Remarks 1 and 2, the antecedent in Eq. 0.1 becomes irrelevant to the truth table result in 1.2 and 2.2.
> For example, we rewrite Eqs. 1.2 and 2.2 without the (r@s) as:
>
> (((p&r)&(p&s))=((p&s)&(p&r)))>((q&(r&s))=(q&(s&r))) ;
> TTTT TTTT TTTT TTTT (3.1.2)
>
> (((p&r)&(p&s))=~((p&s)&(p&r)))>((q&(r&s))=~(q&(s&r))) ;
> TTTT TTTT TTTT TTTT (3.2.2)
>
> What follows is that the relation of x and y in Eq. 0.1.1 is irrelevant.
>
> What further follows is that the subsequent matrix machinations in the cited text are specious. In fact, the text admits this in so many words with:
>
> > "Naively, neither [Eqs. 1.1 or 2.1] has anything to do with the spin, which determines the rotation properties of the particles, not the exchange properties."

The results of Eqs. 3.1.2 and 3.2.2 refute the spin-statistics theorem. This implies that quantum field theory (QFT) does not have a stable foundation in bivalent, mathematical logic.