Refutation of mapping definable functions as neighbourhood functions

Abstract: We evaluate two applied equations of stopping and closure functions of bar recursion. None is tautologous. This refutes the approach of mapping functions as such.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x :
D, T, ρ, σ, τ, τ*, T, N, N*;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⟺, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z<#z) C as contingency, Δ, ordinal 1; (%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

From: Kawai, T. (2019). Representing definable functions of HA^{ω} by neighbourhood functions. arxiv.org/pdf/1901.11270.pdf tatsuji.kawai@jaist.ac.jp

We call a function $BR^{\tau,\sigma}$ of type

$$((N \to \tau) \to N) \to (\tau * \to \sigma) \to (\tau * \to (\tau \to \sigma) \to \sigma) \to \tau * \to \sigma \qquad (6.3.1.1.1)$$

which satisfies (6.3) a bar recursor of types τ and σ. The first argument of a bar recursor, i.e., a function of type (N → τ) → N), is called a stopping function of bar recursion.

(((w>t)>w)>(u>s))>(((u>(t>s))>s)>(u>s)) ;
FFFF FFFF TTTT TTTT(2),
TTTT TTTT TTTT TTTT(14) (6.3.1.1.2)

Theorem 6.3. Closure under the rule of bar induction
For any type σ and a closed term Y: NN→N, there exists a closed term ξ of type

$$(N* \to \sigma) \to (N* \to (N \to \sigma) \to \sigma) \to N* \to \sigma \qquad (6.3.1.2.1)$$

((x>s)>((x>(w>s))>s))>(x>s) ;
TTTT TTTT TTTT TTTT(16),
FFFF FFFF TTTT TTTT(16) (6.3.1.2.2)

Eqs. 6.3.1.1.2 and 6.3.1.2.2 are *not* tautologous, hence refuting the mapping approach.