Can the Twin Prime Conjecture Be Proven
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Abstract: Let P_n be the n_th prime. For twin primes $P_n - P_{n-1} = 2$. Let X be the number of $(6j-1, 6j+1)$ pairs in the closed interval $[P_n, P_n^2]$. The number of twin primes (TPA_n) in $[P_n, P_n^2]$ is

$$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X).\ P_3=5,\ 1.7 < a_m < 2.3$$

For $n \geq 3$, we establish a lower bound for TPA_n $\quad (3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

We exhibit a formula showing as P_n increases, the number of twin primes in the interval $[P_n, P_n^2]$ also increases.

Let $P_n - P_{n-1} = c$. For $n \geq 4$, $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

Introduction:

All primes greater than or equal to five are of the form $6j-1$ or $6j+1$.

$m=1$ to n $\prod P_m = J_n$ is the product of the first n primes. For $n \geq 3$, the number of $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in the closed interval $[1, J_n+1]$ is exactly $(1/6)(3/5)(5/7)\ldots((P_n-2)/P_n)(J_n)$.

Closely related to this is (TPA_n) the number of $(6j-1, 6j+1)$ pairs in $[P_n, P_n^2]$ with no factor less than P_{n+1}. They are all twin primes.

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. X_m is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with no factor in the interval $[P_m, P_n]$. $n \geq m \geq 3$. $(X_{m+1})(P_m - a_m)/P_m = X_m$

(TPA_n) the number of twin primes in $[P_n, P_n^2]$ can be calculated (TPC_n) by the formula

$$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X).\ P_3=5,\ 1.7 < a_m < 2.3$$

Graph 1 shows values of a_m $(P_m - a_m)$ for $[743, 743^2]$, $[3011, 3011^2]$, $[10007, 10007^2]$, $[19993, 19993^2]$

X_{pm} is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with a factor of P_m.
X_{pmj} is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with a factor of P_m that also have no factor in the interval $[P_j, P_n]$. $n \geq j \geq m$. $(X_{pmj+1})(P_j - b_j)/P_j = X_{pmj}$

$$X_m = ((P_n - b_n)/P_n)((P_{n-1} - b_{n-1})/P_{n-1})((P_{n-2} - b_{n-2})/P_{n-2})\ldots((P_m - b_m)/P_m)(X_{pm})$$

H_{pmj} is $X_{pmj+1} - X_{pmj}$. For all the b_j in $(P_j - b_j)$ $n \geq j \geq m$, the weighted average b_{wm} is the sum of all the $(H_{pmj})(b_j)$ $n \geq j \geq m$ (H_{pmj} multiplied by b_j) divided by the sum of all the H_{pmj}, $n \geq j \geq m$.

Graph 2 shows the value of b_{wm} associated with each a_m of $(P_m - a_m)$.

Table 1 Let $P_n - P_{n-1} = c$. For $n \geq 4$, $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

Table 2 shows the actual number of twin primes (TPA_n) versus the calculated number (TPC_n) in $[P_n, P_n^2]$ for $347 \leq P_n \leq 31153$.

Graph 3 shows how the average a_m values cycle around 2.0606 for $29917 \leq P_n \leq 31337$,

X is the number of $(6j-1, 6j+1)$ pairs in $[P_n, P_n^2]$. For $n \geq 3$, $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n < TPA_n$.

Section 1 $m=1$ to n $\prod P_m = J_n$ is the product of the first n primes.

Calculating the number of $(6j-1, 6j+1)$ pairs (F_n) with no factor $< P_{n+1}$ in $[1, J_n+1]$.

For each $(6j-1, 6j+1)$ pair with no factor less than P_n in $[1, J_{n-1}+1]$ there are pairs $(6j-1+mJ_{n-1}, 6j+1+mJ_{n-1})$ for $m = 0$ to $P_n - 1$ in $[1, J_n+1]$. P_n and J_{n-1} are relatively prime. Thus, P_n divides $6j-1+mJ_{n-1}$ and $6j+1+mJ_{n-1}$ each for exactly one different value of m.

$P_3=5$, $P_4=7$. $\underline{F_3 = (1/6)(3/5)(J_3)}$. $F_4 = (5)(F_3)$. $J_4 = (7)(J_3)$ $F_4 / F_3 = (5/7)(J_4 / J_3)$. $\underline{F_4 = (1/6)(3/5)(5/7)(J_4)}$.

The number of $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in the interval $[1, J_n+1]$ for $n \geq 3$, is exactly

$$(1/6)(3/5)(5/7)\ldots((P_n-2)/P_n)(J_n).$$