

# Can the Twin Prime Conjecture be proven

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**Abstract:** Let  $P_n$  be the  $n$ <sub>th</sub> prime. For twin primes  $P_n - P_{n-1} = 2$ . Let  $X$  be the number of  $(6j-1, 6j+1)$  pairs in the interval  $[P_n, P_n^2]$ . The number of twin primes ( $TPA_n$ ) in  $[P_n, P_n^2]$  can be approximated by the formula

$$(a_3/5)(a_4/7)(a_5/11)\dots(a_n/P_n)(X) \text{ for } 3 \leq m \leq n, a_m = P_m - 2.$$

We establish a lower bound for  $TPA_n$   $(3/5)(5/7)(7/9)\dots(P_{n-2}/P_n)(X) = 3X/P_n < TPA_n$ .

We exhibit a formula showing as  $P_n$  increases, the number of twin primes in the interval  $[P_n, P_n^2]$  also increases.

$$\text{Let } P_n - P_{n-1} = c. \text{ For all } n (TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$$

## Introduction: Blue items in Section 1 must be proven.

All primes greater than or equal to five are of the form  $6j-1$  or  $6j+1$ .

$m=1$  to  $n$   $\prod P_m = J_n$  is the product of the first  $n$  primes. The number of  $(6j-1, 6j+1)$  pairs with no factor less than  $P_{n+1}$  in the closed interval  $[1, J_n+1]$  is exactly  $(1/6)(3/5)(5/7)\dots((P_n-2)/P_n)(J_n)$ .

Closely related to this is  $(TPA_n)$  the number of  $(6j-1, 6j+1)$  pairs in  $[P_n, P_n^2]$  with no factor less than  $P_{n+1}$ . They are all twin primes. Let  $X$  be the number of  $(6j-1, 6j+1)$  pairs in the interval  $[P_n, P_n^2]$ .

$(TPA_n)$  the number of twin primes in  $[P_n, P_n^2]$  can be approximated  $(TPC_n)$  by the formula

$$(a_3/5)(a_4/7)(a_5/11)\dots(a_n/P_n)(X) \text{ for } 3 \leq m \leq n, a_m = P_m - 2.$$

**Table 1/2/3** show  $a_m$  for  $3 \leq m \leq n$ , in  $[743, 743^2]$ ,  $[19993, 19993^2]$ ,  $[3011, 3011^2]$ ,  $[19993, 19993^2]$

**Table 4** Let  $P_n - P_{n-1} = c$ . For all  $n (TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

**Table 5,6** show the actual number of twin primes ( $TPA_n$ ) versus the calculated number ( $TPC_n$ ) in  $[P_n, P_n^2]$  for  $347 \leq P_n \leq 31153$  and for  $30013 \leq P_n \leq 31337$ , where the average  $a_m$  values cycle between

$P_m - 2.06$  and  $P_m - 2.07$ .  $X$  equals the number of  $(6j-1, 6j+1)$  pairs in the interval  $[P_n, P_n^2]$ . We show that

$$(3/5)(5/7)(7/9)\dots(P_{n-2}/P_n)(X) = 3X/P_n < TPA_n.$$

## Section 1

### Calculating the number of $(6j-1, 6j+1)$ pairs ( $F_n$ ) with no factor $< P_{n+1}$ in $[1, J_n+1]$ .

For each  $(6j-1, 6j+1)$  pair with no factor less than  $P_n$  in  $[1, J_{n-1}+1]$  there are pairs

$(6j-1+mJ_{n-1}, 6j+1+mJ_{n-1})$  for  $m = 0$  to  $P_n-1$  in  $[1, J_n+1]$ .  $P_n$  and  $J_{n-1}$  are relatively prime.

Thus,  $P_n$  divides  $6j-1+mJ_{n-1}$  and  $6j+1+mJ_{n-1}$  each for exactly one different value of  $m$ .

$$P_3=5, P_4=7. \underline{F_3 = (1/6)(3/5)(J_3)}. F_4 = (5)(F_3). J_4 = (7)(J_3) F_4 / F_3 = (5/7)(J_4 / J_3). \underline{F_4 = (1/6)(3/5)(5/7)(J_4)}.$$

The number of  $(6j-1, 6j+1)$  pairs with no factor less than  $P_{n+1}$  in the interval  $[1, J_n+1]$  is exactly

$$(1/6)(3/5)(5/7)\dots((P_n-2)/P_n)(J_n).$$

This occurs because  $J_n$  is divisible by all primes in the interval  $[P_3, P_n]$ .

All the  $(6j-1, 6j+1)$  pairs with no factor less than  $P_{n+1}$  in which  $6j < P_{n+1}^2$  are twin primes.

### Determining the number of twin primes pairs in the closed interval $[P_n, P_n^2]$ .

Let  $X$  be the number of  $(6j-1, 6j+1)$  pairs in the interval  $[P_n, P_n^2]$ . The number of twin prime pairs in  $[P_n, P_n^2]$  is

$(a_3/5)(a_4/7)(a_5/11)\dots(a_n/P_n)(X)$ . For  $3 \leq m \leq n$   $P_m - 2.5 < a_m \leq P_m - 1.5$ . The absolute value of  $P_m - 2$  for the number of pairs in  $[1, J_m+1]$  with no factor less than  $P_{m+1}$  sets a range for the possible values of  $a_m$  at

$$P_m - (2 + .5) < a_m \leq P_m - (2 - .5)$$

**Table 1/2** ( $P_m$  primes in descending order) illustrate this formula for the intervals (Table 1)  $[743, 743^2]$  and  $[19993, 19993^2]$  (Table 2)  $[3011, 3011^2]$  and  $[10007, 10007^2]$ . For selected  $P_m$  they show the actual number of  $(6j-1, 6j+1)$  pairs with no factor less than  $P_{m+1}$  and the value of  $a$  in  $a_m (P_m - a)$ . Filtering out factors less than  $P_{m+1}$  is a linear process. Each  $P_m$  divides  $6j-1$  and  $6j+1$  terms at regular  $P_m$ -th intervals within  $[P_n, P_n^2]$ . **Similar  $a (P_m - a)$  patterns are found in all sufficiently large  $[P_n, P_n^2]$  intervals ( $P_n > 500$ ).** See the **table 3** graph.

**Table 5** shows the number of twin primes calculated ( $TPC_n$ ) in  $[P_n, P_n^2]$  for  $347 \leq P_n \leq 31153$ , when  $a_m$  equals  $P_m - 2.04$ ,  $P_m - 2.06$ , and  $P_m - 2.08$  for  $3 \leq m \leq n$ . Comparing  $TPC_n$  with  $TPA_n$  shows the average value for  $a_m = P_m - a$  starts out near  $P_m - 2.02$  for  $P_n = 347$  and decreases to slightly less than  $P_m - 2.06$  for  $P_n = 31153$  (see  $TPA_n / TPC_n 2.06$ ).

**Table 6** shows that as  $TPA_n$  gets larger ( $P_n > 31000$ ), the average  $a_m$  values cycle between  $P_m - 2.06$  and  $P_m - 2.07$ .  $X$  equals the number of  $(6j - 1, 6j + 1)$  pairs in the interval  $[P_n, P_n^2]$ .

$$\begin{aligned} & ((P_m - 2)/P_m)(P_m - 2)/(P_m - 4) = (P_m - 4)/P_m < \\ & (P_m - 2.08)/P_m < a_m/P_m, \text{ or for twin primes} \\ & ((P_m - 4)/P_m)(P_m - 4)/(P_m - 6) = (P_m - 6)/P_m < \\ & ((P_m - 4.08)(P_m - 2.08))/((P_m - 2)(P_m)) < \\ & ((a_{m-1})(a_m))/((P_{m-1})(P_m)). \end{aligned}$$

$$\text{Thus, } (3/5)(5/7)(7/9)\dots(P_n - 2)/P_n(X) = 3X/P_n < TPA_n.$$

## Section 2

### Establishing a lower bound for the ratio $TPA_n / TPA_{n-1}$

The number of twin primes in the interval  $[P_n, P_n^2]$  is  $(a_3/5)(a_4/7)(a_5/11)\dots(a_n/P_n)(X)$ .

The average value of  $a_m$ ,  $3 \leq m \leq n$  can be approximated by  $P_m - 2$ .

$$X \text{ is close to } (P_n^2 - P_n)/6. \quad m = 3 \text{ to } n \quad \square \quad P_m - 2 = F_n$$

The number of twin prime pairs in  $[P_n, P_n^2]$  is approximately  $(F_n)(P_n^2) / J_n$

$$TPA_n \text{ is approximately } (TPA_{n-1})((F_n)(P_n^2) / J_n) / ((F_{n-1})(P_{n-1})^2 / J_{n-1}).$$

$$TPA_n \text{ is greater than } (TPA_{n-1})\left(\left(\frac{(F_n)(P_n^2) / J_n}{(F_{n-1})(P_{n-1})^2 / J_{n-1}}\right) + 1\right) / 2.$$

$$\text{Calculating } \left(\left(\frac{(F_n)(P_n^2) / J_n}{(F_{n-1})(P_{n-1})^2 / J_{n-1}}\right) + 1\right) / 2.$$

$$\text{Let } P_n - P_{n-1} = c.$$

$$\left(\frac{(F_n)(P_n^2) / J_n}{(F_{n-1})(P_{n-1})^2 / J_{n-1}}\right) =$$

$$\left(\frac{(F_{n-1})(P_{n-1} + c - 2)(P_{n-1} + c)^2 / ((J_{n-1})(P_{n-1} + c))}{(F_{n-1})(P_{n-1})^2 / J_{n-1}}\right) =$$

$$\frac{(P_{n-1} + c - 2)(P_{n-1} + c)}{P_{n-1}^2} =$$

$$1 + (2c - 2)/P_{n-1} + (c^2 - 2c)/P_{n-1}^2 \quad \text{Table 4 (column D) / (column C)}$$

$$\text{For all } n, (TPA_{n-1})(1 + (2c - 2)/2P_{n-1} + (c^2 - 2c)/2P_{n-1}^2) < TPA_n$$

$$\text{Table 4 (column B)}((\text{column D}/\text{column C}) + 1)/2 = (\text{column F})$$

$$\text{For all } n, TPA_{n-1} < TPA_n$$

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Table 1 - Twin Primes in the interval  $[743, 743^2]$  and  $[19993, 19993^2]$

$X_m$  start is the number of  $(6j-1, 6j+1)$  pairs starting with no factor less than  $P_m$

$X_{m+1}$  remain is the number of  $(6j-1, 6j+1)$  pairs remaining with no factor less than  $P_{m+1}$

$P_m$  in descending order,  $a_m = P_m - a$  ( $X_m$  start) $(P_m - a) / P_m = X_{m+1}$  remain

$743 P_m$	$X_m$ start	$X_{m+1}$ remain	$a$	$19993 P_m$	$X_m$ start	$X_{m+1}$ remain	$a$
743	91885	91638	1.9973	19993	66616677	66610013	2.0000
727	91143	90894	1.9861	19991	66610013	66603350	1.9997
701	90393	90138	1.9775	19973	66596685	66590019	1.9992
677	89622	89365	1.9414	19031	65947444	65940581	1.9805
659	88842	88580	1.9434	18401	65571948	65564917	1.9731
643	88047	87784	1.9207	17921	65189084	65181934	1.9656
619	87258	86985	1.9366	17389	64799032	64791740	1.9568
607	86441	86170	1.9030	16879	64400779	64393329	1.9526
593	85623	85353	1.8699	16301	63993105	63985438	1.9530
571	84788	84501	1.9328	15761	63574585	63566751	1.9422
557	83936	83650	1.8979	15277	63146223	63138201	1.9408
521	82762	82467	1.8571	14767	62707292	62699034	1.9447
503	82158	81847	1.9041	14387	62333157	62324784	1.9325
487	81220	80904	1.8948	13711	61792599	61783841	1.9433
463	80253	79918	1.9327	13177	61313215	61304193	1.9389
449	79255	78916	1.9205	12671	60818186	60808852	1.9447
433	78225	77883	1.8931	12211	60307200	60297613	1.9412
419	77179	76820	1.9490	11717	59778647	59768689	1.9518
397	76088	75716	1.9410	11161	59229208	59218806	1.9601
379	74952	74565	1.9569	10639	58656237	58645381	1.9690
359	73777	73374	1.9610	10139	58057571	58046251	1.9769
347	72535	72124	1.9662	9631	57432279	57420422	1.9883
331	71702	71263	2.0266	8941	56478138	56465440	2.0102
307	69908	69463	1.9542	8689	56093932	56080992	2.0044
281	68505	68020	1.9894	8191	55379685	55366129	2.0050
269	67037	66543	1.9823	7681	54628936	54614676	2.0050
251	65524	65007	1.9805	7207	53840827	53825860	2.0034
233	63937	63390	1.9934	6709	53010359	52994591	1.9956
223	62288	61741	1.9583	6247	52133798	52117159	1.9938
197	60572	59964	1.9774	5779	51205434	51187767	1.9939
181	58745	58118	1.9319	5309	50221808	50203021	1.9860
167	56828	56180	1.9043	4831	49172869	49152739	1.9777
151	54837	54144	1.9083	4363	48047958	48026305	1.9662
137	52729	51994	1.9097	3917	46834227	46810875	1.9531
113	50452	49640	1.8187	3491	45515840	45490403	1.9510
103	47953	47104	1.8236	3023	44072710	44044443	1.9389
89	45414	44496	1.7990	2617	42461592	42430302	1.9285
73	42553	41540	1.7378	2161	40647352	40611278	1.9179
61	39449	38330	1.7303	1747	38565951	38524087	1.8964
47	35938	34594	1.7577	1361	36117063	36067505	1.8675
37	31722	30176	1.8032	971	33121093	33059106	1.8173
23	26522	24294	1.9321	397	26146005	26033510	1.7099
7	12793	8783	2.1942	17	6731087	5832579	2.2718
5	8783	4934	2.1912	5	2632598	1506428	2.1389

Table 2 - Twin Primes in the interval  $[3011, 3011^2]$  and  $[10007, 10007^2]$

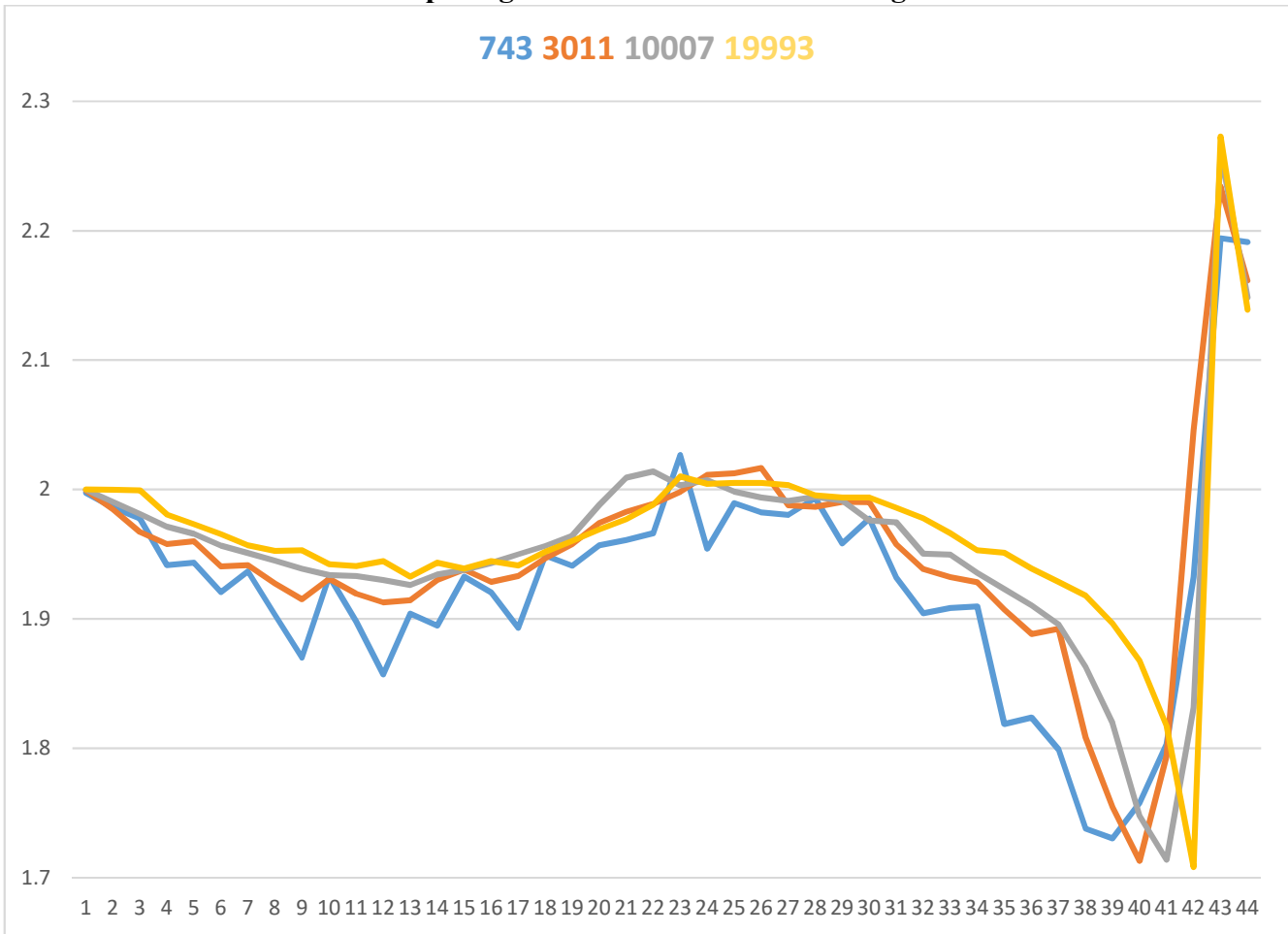
$X_m$  start is the number of  $(6j-1, 6j+1)$  pairs starting with no factor less than  $P_m$

$X_{m+1}$  remain is the number of  $(6j-1, 6j+1)$  pairs remaining with no factor less than  $P_{m+1}$

$P_m$  in decending order,  $a_m = P_m - a$   $(X_m \text{ start})(P_m - a) / P_m = X_{m+1} \text{ remain}$

$3011 P_m$	$X_m$ start	$X_{m+1}$ remain	$a$	$10007 P_m$	$X_m$ start	$X_{m+1}$ remain	$a$
3011	1510519	1509516	1.9993	10007	16688341	16685006	1.9998
2909	1499387	1498364	1.9847	9769	16604272	16600889	1.9904
2833	1489085	1488051	1.9672	9497	16508869	16505425	1.9812
2749	1478618	1477565	1.9577	9281	16411971	16408485	1.9713
2689	1468007	1466937	1.9600	9013	16313525	16309967	1.9657
2621	1457316	1456237	1.9406	8761	16213235	16209614	1.9566
2539	1446392	1445286	1.9415	8539	16111222	16107541	1.9509
2437	1435167	1434032	1.9273	8269	16007030	16003265	1.9449
2371	1423671	1422521	1.9152	8011	15900547	15896699	1.9387
2293	1411994	1410805	1.9309	7727	15791394	15787442	1.9338
2221	1400037	1398827	1.9195	7523	15679498	15675469	1.9331
2111	1383926	1382672	1.9128	7237	15565050	15560899	1.9300
2063	1375038	1373762	1.9144	7019	15472535	15468289	1.9262
1993	1361991	1360672	1.9301	6737	15325588	15321188	1.9342
1901	1348588	1347213	1.9382	6481	15200767	15196222	1.9378
1823	1334725	1333313	1.9285	6011	14938546	14933717	1.9431
1741	1320281	1318815	1.9332	5749	14800529	14795509	1.9499
1663	1305307	1303779	1.9467	5519	14657855	14652659	1.9564
1597	1289701	1288120	1.9577	5281	14509943	14504546	1.9643
1523	1273645	1271994	1.9742	4793	14194722	14188834	1.9881
1453	1256836	1255121	1.9827	4549	14026415	14020220	2.0091
1381	1239496	1237711	1.9888	4363	13894987	13888573	2.0140
1297	1221070	1219189	1.9980	4073	13666055	13659334	2.0031
1229	1201942	1199975	2.0113	3847	13473631	13466601	2.0072
1153	1182002	1179939	2.0124	3613	13271990	13264649	1.9984
1093	1165263	1163113	2.0167	3389	13061960	13054276	1.9937
1019	1139139	1136917	1.9877	3169	12841640	12833571	1.9912
947	1116381	1114039	1.9867	2909	12607781	12599138	1.9942
877	1092296	1089817	1.9904	2699	12359352	12350233	1.9914
811	1066879	1064261	1.9901	2467	12098595	12088904	1.9761
739	1040001	1037246	1.9576	2269	11819485	11809199	1.9746
661	1011475	1008509	1.9383	2039	11522272	11511251	1.9503
607	981145	978022	1.9321	1823	11202910	11190929	1.9496
547	949136	945790	1.9283	1607	10852041	10838970	1.9356
467	914065	910332	1.9072	1427	10471073	10456962	1.9231
419	875871	871924	1.8882	1201	10048332	10032350	1.9102
353	834022	829551	1.8924	1009	9570992	9553010	1.8957
283	787369	782338	1.8083	811	9030446	9009704	1.8628
179	674116	667509	1.7544	617	8395694	8370936	1.8195
131	609444	601475	1.7129	439	7632758	7602369	1.7478
73	503066	490702	1.7941	307	6886094	6847652	1.7138
31	346963	324065	2.0459	109	5064667	4979565	1.8315
11	174552	139096	2.2344	17	1977781	1714720	2.2611
5	95299	54102	2.1615	5	772670	440666	2.1484

Table 3 - Comparing  $a$  values for  $P_m$  in descending order



**1-743 1.9973 12-521 1.8571 23-331 2.0266 39-61 1.7303 43-7 2.1942 44-5 2.1912**

<b>1.9973</b>	1.9861	1.9775	1.9414	1.9434	1.9207	1.9366	1.9030	1.8699	1.9328	1.8979
<b>1.8571</b>	1.9041	1.8948	1.9327	1.9205	1.8931	1.9490	1.9410	1.9569	1.9610	1.9662
<b>2.0266</b>	1.9542	1.9894	1.9823	1.9804	1.9934	1.9583	1.9774	1.9319	1.9043	1.9083
1.9097	1.8187	1.8236	1.7990	1.7378	<b>1.7303</b>	1.7577	1.8032	1.9321	<b>2.1942</b>	<b>2.1912</b>

**1-3011 1.9993 12-2111 1.9128 26-1093 2.0167 40-131 1.7129 43-11 2.2344 44-5 2.1615**

<b>1.9993</b>	1.9847	1.9672	1.9577	1.9600	1.9406	1.9415	1.9273	1.9152	1.9309	1.9195
<b>1.9128</b>	1.9144	1.9301	1.9382	1.9285	1.9332	1.9467	1.9577	1.9742	1.9827	1.9888
1.9980	2.0113	2.0124	<b>2.0167</b>	1.9877	1.9867	1.9904	1.9901	1.9576	1.9383	1.9321
1.9283	1.9072	1.8882	1.8924	1.8083	1.7544	<b>1.7129</b>	1.7941	2.0459	<b>2.2344</b>	<b>2.1615</b>

**1-10007 1.9998 12-7019 1.9262 22-4363 2.0140 41-307 1.7138 43-17 2.2611 44-5 2.1484**

<b>1.9998</b>	1.9904	1.9812	1.9713	1.9657	1.9566	1.9509	1.9449	1.9387	1.9338	1.9331
1.9300	<b>1.9262</b>	1.9342	1.9378	1.9431	1.9499	1.9564	1.9643	1.9881	2.0091	<b>2.0140</b>
2.0031	2.0072	1.9984	1.9937	1.9912	1.9942	1.9914	1.9761	1.9746	1.9503	1.9496
1.9356	1.9231	1.9102	1.8957	1.8628	1.8195	1.7478	<b>1.7138</b>	1.8315	<b>2.2611</b>	<b>2.1484</b>

**1-19993 2.0000 13-14387 1.9326 23-8941 2.0102 42-397 1.7081 43-17 2.2728 44-5 2.1389**

<b>2.0000</b>	1.9997	1.9992	1.9805	1.9731	1.9656	1.9568	1.9526	1.9530	1.9422	1.9408
1.9447	<b>1.9326</b>	1.9433	1.9389	1.9447	1.9412	1.9518	1.9601	1.9690	1.9769	1.9883
<b>2.0102</b>	2.0044	2.0050	2.0050	2.0034	1.9956	1.9938	1.9939	1.9860	1.9777	1.9662
1.9531	1.9510	1.9389	1.9285	1.9179	1.8964	1.8675	1.8173	<b>1.7081</b>	<b>2.2728</b>	<b>2.1389</b>

Table 4

A	B	C	D	E	F	G	H	I
<i>prime</i>	$TPA_{n-1}$	$(F_{n-1})(P_{n-1})^2/J_{n-1}$	$(F_n)(P_n)^2/J_n$	$(D/C+I)/2$	$(B)(E)$	$TPA_n$	$F/G$	$B/G$
71	120	109.0	112.1	1.01408	121.7	123	0.989483	0.97561
73	123	112.1	127.9	1.07047	131.7	138	0.954117	0.89130
1019	8420	8935.3	8952.8	1.00098	8428.2	8450	0.997425	0.99645
1021	8450	8952.8	9111.3	1.00885	8524.8	8586	0.992872	0.98416
2087	28819	30850.0	30879.6	1.00048	28832.8	28867	0.998816	0.99834
2089	28867	30879.6	31146.2	1.00432	28991.6	29106	0.996070	0.99179
3461	68804	74874.0	74917.3	1.00029	68823.9	68872	0.999302	0.99901
3463	68872	74917.3	75047.1	1.00087	68931.7	69019	0.998735	0.99787
4637	114316	125244.7	125298.7	1.00022	114340.6	114394	0.999534	0.99932
4639	114394	125298.7	125460.9	1.00065	114468.0	114580	0.999023	0.99838
6299	195208	215150.4	215218.7	1.00016	195239.0	195319	0.999590	0.99943
6301	195319	215218.7	215833.9	1.00143	195598.2	195879	0.998566	0.99714
8009	297317	329810.8	329893.1	1.00012	297354.1	297454	0.999664	0.99954
8011	297454	329893.1	330305.0	1.00062	297639.7	297851	0.999291	0.99867
9857	428957	476792.2	476889.0	1.00010	429000.5	429089	0.999794	0.99969
9859	429089	476889.0	477953.7	1.00112	429568.0	430004	0.998986	0.99787
11777	588001	656535.4	656646.9	1.00008	588050.9	588163	0.999809	0.99972
11779	588163	656646.9	656981.4	1.00025	588312.8	588502	0.999679	0.99942
13931	791507	885279.3	885406.4	1.00007	791563.8	791704	0.999823	0.99975
13933	791704	885406.4	889096.0	1.00208	793353.6	794778	0.998208	0.99613
16187	1033547	1158651.2	1158794.4	1.00006	1033610.9	1033796	0.999821	0.99976
16189	1033796	1158794.4	1159223.9	1.00019	1033987.6	1034307	0.999691	0.99951
18041	1254327	1408473.1	1408629.2	1.00006	1254396.5	1254586	0.999849	0.99979
18043	1254586	1408629.2	1409097.7	1.00017	1254794.6	1255094	0.999761	0.99960
20147	1527206	1717720.9	1717891.4	1.00005	1527281.8	1527479	0.999871	0.99982
20149	1527479	1717891.4	1719767.6	1.00055	1528313.1	1529106	0.999481	0.99894
21839	1763993	1985940.5	1986122.3	1.00005	1764073.7	1764289	0.999878	0.99983
21841	1764289	1986122.3	1987759.5	1.00041	1765016.2	1765719	0.999602	0.99919
23741	2047968	2308071.0	2308265.5	1.00004	2048054.3	2048281	0.999889	0.99985
23743	2048281	2308265.5	2308848.8	1.00013	2048539.8	2048899	0.999825	0.99970
26861	2555034	2883638.7	2883853.4	1.00004	2555129.1	2555371	0.999905	0.99987
26863	2555371	2883853.4	2887074.9	1.00056	2556798.3	2558027	0.999520	0.99896
28619	2861908	3233814.5	3234040.4	1.00003	2862008.0	2862279	0.999905	0.99987
28621	2862279	3234040.4	3235170.5	1.00017	2862779.1	2863372	0.999793	0.99962
31319	3365123	3806114.0	3806357.0	1.00003	3365230.4	3365489	0.999923	0.99989
31321	3365489	3806357.0	3807572.4	1.00016	3366026.3	3366653	0.999814	0.99965

**Table 5 – Twin Primes in the interval  $[P_n, P_n^2]$  for  $347 \leq P_n \leq 31153$**

$(a_3/5)(a_4/7)(a_5/11)\dots(a_n/P_n)(X)$ . For  $3 \leq m \leq n$

$a_m$  is replaced by  $P_m - 2.04$   $P_m - 2.06$   $P_m - 2.08$  for  $m = 3$  to  $n$ .

$P_n$	$TPA_n$	$TPA_n /$ $TPC_n 2.06$	$TPC_n$ $2.04$	$TPC_n$ $2.06$	$TPC_n$ $2.08$	$3X/P_n$
347	1405	1.0637	1360.2	1320.8	1282.4	173
349	1419	1.0683	1368.0	1328.2	1289.6	174
1151	10387	1.0430	10293.6	9958.4	9633.4	575
1153	10408	1.0434	10311.2	9975.2	9649.5	576
1997	26735	1.0369	26690.2	25783.3	24905.1	998
1999	26777	1.0375	26716.5	25808.3	24929.1	999
2969	52817	1.0302	53125.5	51267.4	49470.3	1484
2971	52877	1.0307	53160.6	51300.9	49502.3	1485
3851	82712	1.0224	83885.3	80901.1	78016.6	1925
3853	82802	1.0230	83928.1	80941.9	78055.5	1926
4649	114842	1.0196	116843	112636	108572	2324
4651	114919	1.0198	116892	112683	108617	2325
5849	171367	1.0156	175132	168737	162561	2924
5851	171471	1.0159	175191	168793	162614	2925
6947	231582	1.0123	237533	228770	220312	3473
6949	231708	1.0126	237600	228834	220373	3474
8387	322646	1.0100	331826	319452	307514	4193
8389	322805	1.0103	331903	319526	307585	4194
9677	415267	1.0091	427606	411530	396024	4838
9679	415417	1.0092	427693	411613	396103	4839
10937	515723	1.0078	531882	511751	492342	5468
10939	515884	1.0079	531977	511842	492428	5469
12251	630469	1.0059	651581	626775	602859	6125
12253	630646	1.0060	651685	626874	602954	6126
13997	798218	1.0048	826113	794435	763902	6998
13999	798427	1.0049	826228	794545	764006	6999
15731	982287	1.0039	1017750	978483	940646	7865
15733	982497	1.0040	1017877	978604	940761	7866
17291	1162662	1.0026	1206394	1159636	1114583	8645
17293	1162911	1.0027	1206531	1159767	1114707	8646
18251	1280482	1.0031	1328116	1276491	1226753	9125
18253	1280728	1.0032	1328259	1276627	1226882	9126
19991	1506151	1.0015	1564964	1503866	1445014	9995
19993	1506427	1.0016	1565117	1504011	1445152	9996
21191	1671686	1.0015	1737182	1669161	1603649	10595
21193	1671950	1.0016	1737343	1669314	1603795	10596
22541	1866304	1.0009	1940784	1864560	1791151	11270
22543	1866615	1.0010	1940953	1864721	1791304	11271
23831	2061886	1.0010	2144230	2059785	1978464	11915
23833	2062203	1.0011	2144407	2059953	1978623	11916
26111	2428375	1.0000	2528479	2428472	2332181	13055
26113	2428739	1.0000	2528668	2428652	2332353	13056
27689	2697588	0.9992	2811333	2699839	2592502	13844
27691	2697935	0.9992	2811532	2700028	2592683	13845
29207	2968309	0.9994	3093224	2970220	2851812	14603
29209	2968674	0.9994	3093432	2970418	2852000	14604
31151	3333028	0.9987	3476151	3337515	3204082	15575

**Table 6 – Twin Primes in the interval  $[P_n, P_n^2]$  for  $30013 \leq P_n \leq 31337$**

$(a_3/5)(a_4/7)(a_5/11)\dots(a_n/P_n)(X)$ . For  $3 \leq m \leq n$

$a_m$  is replaced by  $P_m - 2.04$   $P_m - 2.06$   $P_m - 2.08$  for  $m = 3$  to  $n$ .

$P_n$	$TPA_n$	$TPA_n /$ $TPC_n 2.06$	$TPC_n$ $2.04$	$TPC_n$ $2.06$	$TPC_n$ $2.08$	$3X/P_n$
30013	3117235	0.9989	3250135	3120738	2996185	15006
30059	3125815	0.9988	3259441	3129667	3004753	15029
30089	3131419	0.9987	3265507	3135488	3010337	15044
30103	3134041	0.9988	3267881	3137764	3012514	15051
30119	3137071	0.9989	3270689	3140456	3015091	15059
30187	3149843	0.9990	3283922	3153143	3027259	15093
30253	3162312	0.9989	3296961	3165651	3039257	15126
30319	3174656	0.9990	3309802	3177967	3051063	15159
30367	3183673	0.9989	3319399	3187171	3059893	15183
30403	3190385	0.9988	3326604	3194082	3066524	15201
30449	3199039	0.9987	3336006	3203106	3075180	15224
30491	3206878	0.9986	3344543	3211298	3083037	15245
30529	3214059	0.9987	3351766	3218220	3089670	15264
30649	3237009	0.9986	3375913	3241386	3111894	15324
30707	3247989	0.9987	3387130	3252136	3122198	15353
30817	3269008	0.9986	3409404	3273506	3142697	15408
30853	3275851	0.9987	3416246	3280064	3148981	15426
30941	3292731	0.9988	3433723	3296821	3165051	15470
30983	3300720	0.9987	3442143	3304899	3172798	15491
31063	3315985	0.9986	3458580	3320666	3187921	15531
31151	3333028	0.9987	3476151	3337515	3204082	15575
31189	3340292	0.9988	3483266	3344340	3210620	15594
31231	3348275	0.9987	3491742	3352469	3218415	15615
31249	3351726	0.9988	3495084	3355671	3221481	15624
31271	3355977	0.9989	3499095	3359511	3225158	15635
31321	3365489	0.9988	3509378	3369373	3234622	15660
31337	3368570	0.9989	3512277	3372152	3237284	15668