This occurs because \( J_n \) is divisible by all primes in the interval \([P_5, P_n]\).

All the \((6j-1, 6j+1)\) pairs with no factor less than \(P_{n+1}\) in which \(6j < P_{n+1}^2\) are twin primes.

### Determining the number of twin primes pairs in the closed interval \([P_n, P_n^2]\).

Let \(X\) be the number of \((6j-1, 6j+1)\) pairs in the interval \([P_n, P_n^2]\). The number of twin prime pairs in \([P_n, P_n^2]\) is \(((P_n - a_m)/P_m)((P_{n-1} - a_{m-1})/P_{m-1})(P_{n-2} - a_{m-2})/P_{m-2})\ldots((5 - a_3)/5)(X)\) for \(n \geq m \geq 3\), \(P_5=5, 1.7 < a_m < 2.3\). The \(P_m-2\) for \(n \geq m \geq 3\) used in calculating the number of \((6j-1, 6j+1)\) pairs in \([I, J_m+1]\) with no factor less than \(P_{n+1}\) sets a range for the values of \(a_m\) at \(2 < a_m < 2.3\) as explained under Graph 1 and Graph 2.

### Graph 1

\((P_m\) in descending order\) plots of \(a_m\) values for 44 equally spaced \(P_m\) in the intervals \([5, 743], [5, 3011], [5, 10007]\) and \([5, 19993]\) illustrate this formula for \([743, 743^2], [3011, 3011^2], [10007, 10007^2]\) and \([19993, 19993^2]\). For selected \(P_m\) they show the value of \(a_m (P_m - a_m)\). As explained under Graph 2, similar \(a_m\) patterns to the four plots in graph 1 are found in \([P_n, P_n^2]\) intervals for \((P_n > 500)\).

### Graph 2

\((P_m\) in descending order\) The \(b_m\) values used in graph 2 are for the same \(P_m, n-1 \geq m \geq 3\) values used in graph 1. There are \((P_n^2 - P_n)/6+1=X (6j-1, 6j+1)\) pairs. \(P_m\) divides the \(6j-1 \) and \(6j+1\) at regular intervals a total of the largest integer divisor \(S\), or \(S+1\) of \(X/P_{m}\) times. For \(P_t, m \leq t \leq n\) Largest integer divisor \(Y\) or \(Y+1\) of \(S (S+1)/P_t\) (of the \(6j-1 \) and \(6j+1\) that are divisible by \(P_m\)) are divisible at regular intervals by \(P_t\). Not all divisions are unique because the same \(6j-1 \) and \(6j+1\) may be divisible by more than one \(P_t, m \leq t \leq n\). Because all these divisions are linear, similar \(b_m\) patterns to the four plots in graph 2 are found in \([P_n, P_n^2]\) intervals for \((P_n > 500)\). The values of \(b_m\) determine the values of \(a_m (P_m - a_m)\), which explains why similar \(a_m\) patterns to the four plots in graph 1 are found in \([P_n, P_n^2]\) intervals for \((P_n > 500)\).

### Table 1

| \(|P_n, P_n^2|\) for \(347 \leq P_n \leq 31153\), when \(a_m\) equals 2.04, 2.06, and 2.08 for \(3 \leq m \leq n\). Comparing TPC\(_n\) with TPA\(_n\) shows the average value for \(P_m - a_m\) starts out near \(P_m - 2.02\) for \(P_n = 347\) and decreases to slightly less than \(P_m - 2.06\) for \(P_n = 31153\). The \(a_m\) values increase as \(P_n\) gets larger \(1\), making the \(P_m - a_m\) averages less.

For \(TPA_n, P_n > 30000\), the average \(a_m\) values cycle around 2.0606 \((Graph 3)\). This happens because similar \(a_m\) patterns to the four plots in graph 1 are found in \([P_n, P_n^2]\) intervals for \((P_n > 500)\). This puts a cap \(< 2.07\) on the average value of \(a_m\) for all \(P_n\).

\(X\) = the \((6j-1, 6j+1)\) pairs in \([P_n, P_n^2]\). For \(n \geq 3, (3/5)(5/7)(7/9)...(P_n-2)/P_n)(X) = 3X/P_n < TPA_{n}.

\(((P_n - 4)/P_n - 2))((P_{n-1} - 4)/P_{n-1}) \ldots((P_{m} - 4)/P_{m}) = (P_m - 4)/P_m <\)

\((P_m - 2.3)/P_m < (P_m - a_m)/P_m, or for twin primes\)

\(((P_m - 6)/(P_m - 4))/P_m = (P_m - 6)/P_m <\)

\(((P_m - 4.3)/(P_m - 2))\)

Thus, for \(n \geq 3, (3/5)(5/7)(7/9)...(P_n-2)/P_n)(X) = 3X/P_n < TPA_{n}.

### Section 2

Establishing a lower bound for the ratio \(TPA_n / TPA_{n-1}\)

Let \(X\) be the number of \((6j-1, 6j+1)\) pairs in the interval \([P_n, P_n^2]\).

The number of twin primes in the interval \([P_n, P_n^2]\) is \((a_3/5)(a_4/7)(a_5/11)...(a_n/P_n)(X)\).

The average value of \(a_m, 3 \leq m \leq n\) can be approximated by \(P_m - 2\).

\(X\) equals \((P_n^2 - P_n)/6+1. m =3 to n \prod P_m - 2 = F_n m =1 to n \prod P_m = J_n\)

The number of twin prime pairs in \([P_n, P_n^2]\) is approximately \((F_n)(P_n^2)/J_n\)

\(TPA_n\) is approximately \(((TPA_{n-1})(F_n)(P_n^2)/J_n)/((TPA_{n-1})(F_{n-1})(P_{n-1})^2/J_{n-1})\).